**High-order Error-Feedback Mismatch Error Shaping for Continuous-time DACs**

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DAC mismatch is the bottleneck for data converters to achieve high resolution. The error-feedback mismatch error shaping (EF-MES) is an effective method to address the DAC mismatch. However, the existing EF-MES can only achieve the 1st-order shaping for continuous-time (CT) DACs. This letter proposes the methodology and implementation details of a 2nd-order EF-MES for CT DACs. This method can also be generalized to arbitrarily high-order EF-MES.

*Introduction:* High-resolution data converters have a wide range of applications. However, device mismatch is the resolution bottleneck. The classic mismatch solution includes calibration and DEM. Calibration relies on the accurate measurement and compensation of DAC mismatch. However, it greatly increases the circuit complexity and testing cost. And, the foreground calibration needs to be repeated as environment changes (e.g., temperature variation). The background calibration can track the mismatch drift, but it requires long convergence time which restricts its application [1]. Another method is to use DEM. The rationale of DEM is not to remove the mismatch error, but rather spectrally shape it in such a way that it does not affect the overall linearity [2, 3]. However, the circuit complexity of DEM increases exponentially with DAC resolution, making it not suitable for high-resolution DACs.

*Previous 1st- & 2nd-order EF-MES:* The work [4] proposed 1st-order EF-MES for CT DACs as shown in Fig. 1*a*. The DAC combines one MSB DAC and two LSB DACs. The previous LSB result, z-1LSBs, is added with the digital input. The addition result sets the MSB DAC and one LSB DAC. To recover the input signal, the inversion of the previous LSB result, −z-1LSBs, is used to set the another LSB DAC. In this manner, the LSB mismatch error in the present cycle, *E*(n), is subtracted from the previous mismatch error, *E*(n−1). Assuming that the MSB DAC is ideal (or addressing MSB mismatch by DEM), the total error is *E*(n−1) −*E*(n), realizing the 1st-order shaping.

1st-order shaping is often insufficient for high-resolution and/or larger DAC mismatch errors. To enhance the mismatch shaping effect, a 2nd-order EF-MES for DT DACs, such as the switched-capacitor DAC, is proposed in [5]. By leveraging the memory capability and operating the DT DACs in sampling and hold phases, every LSB DAC can contributes two mismatch errors. As shown in Fig. 1(b), one LSB DAC generates the mismatch error of −2*E*(n−1) while another LSB DAC generates *E*(n)+*E*(n−2), the total mismatch error becomes −*E*(n)+2*E*(n−1) −*E*(n−2) and the second-order shaping is obtained. However, this method cannot be directly applied to the memory-less CT DACs.

*Proposed high-order EF-MES:* Inspired by [4] and [5], this letter proposes a high-order EF-MES for CT DACs. As illustrated in Fig. 2, the DAC consists of one MSB DAC and four LSB DACs for 2nd-order EF-MES implementation. The 4 LSB DACs receives the digital codes from the present and previous 3 conversion cycles (LSBs, −z-1LSBs, −z-2LSBs and z-3LSBs), respectively. To compensate for the additional −z-1LSBs, −z-2LSBs and z-3LSBs in the analog domain, the (z-1+ z-2 −z-3) LSBs are added to the digital input to recover the DAC output level, VOUT or IOUT. In this manner, the total mismatch error becomes −*E*(n)+*E*(n−1)+*E*(n−2)−*E*(n−3). The mismatch error transfer function (METF) can be expressed as:

METF = 1 − z-1 − z-2 + z-3 = (1 − z-1)2 (1 + z-1) (1)

It can be seen that the METF has two zeroes at DC and one zero at fs/2, which means that it has a 2nd-order high-pass filtering and a 1st-order low-pass filtering effect.



