**Image transmission over quantum communication systems with three-qubit error correction**

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Quantum communication is expected to become the cornerstone of global communication systems, addressing critical issues of classical communication while providing unprecedented security and efficiency. A crucial aspect of advancing this field is quantum channel coding, which ensures data integrity by detecting and correcting errors specific to quantum systems. This research evaluates the performance of the three-qubit error correction code, the fundamental and simplest technique in quantum channel coding, for image transmission over error-prone channels. JPEG and HEIF format images are encoded using the three-qubit error correction method and compared to the 1/3 rate polar codes. Our results demonstrate that the three-qubit error correction code significantly outperforms advanced classical polar codes in both classical and quantum domains, achieving a maximum PSNR of 61.5 dB (SSIM = 0.9997) in HEIF and 58.3 dB (SSIM = 0.9994) in JPEG. This showcases its potential as a robust solution for quantum communication.

*Introduction:* Quantum communications represent a revolutionary advancement in the field of information and communication technology, leveraging principles of quantum mechanics to provide an alternative paradigm for secure and efficient data transmission [1]. As classical communication technologies are reaching their saturation point, quantum communication is poised to become the primary communication process, overcoming the inherent limitations of classical systems. Therefore, researchers worldwide are dedicating their efforts to advancing various aspects of quantum communication, addressing challenges, and exploring the full potential of quantum technologies.

One critical area of focus when developing quantum communication systems is quantum channel coding, aspiring to achieve the same objectives as channel coding [2] in classical communication systems, but in quantum channels. Because this is a vital concern since quantum communication systems are also susceptible to errors due to decoherence, and noise, as well as other quantum-specific disturbances. These errors can significantly impact the fidelity of the transmitted quantum information, making error correction a crucial component for practical quantum communication systems [3]. However, direct adaptation of existing classical error correction techniques for quantum communication is challenging due to the no-cloning theorem [4], which prohibits the duplication of qubits similar to classical bits.

As an alternative, several quantum error correction codes have been proposed, reflecting significant progress in addressing errors in quantum information theory. The initial breakthroughs in quantum error correction introduced the concept of using multiple qubits to protect a single qubit against various types of errors. For instance, the Shor code demonstrated that nine qubits could safeguard a single qubit [5], while the Steane code employed seven qubits for the same purpose [6]. Following these foundational works, many other error correction codes were developed, including the three-qubit error correction code [7], the five-qubit error correction code [8], the surface code [9], and stabilizer codes [10].

Among these, the three-qubit error correction code is the most fundamental and simplest method, capable of detecting and correcting a single qubit error. The effectiveness of the three-qubit error correction code has been extensively investigated in the context of quantum computing [11-13]. However, despite these advances, none of these methods have been analysed for their effectiveness in quantum communication processes involving image transmission.

In this research, we focus on analysing the performance of the three-qubit error correction code in the context of image transmission using quantum communications over error-prone channels. To investigate its capabilities, images are encoded using the joint photographic experts group (JPEG) codec and the higher efficiency image format (HEIF). These images are then channel coded using the three-qubit error correction method in the quantum communication process. We compare its performance in quantum channels against rate 1/3 polar codes [14] in both quantum and classical channels. Simulation results clearly show that three-qubit error correction can significantly outperform rate 1/3 polar codes over both quantum and classical systems in terms of peak signal-to-noise ratio (PSNR) and structural similarity index measure (SSIM).

Therefore, the key novel contributions of this paper are:

* Analyse the performance of the three-qubit error correction code.
* Compare its performance against modern classical error correction methods in quantum communication process.
* Propose a more robust quantum communication system for image transmission.

A diagram of a process flow

Description automatically generated*Proposed Framework:* The proposed quantum communication process with the three-qubit error correction code is illustrated in Figure 1.

Fig. 1 End-to-end quantum communication system with three-qubit error correction method.

In the proposed system, the information source consists of original images sourced from the Microsoft COCO dataset [15]. These undergo initial processing using a source encoder (JPEG or HEIF), which compresses and formats the images into a stream of classical bits. These classical bits are then fed into a quantum encoder, which transforms them into qubits as we proposed in [16]. The uniqueness of this research lies in the quantum channel coding, where the three-qubit error correction method is used.

A diagram of a music system

Description automatically generated with medium confidenceThe encoding and decoding processes of the three-qubit code can be represented as shown in Figure 2. The initial quantum information is represented by |𝜓⟩, which is the general superposition state corresponding to |0⟩ or |1⟩. In the three-qubit quantum channel coding process, each qubit is encoded into three qubits. Therefore, the three-qubit general superposition state can be generated using two additional |0⟩ qubits.

Fig. 2 Encoding and decoding process of the three-qubit error correction method.

The state of the three-qubits before the controlled not (CNOT) gates can be represented as shown in (1).

(1)

Here α and β are probability amplitudes [16]. Then the first CNOT gate is applied from the first qubit to the second qubit, and the second CNOT gate is applied from the first qubit to the third qubit. The final state after the CNOT gates is represented in (2).

