Vulnerable Node Identification Method for Distribution Networks Based on Complex Networks and Improved TOPSIS Theory

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**Abstract:** A method for identifying vulnerable nodes in distribution networks is proposed, which is based on complex networks and optimized TOPSIS. This method aims to address the issues of one-sided evaluation indicators and inaccurate indicator weights that are present in existing methods for identifying vulnerable nodes in distribution networks. Based on the theory of complex networks, a comprehensive set of vulnerability indicators for distribution network nodes is constructed by considering both the topology structure and system operation status of the distribution network. The TOPSIS comprehensive evaluation model for optimization is proposed to enhance the selection process of optimal and worst indicator values. The advantages and disadvantages of each indicator are characterized using Mahalanobis distance. The calculation of proximity is optimized by establishing a virtual negative ideal solution, which makes the identification of vulnerable nodes more reasonable. The simulation results demonstrate that this method is more effective in identifying vulnerable nodes in the power grid compared to traditional methods, and has significant practical applications.

**1 Introduction**

The power system has undergone continuous expansion, resulting in increasingly complex connection types [1, 2]. In the event of a power system accident or a sequence of chain failures, the consequences extend beyond human casualties to encompass substantial societal property damage [3, 4]. Within a power network, the presence of vulnerable nodes can exert a substantial influence on the incidence and propagation of faults. Hence, the precise and efficient identification of susceptible nodes in the power grid holds immense practical importance in mitigating operational hazards and guaranteeing secure and dependable functioning [5].

The current research on vulnerable nodes in the power grid primarily focuses on analyzing the power system topology structure from both static and dynamic perspectives of the operating state. Indices that can characterize the vulnerability of nodes are established, and identification of vulnerable nodes is carried out. Through a single index or a comprehensive set of multiple indices. [6] introduced virtual nodes in the power network, established a transmission self-linking matrix between nodes, and applied an improved PageRank algorithm to identify vulnerable nodes. [7] was based on a complex network model that considered the weighted transmission network distance. The study proposed an identification method to characterize the vulnerability of nodes. [8] proposed evaluating nodes based on the impact of non-adjacent nodes. This method integrates both the local and overall importance of nodes. However, this method only considers the topology of the power system, without taking into account the operational factors that can influence its performance.

From the perspective of power grid operation, the development of methods to identify vulnerable nodes primarily involves analyzing changes in the power system's operations. [9] conducted an assessment of power system flow techniques and identified vulnerable nodes by utilizing vulnerability indices. A study conducted by [10] assessed each node by defining highly influential (HI) nodes, and combining coupling value and risk indices. [11] evaluated the vulnerability by considering the balance of power flow distribution, using the Taylor entropy as an index.

However, previous methods fail to fully consider the operational parameters of power system nodes and the interconnectivity between them. A single index is insufficient to fully characterize the vulnerability of each node.

To address the aforementioned issues, this article comprehensively examines the topology structure and operational characteristics of the power grid and establishes a comprehensive evaluation index system. Based on this premise, a comprehensive evaluation model is established using the optimized TOPSIS method to optimize the selection of the optimal and worst values of the evaluation object's indicators. The Mahalanobis distance is used to assess the strengths and weaknesses of each indicator in the distribution network, while minimizing the impact of correlations between them. By establishing a virtual negative ideal solution, the calculation of proximity is optimized, thereby enhancing the effectiveness of the evaluation model. The correctness and feasibility of the designed method will be verified through simulation using the IEEE39 node model and physical verification.

**2 Evaluation index based on complex network theory**

A network is a collection of individuals with specific relationships, and a complex network is a model that includes a significant number of nodes and intricate topological structures between them [12, 13]. The study of complex networks involves analyzing actual networks based on their static geometric characteristics.

(1) Improved Node Degree Index

The structure of the distribution network exhibits a certain level of density. In the process of calculating node degree, it is necessary to enhance the original degree index by incorporating the concept of a node's influence on its surrounding nodes. Therefore, the proposed index for improved node degree is:

 (1)

In the formul,  represents the number of edges connected to a node ,  represents the average degree of nodes in the system,  epresents all node sets with edges between node , and  represents the degree of the node connected to it.

After the improvement, not only is the relationship between edges and nodes in the original index considered, but the influence of surrounding connected nodes is also taken into account. The improved indices are more aligned with the power system network.

1. Node Tightness Index

This index represents the degree of proximity of each node in the system. A higher value of this index indicates that the node is in a relatively central position within the entire system.

Based on the theory of proximity centrality in complex networks, the gas distance model is proposed.

 (2)

In the formula,  represents the total impedance of the shortest path between two node.

1. Improved Node Intermediate Index

The analysis above only focuses on the topological structure. The analysis does not take into account the possible variations in identifying susceptible nodes that may result from the practical functioning of the power system. Under the same topology structure, the vulnerability level may vary when the operating level changes, even if it experiences the same disturbance. As a result, the impact of power outage accidents can also differ.