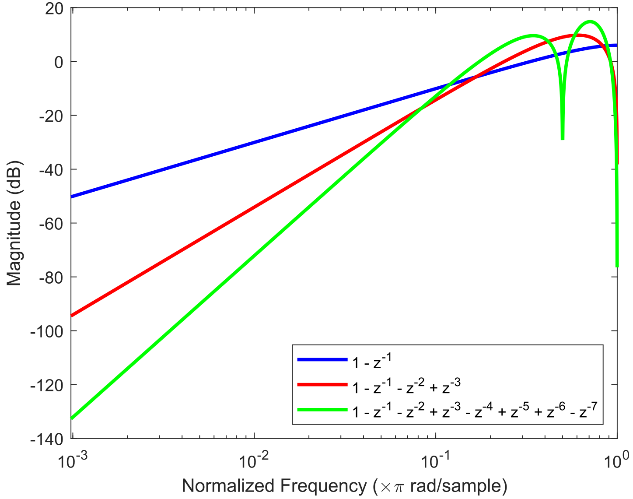
**Fig. 1** *Previous* *EF-MES.*

*a* 1st-order EF-MES for CT-DACs

*b* 2nd-order EF-MES for DT-DACs

**Fig. 2** *Proposed 2nd-order EF-MES for CT DACs.*

The proposed EF-MES method can be generalized to arbitrarily high-order shaping*.* 2N LSB DACs are required to implement the Nth-order EF-MES. Taking the 3rd-order MES as an example, 8 LSB DACs can be used to realize the 1−z-1−z-2+z-3−z-4+z-5+z-6−z-7 3rd-order shaping. Fig. 3 shows the frequency response of METF for 3rd-order EF-MES, and compares it with that of the 1st-order and 2nd-order ones. It can be seen that it has 3 zeros at DC, two zeros at fs/2 and one zero at fs/4.



**Fig. 3***Frequency response of METFs of 1st-, 2nd- & 3rd-order EF-MES.*

*Implementation & simulation results:* The implementation detail of the proposed 2nd-order EF-MES is illustrated in Fig. 4. In the nth cycle, 3 LSBs DACs receive the digital codes from the previous 3 cycles, *DLSB*(n−3), −*DLSB*(n−2) and −*DLSB*(n−1), respectively. The remaining LSB DAC and MSB DAC receive the digital codes from the present cycle, *DLSB*(n) and *DMSB*(n), respectively. *DMSB*(n) and *DLSB*(n) are determined by the following equation:

*DMSB*(n) + *DLSB*(n) = *DIN*(n) + *DLSB* (n − 1)

+ *DLSB* (n − 2) − *DLSB* (n − 3) (2)