(2)

This three-qubit superposition state can be sent through a noisy quantum channel, which introduces noise into it. We use a simple quantum channel model to simulate the process, applying random quantum noise at different variance levels [17]. This leads to 8 possible states to be received, as shown in Table 1.

**Table 1:** Relationship between the received state, position of the error and ancilla state

|  |  |  |
| --- | --- | --- |
| Received state | Position of the Error | Ancilla state |
|  | No error | |00⟩ |
|  | Error in first bit | |11⟩ |
|  | Error in second bit | |10⟩ |
|  | Error in third bit | |01⟩ |
|  | Error in first two bits | |01⟩ |
|  | Error in first and third bits | |10⟩ |
|  | Error in last two bits | |11⟩ |
|  | 3-bit error | |00⟩ |

In the error correction process as in Figure 2, two additional qubits in the ∣00⟩ state, called ancilla bits, can be employed to detect errors more efficiently and gather noise information. Using CNOT gates, the first and second qubits are connected with the first ancilla bit, and the first and third qubits with the second ancilla bit. This transforms the ancilla qubits into specific states as represented in Table 1. The ancilla qubits are then measured, producing two classical bits based on Table 2, which are used to diagnose and correct any errors in the qubits.

**Table 2:** Action to correct errors

|  |  |
| --- | --- |
| Measured Syndrome | Action to correct errors |
| 00 | Don’t need to correct |
| 01 | Apply Pauli x gate to the third qubit |
| 10 | Apply Pauli x gate to the second qubit |
| 11 | Apply Pauli x gate to the first qubit |

In the three-qubit channel decoder, a CNOT gate is applied from the first qubit to the third qubit, and another from the first qubit to the second qubit to retrieve the original qubit's superposition state, as depicted in Figure 2.

This theoretical process is simulated using matrix representations. The tensor product is used to calculate matrices for multi-qubit systems, with |0⟩ and |1⟩ matrices, which are shown in equations (3) and (4).

|0⟩ =  (3)

|1⟩ = (4)

Also, three-qubit multiple systems can be represented as in (5) and (6).

|000⟩ = ⨂ ⨂ =T (5)

|111⟩ = ⨂ ⨂ = T (6)

Then, the three-qubit superposition state is mapped to a matrix S as shown in (8).

S = T + T (8)

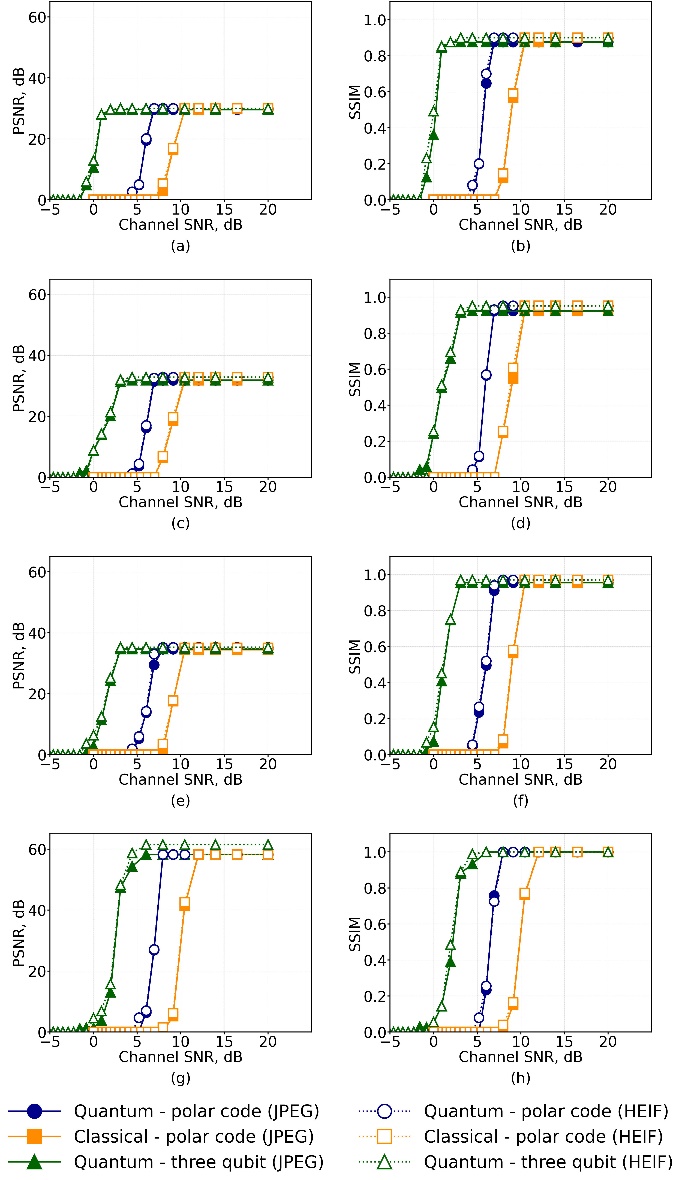
At the receiver, the received matrix is compared with 8 possible received state matrices. By examining the signs of the values, the results can be extended to 16 possible state matrices corresponding to 0 and 1, as shown in Table 3.