The intermediate index represents the shortest path through a node [14]. When applying the index to the power system, it is necessary to consider its specific operational situation. The improved intermediate index for nodes is defined as follows.

 (3)

 (4)

 (5)

In the formula,  represents the power node;  represents the load nodes; represents a set of load nodes;  and  are sets of power nodes and load nodes, respectively;  and  represent the actual power of the power node and the load node, respectively.  represents the current at node  after node  is connected to a unit current,  represents the power transmission limit value of the node ;  represents the injecting power to nodes.

The traditional intermediate index only considers the shortest path propagation in a simple system, whereas the improved node intermediate index takes into account the actual circuit equation. In actual situations, different nodes also have varying capacities in the energy transmission process. If a node transmits more power, the impact on the power system will also be greater in the event of a node failure. Therefore, this article also considers the injection power of nodes, which is more aligned with practical operating conditions.

1. Improved voltage crossing index

Propose an improvement to the node voltage threshold crossing index. The expression for crossing the node voltage threshold is:

 (6)

In the formula,  is the unit voltage value of a node,  and  represent the ratios of the upper and lower voltage values of the nodes to their corresponding actual voltage values.

As can be seen from the above equation, as the voltage gradually changes, its value also increases..

**3 Comprehensive evaluation method based on improved TOPSIS method**

This article presents a comprehensive index set for identifying vulnerable nodes. The intuitionistic fuzzy analytic hierarchy process (IFAHP) method is employed to calculate subjective weights , while the entropy weight method is used to calculate objective weights . The weighted averages of each index are obtained as follows:

 (7)

In the formula,  and  are the allocation factors.

This article utilizes the least squares method in combination with the weighting model [15] to ascertain the allocation factors. The formula is as follows:

 (8)

  (9)

Here, indicates the number of types of node vulnerability indices;  represents the matrix formed by each index after normalization.

Comprehensive evaluation of vulnerable nodes in distribution networks using optimized TOPSIS method.

**3.1 Subjective weight calculation method**

The IFAHP method is based on the AHP and incorporates intuitive fuzzy sets, as described by [16, 17]. When evaluating different indices of the distribution network, the IFAHP method can utilize the three concepts of membership, non-membership, and hesitation in the intuitionistic fuzzy set principle to create a judgment matrix. The specific steps are as follows:

1. Constructing matrices using intuitionistic fuzzy logic

When experts evaluate each index, they use their experience and understanding to determine the priority of each index at the same level. They then establish an intuitionistic fuzzy matrix , where  and  represent the priority relationship between two indices under the same level compared to each other, indicating membership and non-membership respectively.  indicates the degree of hesitation experienced during expert evaluation. The table is an intuitive fuzzy evaluation table 1.

**TABLE 1** Rule of judgment criterion

|  |
| --- |
| Evaluation level Intuitive fuzzy evaluation number |
| Index  is significantly (0.70,0.20,0.10)  more important than indiex  Index isas important as (0.50,0. 30,0. 20)  index  Index  is significantly (0.30,0.60,0.10)  more than indiex |

2) Inspection and correction for consistency

Before calculating the weights for each index, it is important to first check the consistency of the matrix. Firstly, construct a matrix for judging product consistency and adjust the original intuitive fuzzy judgment matrix  to . Then, use the information distance measurement formula for intuitive fuzzy numbers to obtain the consistency value . When it is less than 1, it is considered to be within the standard.

3) Calculating the weight

The formula for calculating index weight is as follows:

 (10)

According to the formula, the weight value of a certain index can be calculated. However, the expression is two-dimensional, which may not be suitable for intuitive evaluation in real-world scenarios. It needs to be integrated and normalized as follows:

 (11)

 (12)

**3.2 Objective weight method**

Using entropy weights, each index is assigned a weight based on its level of variability [18]. The method can be found in [19].

**3.3 Based on improved TOPSIS method comprehensive evaluation method**

The TOPSIS method primarily utilizes the information within the data to process it in a forward direction. By analyzing the optimal and worst solutions of the evaluation scheme, and characterizing the advantages and disadvantages of each scheme by comparing their differences (as demonstrated in studies by [20, 21, 22], we can determine the optimal evaluation scheme. It is often used to solve decision-making problems for multiple evaluation objects.

However, the research focus of this article is the power network within a single region. Ranking and selecting alternatives based solely on their distance from ideal solutions is not feasible. Therefore, it is necessary to optimize the selection of the best and worst values for traditional evaluation criteria. However, in traditional calculation schemes that use Euclidean distance as the measure of distance, the coupling problem between data can result in a decrease in evaluation accuracy. Additionally, there may be situations where the calculated distances are equidistant. The Mahalanobis distance is utilized as a replacement for the Euclidean distance in characterizing the strengths and weaknesses of different indices in the distribution network. This helps to reduce the interference caused by correlations between indices and introduces a virtual negative ideal solution to optimize the calculation of closeness. The optimized TOPSIS method can provide a more accurate and comprehensive evaluation of indices for individual distribution networks.

