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Quantum gravity by relativization of Quantum Field Theory.

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ABSTRACT

The question is how can we make quantum field theory part of General Relativity instead of how we can quantize gravity. Here it will be shown that a Hilbert space can be defined such that the bra-ket is a four vector in $\mathscr{M}$inkowski space-time, $<\psi|\phi>=v^a\in\mathscr{M}$. Similar to a Hilbert space over the field of quarternions. Minkowski space is the tangent space at an arbitrary point on a Riemannian manifold. It will then be shown that the Riemann curvature connects these spaces by operating on the probability density 4-current j^a of the local QFT. Choosing the Klein-Gordon field as a simple example QFT, the quantized Einstein-Hilbert action will then be derived. From there the expected Feynman diagrams for General Relativity can be read off. In this way one may calculate the gravitational effect due to a quantum field theoretical event.

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1 INTRODUCTION

The motivating question for this study was can Hilbert space can be defined such that the $<\psi|\phi>=v^a\in\mathscr{M}$. What kind of physics can be modeled with such a structure. My thought was that QFT could be incorporated into General Relativity, and by that method a new hypothesis of short length scale gravity or possibly quantum gravity could be proposed. In this model QFT would be exactly correct in the tangent spaces of a Riemannian manifold, while gravity would physically connect these spaces.

Quantum field theory has been the most successful program of theoretical physics since General Relativity. The one failure of Quantum Field theory has been an inability to incorporate General Relativity or handle dynamically curving space time. The idea expressed in this short paper is that one could try incorporating QFT into GR instead of the other way around. Instead of quantizing General Relativity we relativize QFT by demanding strict adherence to the Einstein Equivalence Principle [(2)]. That for an infinitely small four-dimensional region, the relativity theory is valid in the special sense when the axes are suitably chosen.
Each point on a curved space time has a flat Minkowski space time$\mathscr{M}$, called a tangent space, in which special relativity is exactly valid. With an appropriate inner product in the Hilbert space, four vectors may act as scalars in the Hilbert space. Thus quantum field theory would automatically obey the Einstein Equivalence Principle.

What is proposed in this paper is not completely without precedent. Standard quantum mechanics is formulated in terms of operators on a Hilbert space over the field of complex scalars. It has been shown that the set of quaternions can also be used as the set of scalars over which the Hilbert space is defined[(3)]. The idea that the key to quantum gravity lies in the formulation of the theory in terms of new variables can be found in the literature [(1)]. There are also many works on the subject of algebraic local quantum field theory(4).

What is new in this paper is the concept of $\mathscr{H}$ over the space $\mathscr{M}$ (which has the Clifford algebra $Cl_{1,3}(R)$). Treating the tangent space attached to each point on a Riemannian manifold as an algebra over which Hilbert space may be defined enables a new approach to quantum gravity. Instead of assuming QFT is fundamental and incorporating gravity into it, we could incorporate QFT into Relativity.

2 HILBERT SPACE OVER THE SPACE OF 4-VECTORS

Latin indicies will indicate vectors in the tangent space at each point on a Riemannian manifold. Greek indicies will indicate vectors in the curved global space-time of the manifold and ket's will be vectors in Hilbert space. In the following the vierbein formulation of General Relativity will be used.
Let
$\mathscr{H}$ be a vector space with elements $|\psi>$ and $|\phi>$ over the space of 4-vectors with $v^a \in \mathscr{M}$ and the inner product defined as follows.

$$<\psi|\phi>=\frac{1}{2}\left(\bar{\psi}\gamma^a\phi+\bar{\phi}\gamma^a\psi\right)(1)$$

$\mathscr{H}$ is a Hilbert space if the inner product satisfies certain conditions. I will now demonstrate that this is a Hilbert space starting with equation [2].

$$\overline{<\psi|\phi>}=<\phi|\psi>(2)$$

Taking the hermitian adjoint of equation [1] results in equation [3]

$$\overline{<\psi|\phi>}=\overline{\frac{1}{2}\left(\bar{\psi}\gamma^a\phi+\bar{\phi}\gamma^a\psi\right)}=\frac{1}{2}\left(\bar{\phi}\gamma^a\psi+\bar{\psi}\gamma^a\phi\right)=<\psi|\phi>(3)$$

Therefore the condition set out in equation [2] will be satisfied. The adjoint of the proposed inner product equals the inner product. Now I will consider the norm of a vector in $\mathscr{H}$, which has to be finite and non zero in the space $\mathscr{M}$. Applying equation [1] results in equation [4] which shows that the normalizability criteria is satisfied by equation [1] .