**Table 3:** Received states and corresponding matrices.

|  |  |
| --- | --- |
| Received vectors | Corresponding matrices |
|  | T |
|  | T |
|  | T |
|  | T |

By analysing the received 1×8 matrix and matching it with the corresponding matrix, we map the result to either |0⟩ or |1⟩. After this single-qubit extraction, the qubits are converted to classical bits and used to reconstruct the image. To evaluate the performance of the three-qubit error correction method, we compare it with a classical channel model that uses binary phase-shift keying (BPSK) modulation and rate 1/3 polar codes. These code rates and modulation scheme are chosen in order to maintain a similar bandwidth utilisation in both systems. Additionally, for simulation purposes, we consider a range of different images with varying spatial information content, allowing to analyse the results of the proposed quantum communication setup and compare them with the reference method using averaged image quality metrics. This comparison helps assess how the quantum error correction method performs relative to a classical channel coding system under similar conditions. Additionally, the results of the three-qubit error correction method are compared with the 1/3 code rate polar codes within a quantum communication setup.

The performance of image transmission in this three-qubit channel-coded quantum communication system is analysed using PSNR and SSIM, which are widely accepted quality matrices in analysing the quality of reconstructed images in image coding and transmission. These metrics are used to evaluate the quality and reliability of the transmitted images in both quantum and classical communication systems.

*Results and Discussion:* This study analyses the performance of the three-qubit error correction method in quantum communication and compares it with 1/3 code rate polar codes in both classical and quantum domains.

**Fig. *3*** *Rate distortion performance measured by the averaged PSNR and averaged SSIM of the test images over classical and quantum channels with error corrections for different Q values (Q = 25, 50, 75, and 100) and JPEG & HEIF image formats: (a) Q25 - PSNR, (b) Q25 - SSIM, (c) Q50 - PSNR, (d) Q50 - SSIM, (e) Q75 - PSNR, (f) Q75 - SSIM, (g) Q100 - PSNR, (h) Q100 – SSIM*

Figure 3 illustrates the variation in PSNR and SSIM of decoded images at different channel signal-to-noise ratios (SNR) for JPEG and HEIF images across various quantisation parameters (*Q25, Q50, Q75*, and *Q100*) ranging from low quality to very high quality. The results show that the three-qubit error correction method achieves significantly better rate-distortion performance in terms of both PSNR and SSIM at low SNR, compared to both classical and quantum methods utilizing polar codes. The three-qubit quantum approach's superior performance is evident across all quality levels, indicating its robustness in preserving image quality under noisy conditions. Additionally, the performance consistency between HEIF and JPEG images suggests that the three-qubit error correction method's advantages are applicable across different image source coding formats, reinforcing its potential as a versatile solution for high-fidelity image transmission using quantum communications.

A white sheet with black text

Description automatically generatedA sample of decoded images for *Q25* (JPEG) at the SNR of 8 dB from the three-qubit channel coding and polar coding processes is shown in Figure 4. At this SNR value, the output of the conventional system is not decoded due to the higher number of bit errors, while the three-qubit code performs the decoding perfectly.

**Fig. 4** *Sample of a decoded images (JPEG) from the experiment for Q = 25 and SNR 8 dB (a) quantum-three qubit, (b) quantum-polar code*

Also, the results of our study highlight that combined with inherent properties of a quantum channel, the three-qubit error correction code outperforms modern classical error correction methods in the quantum domain, revealing significant potential for future research and development. Although our study utilises a basic quantum communication system, it serves as a foundational step towards exploring more sophisticated systems. By extending our research to compare the three-qubit error correction method with more elaborate classical systems, we can better understand its relative performance and benefits. Additionally, developing comprehensive models for quantum channels and simulating quantum noise more accurately will be crucial for refining this method. These advancements will facilitate the practical application of three-qubit error correction codes in real-world quantum communication scenarios and align with the broader integration of quantum computing technologies. Future works will focus on developing more advanced quantum channel coding methods to further enhance the fidelity of quantum communication systems used in image and video transmission, particularly with realistic quantum channel models. These improvements are expected to not only increase the reliability of quantum communications but also advance quantum communication systems towards mainstream adoption for future media applications.

*Conclusion:* A key obstacle for widespread acceptance of quantum communication systems as a viable alternative for classical communication systems is the unavailability of effective error correction methods for quantum channel coding. This challenge is particularly evident in media transmission. As a solution for this problem, we propose the use of three-qubit error correction codes to improve the quality and the fidelity of the images transmitted over noisy channels. We demonstrate that the three-qubit error correction code significantly outperforms rate 1/3 polar codes in both quantum and classical domains by offering better PSNR (61.5 dB-HEIF and 58.3 dB-JPEG) and SSIM (0.9997-HEIF and 0.9994-JPEG) across various image quality levels and formats. Its robustness in maintaining image quality under very low channel SNR conditions underscores its effectiveness.

Although our research employs only a basic quantum communication system with a basic quantum channel coding method, it lays the groundwork for future exploration of more advanced quantum communication systems for media applications. Further research focus on developing accurate quantum channel models and integrating advanced quantum channel coding techniques will enable the advancement of quantum communication technologies towards real-world use.

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