1. Establishing an evaluation matrix.

Establish an initial evaluation matrix based on the evaluation criteria . Define the initial matrix M using the following formula: In the formula,  represents the number of evaluation indices;  is the actual observed or calculated value of the index;  and  represent the optimal and worst values for this index based on operational and industry standards, or expert experience.

 (13)

In the equation, ， and  represent the actual value, optimal value, and worst value of the index, respectively.

1. Normalizing the Evaluation Matrix.

Normalize each index in the original matrix using the formula to obtain the norm matrix .

Here,

, (14)

1. Weighting the normative matrix.

Obtain the matrix  with weighted norms based on the combination weights calculated below.

,  (15)

1. Determining the value vector.

The optimal value vector is located in the second row  of matrix , while the worst value vector is located in the third row , and virtual negative ideal vectors are introduced.

 (16)

1. Calculating the distance and ratio between the vectors of actual values, optimal values, and virtual negative ideal values of the indices.

 (17)

 (18)

In the formula,  represents the inverse matrix of the covariance matrix formed between different indices.

The relative closeness is as follows.

 (19)

In the formula,  represents the degree of proximity between a specific index and its corresponding optimization index. If sorted from smallest to largest, the closer a value is to the optimal level, the better its current operation. Conversely, the farther away a value is, the worse its operation. Based on this, the current performance level of each index can be determined.

**4 Vulnerable nodes identification process**

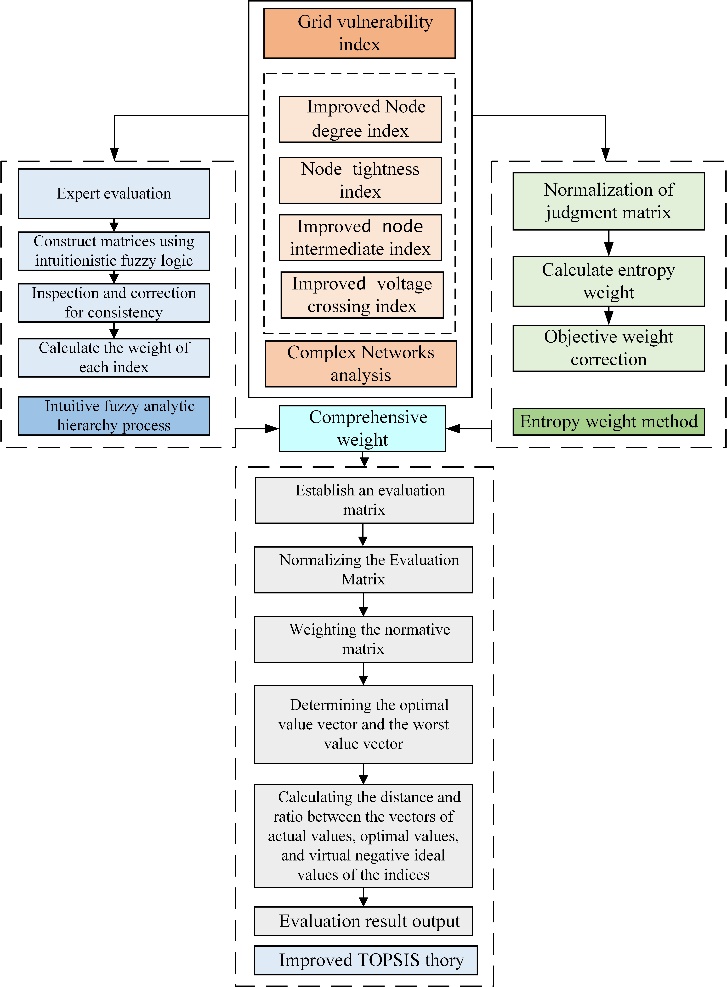
The steps for the calculation are illustrated in Figure 1.

Step 1: Analyze the network structure of the power system and establish a weighted power grid topology.

Step 2: Analyze the indices of the distribution network and establish a set of vulnerability indices for the system.

Step 3: Calculate the weights for each index and construct a combination of indices.

Step 4: Utilize the optimization TOPSIS method to rank each node in descending order based on their vulnerability index value.