$$<\psi|\psi>=\frac{1}{2}\left(\bar{\psi}\gamma^a\psi+\bar{\psi}\gamma^a\psi\right)=\bar{\psi}\gamma^a\psi\neq 0^a\in\mathscr{M}(4)$$

The last requirement is that the inner product would needs to be linear in at least one of it's arguments. To show this let $|\psi_1>,|\psi_2>,|\phi>\in\mathscr{H}$, and let $v^a,y^a\in\mathscr{M}$.

$$\begin{aligned}<v^a\psi_1+y^a\psi_2|\phi> &= \frac{1}{2}\left(\overline{(v^a\psi_1+y^a\psi_2)}\gamma^a\phi+\bar{\phi}\gamma^a(v^a\psi_1+y^a\psi_2)\right) \\ &= v^a<\psi_1|\phi>+y^a<\psi_2|\phi>\end{aligned}(5)$$

The major physical advantage of this Hilbert space lay in the norm of a ket in this space results in the conserved Noether current (equation [6]).

$$<\psi|\psi>=\frac{1}{2}\left(\bar{\psi}\gamma^a\psi+\bar{\psi}\gamma^a\psi\right)=\bar{\psi}\gamma^a\psi=j^a(6)$$

3 RIEMANN CURVATURE OPERATOR.

What is gained by this formulation is a hypothesis which is valid in the flat space-time tangent to every point on a Riemannian manifold. Using the vierbein formulation of General Relativity I can start from the Cartan structure equations using the gamma matrices as basis

$$R^a_b = d\omega^a_b + \omega^a_c \wedge \omega^c_b$$

$$T^a = d\gamma^a + \omega^a_b \wedge \gamma^b.$$

Note the Greek indices's are suppressed but they are still in effect.

Then solve for the Cartan connection.

$$T^a = d\gamma^a + \omega^a_b \wedge \gamma^b \Rightarrow \left(T^a - d\gamma^a\right) \wedge \gamma_b = \omega^a_b \text{(7)}$$

Set the torsion equal to the probability current for the Dirac field $|\psi>$, $j^a =<\psi|\psi>$, and simplify. This results in the exterior derivative of the Cartan connection.

$$d\omega^a_b = dj^a \wedge \gamma_b \text{(8)}$$

Next we can solve for the Riemann curvature in terms of Dirac gamma matrices and probability currents.

$$R^a_b = dj^a \wedge \gamma_b + j^a \wedge \gamma_c \wedge j^c \wedge \gamma_b \text{(9)}$$

To get a valid operator on the Hilbert space use $j^a =<\psi|\psi>$. Also use the outer product $|\psi> \;\; <\psi|$. Then I define $R^a_b\left(<\psi|\right)$.

$$R^a_b\left(<\psi|\right) = \left(d<\psi| \wedge \gamma_b + <\psi| \wedge \gamma_c \wedge <\psi| \wedge \gamma_b\right) \text{(10)}$$

$$\widehat{R^a_b} = R^a_b\left(<\psi|\right)|\psi> \;\; <\psi| = \left(d<\psi|\psi> \wedge\gamma_b + <\psi|\psi> \wedge\gamma_c \wedge <\psi|\psi> \wedge\gamma_b\right) <\psi| \text{(11)}$$

The Riemann operator, with Greek indices's suppressed, is $\widehat{R^a_b}$ with eigenvalues and eigenstates given by equations [12].

$$\widehat{R^a_b}|R^a_b> = \left(R^a_b\left(|\psi>\right)|\psi> \;\; <\psi|\right)|R^a_b> \text{(12)}$$

Equation [12] relates the curvature of space time near a point to the probability four currents due to local non-gravitational fields. Equation [12] provides eigenvalues and eigenstates of space-time curvature. This is a quantization of gravity which will be as renormalizeable as the theories which go into computing the state of the system.

4 IF ALL THE WORLD WAS ESSENTIALLY KLEIN-GORDON, HOW WOULD GRAVITY BEHAVE?

The Klein-Gordon field is the simplest field which could be used in this theory. I will approximate all the matter in space with a Klein-Gordon field. Klein-Gordon is a simple field to work with and will illustrate the basics of this model well. However, a more realistic model may result from use of the Higgs field. The possibility of using the Higgs field will be explored in a future paper.

$$\psi = e^{ik_a x^a} \text{(13)}$$

Knowing the curvature operator equation [11], and the solution to the Klein-Gordon equation [13] I can find the curvature eigenvalues.

$$(\gamma^a \wedge \gamma_c \wedge \gamma^c \wedge \gamma_b) e^{-ik_a x^a} \gamma^a e^u = R^a_b e^u \quad (14)$$