**Fig.4** *Operational details of proposed 2nd-order EF-MES.*

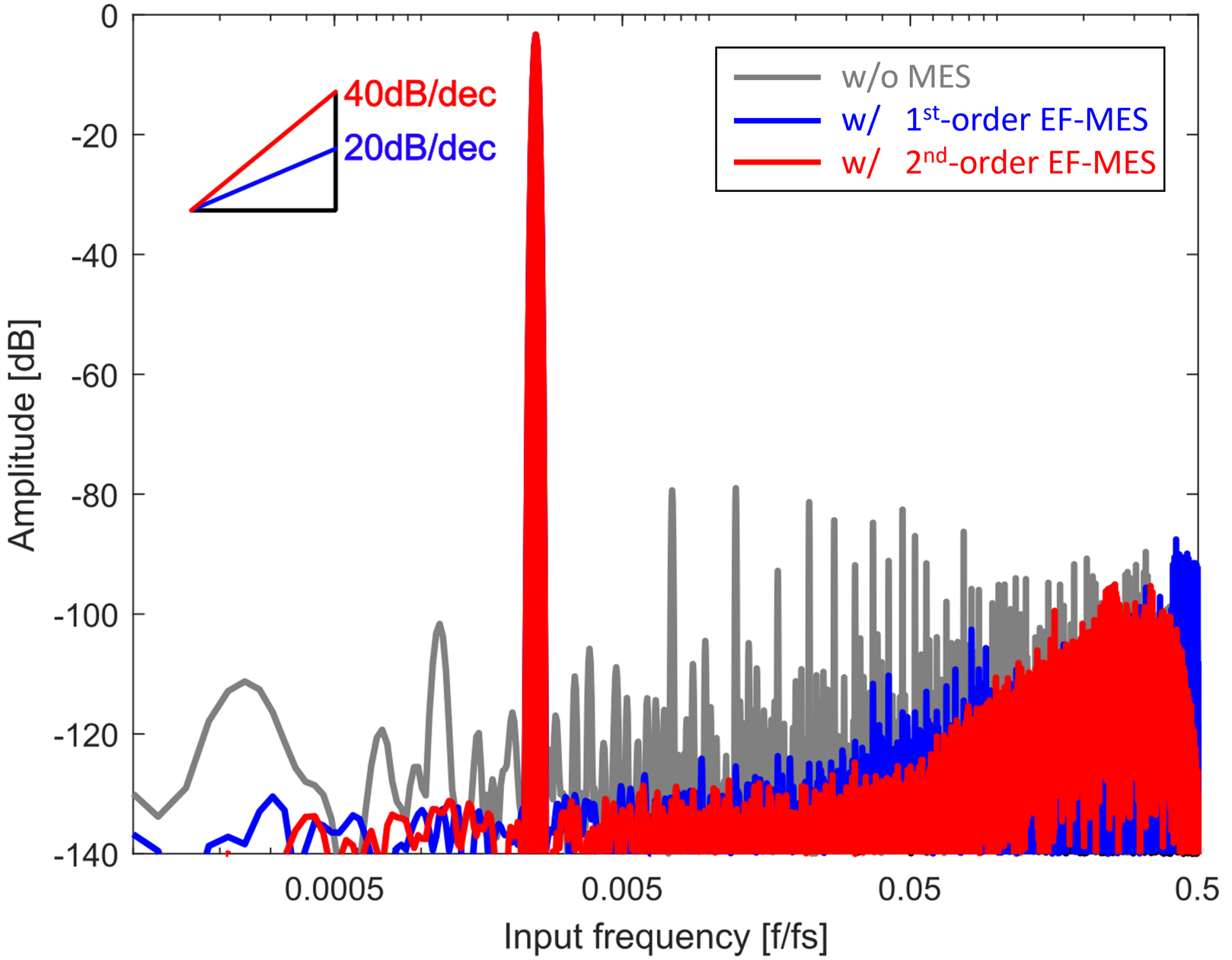
*Table 1*. *DAC mismatch error, 1st- & 2nd-order integrated error sequences with proposed 2nd-order EF-MES.*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cycle | Error of LSB1 | Error of LSB2 | Error of LSB3 | Error of LSB4 |
| 1 | E(1) | 0 | 0 | 0 |
| 2 | -E(1) | E(2) | 0 | 0 |
| 3 | -E(1) | -E(2) | E(3) | 0 |
| 4 | E(1) | -E(2) | -E(3) | E(4) |
| 5 | E(5) | E(2) | -E(3) | -E(4) |
| … | … | … | … | … |
| 4n | E(4n-3) | -E(4n-2) | -E(4n-1) | E(4n) |
| 4n+1 | E(4n+1) | E(4n-2) | -E(4n-1) | -E(4n) |
| 4n+2 | -E(4n+1) | E(4n+2) | E(4n-1) | -E(4n) |
| 4n+3 | -E(4n+1) | -E(4n+2) | E(4n+3) | E(4n) |
| 4n+4 | E(4n+1) | -E(4n+2) | -E(4n+3) | E(4n+4) |
| ∑ELSB | 0 | -E(4n+2) | 0 | E(4n+4) |
| ∑∑ELSB | 0 | 0 | E(4n+3) | E(4n+4) |

Table 1 lists the mismatch errors with every LSB DACs. It can be seen that the 2nd-order integrated mismatch error of each LSB DAC is bounded, which proves that the DAC mismatch error is 2nd-order shaped.

The proposed 2nd-order EF-MES is verified by MATLAB simulations. The 16-bit DAC combines a 4-bit MSB DAC and four 12-bit LSB DACs. The MSB DAC is assumed to be ideal and the unit element of LSB DAC has a 10% standard deviation. The input signal is -3.3dBFS to prevent overload [5].

Fig. 5 compares the output spectra of the CT DAC without MES, with 1st-order EF-MES and with the proposed 2nd-order EF-MES. It clearly shows that the proposed technique significantly reduces the harmonics caused by the DAC mismatch and realizes the 40dB/decade shaping effect at low frequencies. As listed in table 2, with OSR=8, comparing to the DAC without MES and with 1st-order EF-MES, the SFDR/SNDR improvements are 42.4dB/31.5dB and 18.5dB/6.2dB, respectively.



**Fig.5** *Simulated* *spectra of CT DAC without MES, with 1st- & the proposed 2nd-order EF-MES.*

*Table 2. Dynamic performances of CT DAC without MES, with 1st- & the proposed 2nd-order EF-MES.*

|  |  |  |  |
| --- | --- | --- | --- |
|  | w/o MES | w/ 1st-order  EF-MES | w/ 2nd-order  EF-MES |
| SFDR | 75.6dB | 99.5dB | 118dB |
| SNDR | 70.5dB | 95.8dB | 102dB |

*Conclusion:* The principle, implementation and verification results of high-order EF-MES for CT DACs are provided in this letter. Compared to the previous 1st-order EF-MES, the proposed high-order one provides more aggressive mismatch shaping capability. The proposed technique can be useful in high-resolution data converters with large DAC mismatch errors.

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