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**Figure.1** Process for identifying vulnerable nodes

**5 Verification method for vulnerable nodes**

In order to provide a more detailed explanation of the impact of the vulnerable nodes identified in the article on the power system, this study examines the operational capacity of the power grid system in terms of network efficiency. The measure of network efficiency evaluates the ease or difficulty of transmitting information across the network from a global perspective. Its value also fluctuates when the network is attacked. Network efficiency typically refers to the shortest electrical distance between the power source and the load [23]. The specific formula for network efficiency is as follows:

 (20)

In the formula,  represents the number of nodes;  represents the shortest electrical distance between engine nodes  and load nodes ,  represent all engines and load nodes.

After a fault occurs in the power grid, the transmission capacity decreases due to the failure of nodes that affect the system's vulnerability. This results in a decrease in network efficiency. The higher the impact of failed nodes on system vulnerability, the lower the network efficiency becomes.

The power supply capacity is an indicator that characterizes the effective capacity of a distribution network to supply power, and it reflects the network's ability to transmit power during operation [24]. If a node failure results in a significant decrease in the system's power supply capacity, it suggests that the node has a more significant impact. The formula for calculating network power supply capacity is as follows:

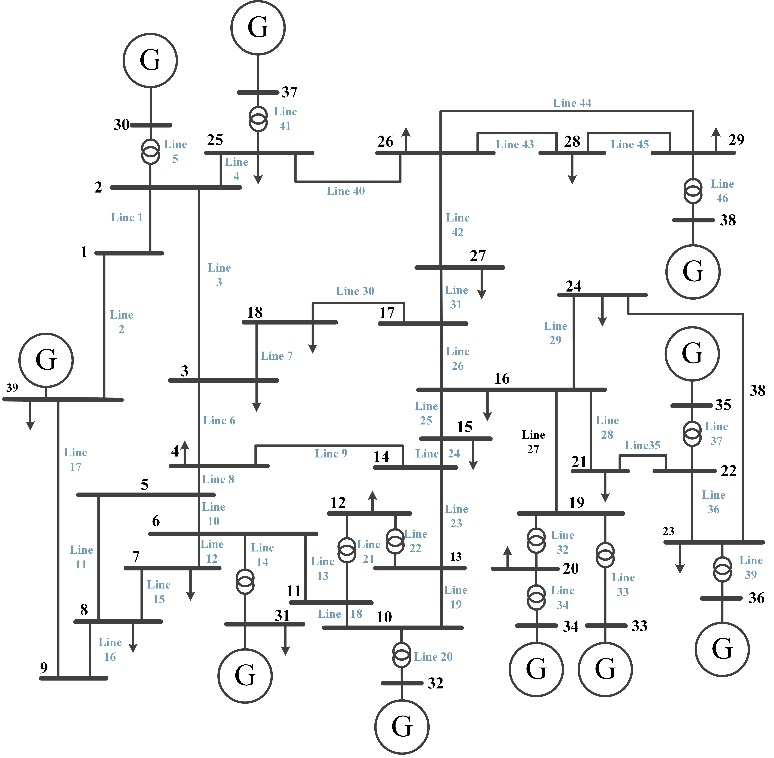
 (21)

In the formula,  and  represent the steady state of the node and the load level of the system after an attack, respectively.

**6 Simulation verification**

**6.1 Identification Result Analysis**

This article validates the proposed method by implementing it on an IEEE 39-bus system. The power system is illustrated in Figure 2, which comprises 39 nodes, 10 generators, 19 loads, and 46 transmission lines [25].



**Figure.2** IEEE 39-bus system

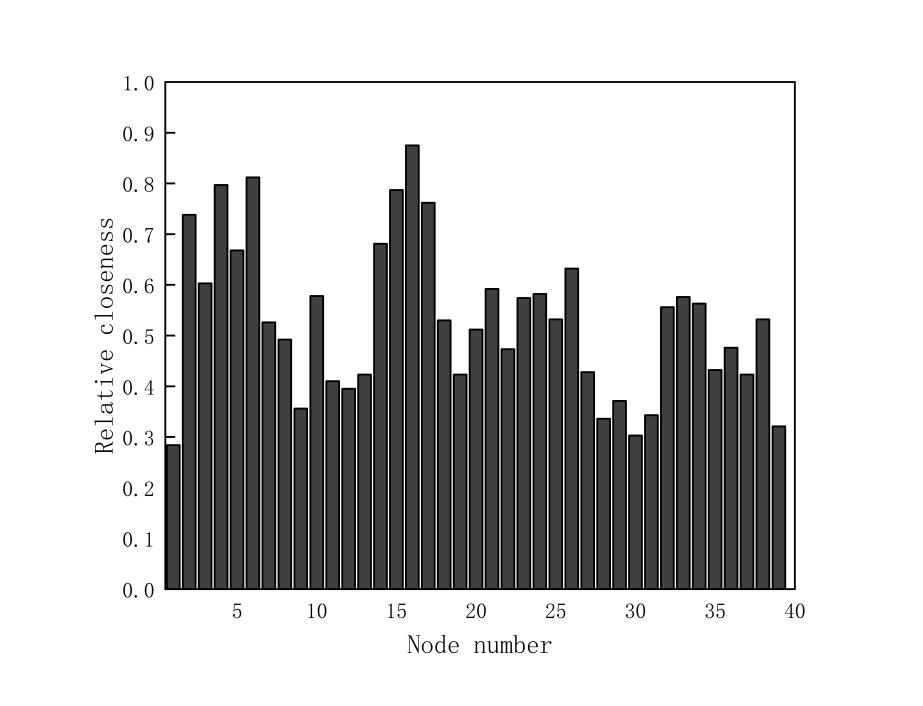
This article focuses on the calculation of subjective weights using the IFAHP method's three-scale approach. The method involves constructing an intuitionistic fuzzy matrix to determine the subjective weights of each index. Calculating the weights of each index objective using the entropy weighting method. The weights are  and . Next, apply the least squares method to determine the combined weights. The allocation factors are  and . To obtain the weights of each index .