Equation [14] can be solved for e^u by examination and yields equation [16] which gives the eigenvalues and eigenstates of $R^{a\mu}_{b\nu}$.

$$\left(\gamma^{a\mu} \wedge \gamma_{c\lambda} \wedge \gamma^{c\lambda} \wedge \gamma_{b\nu}\right) = R^{a\mu}_{b\nu} \quad (15)$$

$$e^u = e^{ik_a x^a} = \left|R^{a\mu}_{b\nu} > \right. \quad (16)$$

Thus the simplest curvature field is the Klein-Gordon field times the curvature eigenvalue. The resulting equation is the quantized Riemann curvature tensor field.

$$R^{a\mu}_{b\nu}(x^a) = \left(\gamma^{a\mu} \wedge \gamma_{c\lambda} \wedge \gamma^{c\lambda} \wedge \gamma_{b\nu}\right) e^{ik_a x^a} \quad (17)$$

Contraction of the appropriate indexes allows me to find the quantized Ricci tensor and quantized Ricci scalar. Then the Einstein-Hilbert action can be written down.

$$S = \frac{1}{2\kappa} \int \eta^b_a \gamma^a_\mu \gamma^\nu_b \eta^b_a \left(\gamma^{a\mu} \wedge \gamma_{c\lambda} \wedge \gamma^{c\lambda} \wedge \gamma_{b\nu}\right) e^{ik_a x^a} \sqrt{- \parallel \eta^b_a \gamma^a_\mu \gamma^\nu_b \parallel}\, \mathrm{d}^4 x \quad (18)$$

Treating the position as an operator requires that I Taylor expand equation [18]. This expansion leads to a formula for computing the quantum gravitational action to arbitrary precision using a series that is known to converge uniformly (equation [19]).

$$S = \frac{1}{2\kappa} \int \eta^b_a \gamma^a_\mu \gamma^\nu_b \eta^b_a \left(\gamma^{a\mu} \wedge \gamma_{c\lambda} \wedge \gamma^{c\lambda} \wedge \gamma_{b\nu}\right) \left(\sum_{n=0}^{\infty} \frac{(ik_a x^a)^n}{n!}\right) \sqrt{- \parallel \eta^b_a \gamma^a_\mu \gamma^\nu_b \parallel}\, \mathrm{d}^4 x \quad (19)$$

To get Feynman diagram style rules for this theory the first few terms in the Taylor series will be useful. Let $R_0 = \eta^b_a \gamma^a_\mu \gamma^\nu_b \eta^b_a \left(\gamma^{a\mu} \wedge \gamma_{c\lambda} \wedge \gamma^{c\lambda} \wedge \gamma_{b\nu}\right)$ in equation [20]. The resulting rules are in figure [1].

$$S = \frac{1}{2\kappa} \int R_0 \left(1 + ik_a x^a - \frac{k^2 x^2}{2} - \dots\right) \sqrt{- \parallel \eta^b_a \gamma^a_\mu \gamma^\nu_b \parallel}\, \mathrm{d}^4 x \quad (20)$$

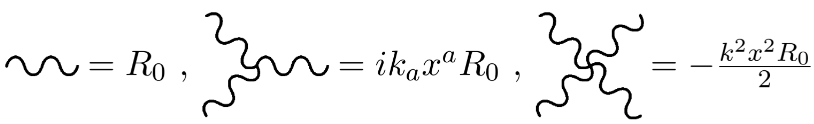

FIGURE 1: FEYNMAN DIAGRAMS OF GRAVITY IN THIS MODEL. HERE WAVY LINES INDICATE GRAVITONS. NOTE THESE RULES RESULT ESSENTIALLY FROM THE CHOICE OF THE KLEIN-GORDON FIELD FOR THE LOCAL QFT.

5 CONCLUSIONS

I have shown that a Hilbert space can be defined such that the $< \psi|\phi >= v^a \in \mathscr{M}$. With Hilbert space so defined one can reasonably write of a Riemann curvature operator. Then an eigenvalue equation can be found. I have also shown that the Einstein Hilbert action and well know Feynman Diagram rules for Einstein gravity can be derived from this formulation. That is to say equation [19] allows one to compute the gravitational effect on interacting particles at any quantum mechanically interesting length scale, and to any desired precision. (It is also interesting that this theory resembles an $f(R)$ model once the Taylor expansion is done.) A more complete treatment of this problem would involve using the full standard model to compute a quantized Einstein Hilbert action, Feynman diagrams etc. However, I believe the essential features of this hypothesis are illustrated well enough by equation [19]. A more complete treatment may introduce new problems of calculation but, as long as the model used to compute ψ is at least renormalized none of those problems could be fatal.

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