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Internal vs Forced Variability Metrics for Geophysical Flows Using Information Theory

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ABSTRACT

11 We propose a metric for measuring internal and forced variability in ensemble atmosphere, ocean,
12 or climate models using information theory: Shannon entropy and mutual information. This metric
13 differs from the standard ensemble-variance approaches. Information entropy quantifies variability
14 by the size of the visited probability distribution, as opposed to variance that measures only its
15 second moment. Shannon entropy and mutual information manage correlated fields, apply to any
16 data, and are insensitive to outliers as well as a change of units or scale. Finally, we use an example
17 featuring a highly skewed probability distribution (Arctic sea surface temperature) to show that
18 the new metric is robust even with a sharp nonlinear cutoff (the freezing point). We apply these
19 two metrics to quantify internal vs forced variability in (1) idealized Gaussian data, (2) an initial
20 condition ensemble of a realistic coastal ocean model, (3) the Community Earth System Model
21 large ensemble. Each case illustrates the advantages of the proposed metric over variance-based
22 metrics. Furthermore, in the coastal ocean model, the new metric is adapted to further quantify
23 the impact of different boundary forcing choices to aid in prioritizing model improvements—i.e.,
24 comparing different choices of extrinsic forcing. The metric can be applied to any ensemble of
25 models where intrinsic and extrinsic factors compete to control variability and can be applied
26 regardless of if the ensemble spread is Gaussian.

27 **1. Introduction**

28 In an ocean or climate model, it is pertinent to understand the cause of variability as it leads to
29 implications for predictability, prioritization of data collections for assimilation, and provides an
30 understanding of the dynamics at play in different regions. In a coastal model, variability can arise
31 from extrinsic factors such as wind forcing, solar and thermal forcing, tides, rivers, evaporation and
32 precipitation, or it can be due to internal chaos inherent to the governing fluid equations (Sane et al.
33 2020). In a climate model, modes of variability such as El Niño, the North Atlantic Oscillation,
34 or the Southern Annular Mode, can conceal or delay the emergence of attributable anthropogenic
35 climate change signals (Milinski et al. 2019). In high-resolution ocean models, internal chaos or
36 intrinsic variability can also be due to eddies (Leroux et al. 2018; Llovel et al. 2018). Accurately
37 quantifying the relative contribution of external and internal factors can help in elucidating the
38 causes responsible for the observed variability in models, help to identify key observable metrics,
39 and help quantify concepts such as the time of emergence of climate signals (Hawkins and Sutton
40 2012).

41 Numerous methods exist in the literature to quantify intrinsic and extrinsic variability using
42 models or observations (e.g., Frankcombe et al. 2015; Schurer et al. 2013; Liang et al. 2020).
43 Model ensembles—i.e., a set of simulations sharing the same forcing—naturally vary because each
44 ensemble member follows the same governing equation (with same external forcings) with identical
45 or similar parameterizations but differ due to intrinsic chaos. Two types of model ensembles are
46 common: initial condition ensembles (where the same model is used repeatedly with perturbed
47 initial conditions and intrinsic variability occurs from model’s chaotic sensitivity to the initial
48 conditions), and multi-model ensembles (where a variety of models differing in numerics and
49 parameterizations are used to simulate change under the same forcing—in this case “intrinsic”

50 variability also includes aspects of model formulations). Most of the discussion here will focus on
51 initial condition ensembles, but the metrics proposed can be adapted to both cases.

52 To help visualize variability, a generic output from an ocean or atmospheric model is shown in
53 Figure 1. Each color represents a different ensemble member and the black solid line is the mean of
54 those members. The black solid line is the signal mostly due to extrinsic factors (aside from finite
55 ensemble size limits) and the model spread (schematized by the double-headed magenta arrow in
56 Figure 1) can be considered due to intrinsic variability or internal chaos.

57 One method of quantifying intrinsic and extrinsic variability is to look at variances (second
58 central statistical moment) of model spread and model mean (Leroux et al. 2018; Llovel et al.
59 2018; Waldman et al. 2018; Yettella et al. 2018). Variance is sufficient to constrain all metrics of
60 variability about the mean when distributions are Gaussian and uncorrelated, but a single statistical
61 moment usually measures only part of a more complex variability. Many climatological variables
62 show non-Gaussian distributions (e.g., Franzke et al. 2020). In fact, generalized variance might
63 be misleading (Kowal 1971). Quantification of variability should be robust to or have known
64 dependence on changes in the units of the quantity or the scale (e.g., changing temperature from
65 Celsius to Fahrenheit or Kelvin). Comparative metrics, such as intrinsic vs. extrinsic variability
66 should not depend on these arbitrary choices of units at all.