The change in the weight of the index can reflect the index's impact on vulnerability. Therefore, this article will compare the weights of the improved node degree index, node tightness index, and improved node intermediate index with those of the node degree index, tightness index, and intermediate index presented in the literature [26].

**TABLE 2** Index weight table

|  |
| --- |
| Index in weight Index in weight  this article literature [26] |
| Improved 0.2632 Degree index 0.1713  degree index  Tightness 0.210 Tightness 0.2393  index index    Improved 0.2818 Intermediate 0.2330  Intermediate index  index |

From Table 2, it is evident that the weight difference between the improved node degree index  and the improved node intermediate index  is decreasing. Additionally, the weight of the improved node intermediate index  is slightly increasing, while the weight of the tightness index  is relatively decreasing. The improved node intermediate index  used in this article takes into account not only the traditional complex network metrics but also the connection relationships and operational capabilities in actual power systems. This approach provides a more accurate reflection of the electrical connections between nodes. This is more in line with the actual situation, as its share is relatively significant.The relative closeness of each node is displayed in Figure 3.



**Figure.3** Relative closeness sorting

To assess the reliability and accuracy of the combined indices, which are based on the node vulnerability index discussed in this article, the vulnerability level of each node in the power system can be determined using four different indices. Using an improved TOPSIS calculation method to comprehensively sort each node.The top 10 nodes in the vulnerability index ranking can then be arranged in descending order, as demonstrated in Table 3.

**TABLE 3** Identification results across various indexes

|  |  |
| --- | --- |
| Rank | Index type |
| Comprehensive  evaluation |
| 1 6 16 6 14 16(0.875)  2 4 4 16 16 6(0.812)  3 17 28 15 29 4(0.797)  4 8 20 24 13 15(0.787)  5 5 2 4 16 17(0.762)  6 2 14 17 11 2(0.738)  7 16 12 13 5 14(0.681)  8 28 10 3 24 5(0.668)  9 3 1 12 15 26(0.632)  10 26 5 26 28 3(0.603) | |

**Note:** The values in () in the table represent the relative closeness of the node

Based on the data presented in Table 3, it is evident that the improved node degree index  and the improved node intermediate index  yield the same results for identifying the most vulnerable node, which is node 6. The node tightness index  identifies node 16 as the most vulnerable. The voltage limit index  identifies node 14 as the most vulnerable.

Each index has different directions for vulnerability analysis, which leads to varying results. The improved node degree index  and improved node intermediate index  reflect the vulnerability of nodes at the local network topology level in the power system. The improved node intermediate index  also considers the power flow relationship and energy transmission ability of the power system operation. The node tightness index  starts from the overall network topology level and considers the connections between each node. The improved voltage crossing index  takes into account the operational condition of the node, with a greater emphasis on the stable relationship between nodes and the power system.

**6.2 Comparison of Different Methods**

To validate the rationality of the proposed method in this article, a comparison was conducted with the vulnerable node identification methods used in literature [27] and literature [28]. The results are presented in Table 4.

**TABLE 4** Comparison results of different methods

|  |
| --- |
| Rank Method of Method of Method of  this article [27] [28] |
| 1 16 6 16  2 6 5 17  3 4 11 19  4 15 8 15  5 17 10 3  6 2 7 10  7 14 13 20  8 5 14 29  9 26 4 2  10 3 15 14 |

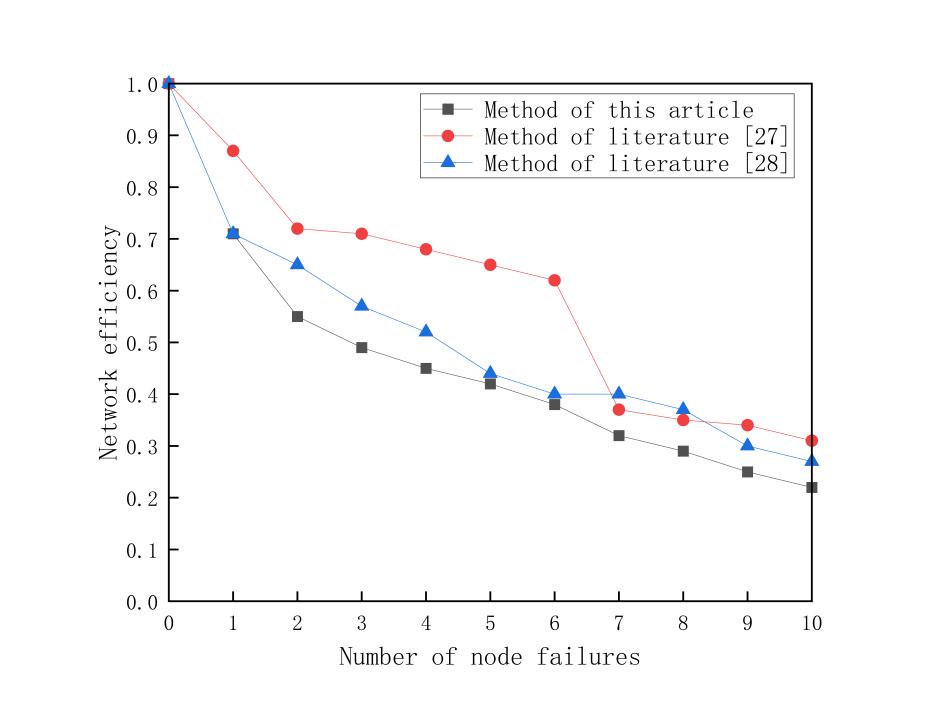
As presented in Table 4, the approach utilized in this article has 5 vulnerable nodes in common with the method employed in [27]. Furthermore, there are a total of 6 identical vulnerable nodes when compared to the method used in [28]. However, there are differences in the arrangement order among the same vulnerable nodes.

From Figure 2, it can be seen that Node 16 is located at the junction of multiple power lines in the system, indicating that it bears a heavy load of power transmission. The lines connected to it are transmission channels for power from engine nodes 33, 34, 35, and 36. If node 16 is removed, it will cause a significant power flow change throughout the entire power system, imposing a substantial burden on the system. This is consistent with the discrimination results reported in the [28]. Nodes 6 and 4 are significant load nodes in the system, situated on the power transmission paths of engine nodes 30, 31, 32, and 37.

Nodes 15, 17, 2, 14, 5, 26, and 3 play a specific role in transmitting engine power within the power system. If they are removed, it will cause frequent power outages, so they are also crucial nodes. Compared to the methods used in other literature, the approach for identifying vulnerable nodes in this article yields slightly different identification results. This discrepancy can be attributed to the use of different indices and research focuses. After analysis, it can be concluded that the methodology employed in this article is reasonably sound.

**6.3 Analysis of Network Efficiency Simulation Results**

To further validate the effectiveness of the methods used in this article, we will analyze and compare changes in the network efficiency indices when vulnerable nodes of the power grid are selected using different methods and subsequently fail. Based on the ranking results from the comparative literature [27], the literature [28], and this article, the top 10 nodes should be removed in the order of their ranking. And the network efficiency value of the power system is calculated.

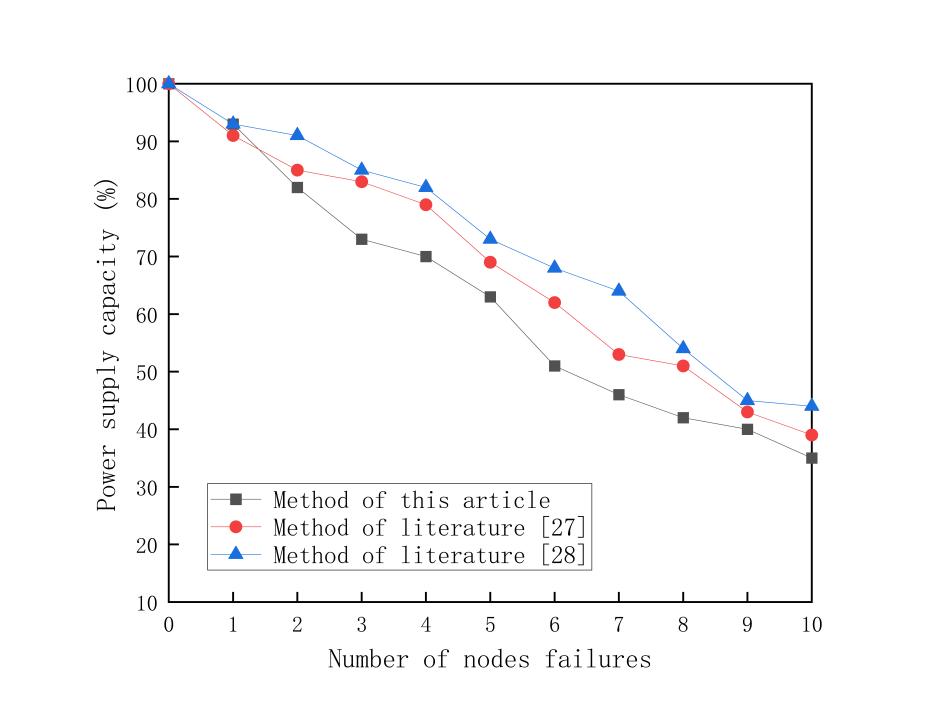


**Figure.4** Network efficiency

Figure 3 illustrates the changes in network transmission efficiency of the IEEE39 node power system. When not under attack, the system's network efficiency remains at 1.

It can be observed that when vulnerable nodes, identified by various methods, are attacked, the network efficiency decreases to varying degrees. In this paper and literature [28], the first node identified is node 16. After an attack on this node, the network efficiency value drops to 71%. In contrast, the identification results in reference [27] only caused a decrease in network efficiency to 87%. After being attacked 10 times, the network efficiency indices reported in [27] and [28] decreased to 31% and 27% of the normal level, respectively. And the network's efficiency indices reported in this article decreased to 22%, resulting in significant damage to the power transmission capacity.

According to the previous method, remove the top ten nodes in order to identify weak nodes and compare the changes in their power supply capacity. The comparison results are displayed in Figure 5.

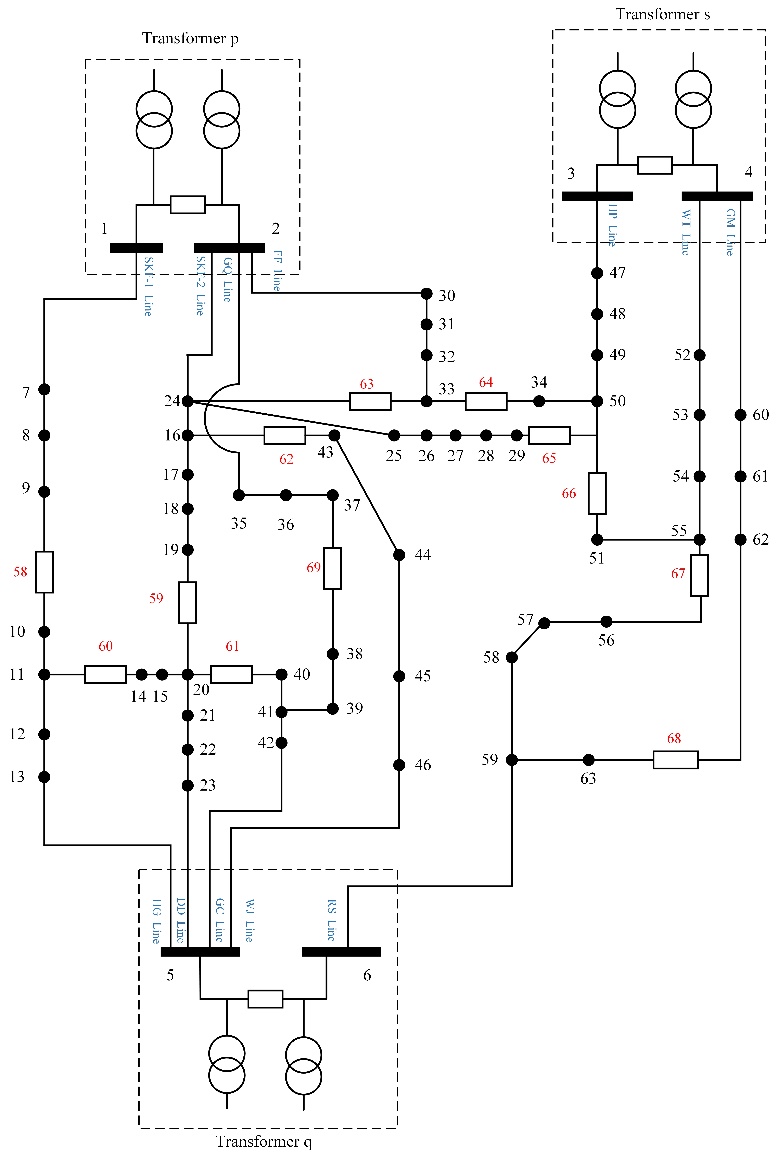


**Figure.5** Power supply capacity

Following the method outlined in this article for identifying weak nodes, the system's power supply capacity decreased to 63% after removing five such nodes in sequence. This resulted in a significant reduction in the system's ability to transmit power. In comparison, the power supply capacity of systems in references [27] and [28] decreased by 69% and 73%, respectively. After removing the 10 weakest nodes, the identification method utilized in this paper maintained the power supply capacity of the system at 35%, which is lower than the 39% and 44% reported in references[27] and [28].

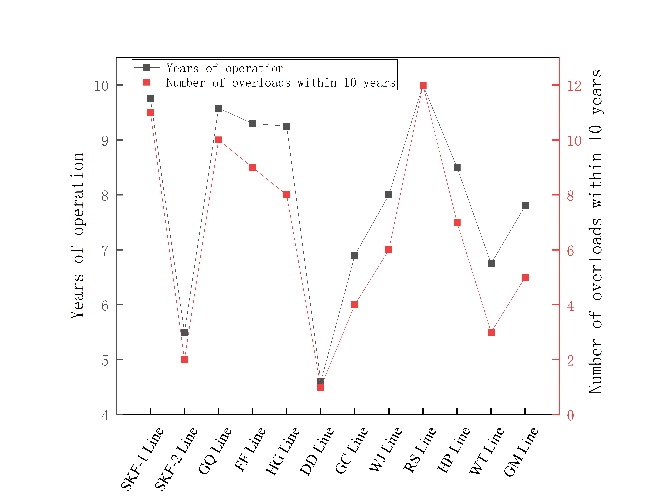
Therefore, compared to the other two methods, the identification method proposed in this paper is more reasonable and effective.

**6.4 Analysis of identification results of actual systems**

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**Figure.7** Power grid structure in a certain region

To evaluate the feasibility of the method proposed in this article for complex systems, a vulnerability analysis was conducted on the 10KV distribution network in Y city [29]. The distribution network model is depicted in Figure 7, and the parameters are listed in the appendix. The ranking of vulnerable nodes is displayed in Table 5. The past overload situation of the system is shown in Figure 8.



**Figure.8** Overload situation of the system

**TABLE 5** Identification results of actual circuit

|  |  |
| --- | --- |
| Rank Node number | Rank Node number |
| 1 59  2 33  3 24  4 16  5 20 | 6 41  7 38  8 60  9 7  10 47 |

According to Table 5, nodes 59, 33, 24, 16, 20, and 41 are identified as the top six vulnerable nodes in the power grid. This is not only because these nodes form the backbone transmission network of the region, but they also undertake the important task of transmitting energy between various lines. If these vulnerable nodes are disturbed, the power flow fluctuations within the system can have a significant impact on the power grid, leading to a reduction in the network's connectivity rate.

As depicted in Figure 8, over the last decade, the RS line that includes node 59 has encountered the most frequent overloads in the entire system. Node 59 carries the heaviest load on the online RS line. This component plays a crucial role in power transmission between the WT line and the GM line. The effectiveness of identifying node 59 as the weakest node has been indirectly confirmed. Node 33 is situated at the end of the FF line and plays a critical role in directly transmitting energy among the FF line, SKF-2 line, and HP line. Nodes 24, 16, and 20 are considered vulnerable based on their height and compactness indices. This indirectly demonstrates the validity of the indicators utilized in this article. Nodes 38, 60, 7, and 47 are closely connected to the substation and carry heavy loads, which significantly impact the power grid. It can be seen that the method for identifying fragile nodes proposed in this article is both reasonable and practical.

**Conclusion**

This article focuses on identifying the vulnerability of distribution network nodes when encountering faults. It takes into account the topological structure and the actual operational factors of the distribution network. The article constructs a set of vulnerability indicators based on complex networks and operational capabilities. A comprehensive evaluation method has been proposed to improve and optimize TOPSIS. This method selects the optimal and worst values of indicators to optimize the evaluation of a single object. The concept of a virtual negative ideal solution is introduced to enhance the calculation method for determining the proximity between the actual value of each index and the optimal value. The Mahalanobis distance is utilized as a replacement for the Euclidean distance in order to assess the strengths and weaknesses of each index. This helps to minimize the impact of correlations between the indexes and reduce interference. By analyzing network efficiency indicators and combining them with specific power grid analysis, the identification results of the method proposed in this article are more reliable for screening weak nodes. Additionally, the evaluation method is more reasonable and objective.

# Author Contributions

Enyu Jiang and Wentao Zhang was responsible for the conceptualization, methodology, MATLAB coding and draft preparation; Ang Xue, for the formal analysis and project administration; Sunfu Lin and Yang Mi , for the resources and data curation; and DL, for the review and editing. All authors have read and agreed to the published version of the manuscript.

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# Conflict of Interest

The authors declare that the research was conducted without any commercial or financial relationships that could be perceived as a potential conflict of interest.

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All authors are acknowledged for their contributions to the article and experiments.

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