67 Variability, in essence, is a function of the number of occurrences or frequency of occurrence
68 (or probability p_i as a fraction over all visited system states) after appropriately binning the data
69 (and thereby making the estimated and visited number of states finite rather than continuous).
70 Information entropy metrics measure variability by taking into account the probability distribution
71 of the binned data, drawing on the statistical mechanics concept of entropy in quantifying the
72 number of microstates that a variable can occupy. The fundamental measure in information theory
73 is the Shannon (1948) or information entropy which characterizes the amount of variability in a

74 variable (Carcassi et al. 2019). The mutual information, another metric introduced by Shannon
75 (1948), measures how much information one variable contains about another variable.

76 Information theory is applied in signal processing, computer science, statistical mechanics,
77 quantum mechanics, etc. It is used to quantify amount of information, disorder, freedom, or lack
78 of freedom (Brissaud 2005). The application of these abstract notions to geophysical flows can
79 have immense practical benefit when information entropy is interpreted as a measure of variability,
80 as entropy does not rely on any particular parametric probability distribution. Metrics from
81 information theory are not new to climate sciences. They have been introduced in predictability
82 studies, evaluating the skill of statistical models, as well as uncertainty studies (e.g., Leung and
83 North 1990; Schneider and Griffies 1999; Kleeman 2002; DelSole and Tippett 2007; Majda and
84 Gershgorin 2010; Stevenson et al. 2013).

85 In this article we bring well-established concepts of information theory to the particular applica-
86 tion of measuring intrinsic and extrinsic variability for ensemble model runs within atmospheric
87 and oceanographic modeling. Our metric uses Shannon entropy and mutual information together,
88 which we explain in section 2. We will apply our metric on three sets of data in section 3: 1.
89 Idealized Gaussian arrays with specified correlation 2. Ensemble output of a regional coastal
90 model (OSOM) (Sane et al. 2020) where output is mostly non Gaussian. 3. Community Earth
91 System Model Large ensemble (Deser et al. 2020). In section 4, we use individual components of
92 our metric to compare simulations from differing forcings applied to OSOM.

93 **2. Information theory**

94 We will introduce information theory concisely assuming the reader has no background
95 knowledge—this section contains standard definitions. Consider a probability distribution p_i ob-
96 tained after binning data into N bins. The user chooses the appropriate number of bins or bin

widths for the range of data. Shannon (1948) identified the average information content in N possible outcomes, equally or not equally likely, as given by:

$$H = \sum_{i=1}^N p_i \log_2(1/p_i), \quad (1)$$

where H is the Shannon entropy with unit of bits when log is base 2 and p_i is the probability of the i^{th} outcome. $\log_2(1/p_i)$ measures the information of the i^{th} outcome as proposed by Hartley (1928) and is also a measure of uncertainty (Cover 1999). The quantum $\log_2(1/p_i)$ measures the information gained by knowing that the i^{th} outcome has happened. Another interpretation for $\log_2(1/p_i)$ is that it measures reduction in uncertainty by knowing the i^{th} event has occurred, or that the variable falls in the i^{th} bin. The term information does not mean knowledge but it means the amount of uncertainty shown by a variable or the freedom that a variable has in falling into the i^{th} out of the N bins. Shannon (1948) found Equation 1 to provide the average information (or uncertainty) for all events in a record. For the entire set of elements, a highly probable event has less uncertainty associated with it and low probability event has high uncertainty associated with it. The prefactor p_i is thus used to weight the information over all possibilities. One way to interpret the need for the prefactor p_i is that in repeated experiments the events with higher probability will occur more often, hence they should contribute more to a quantification of variability than infrequent events.

Stone (2015) gives an intuitive way of understanding Shannon entropy using a binary tree. A binary tree is a tree chart which starts with 1 node and splits to two nodes at each node. At each node you can take a left or right turn to proceed and if there are say 3 levels in the tree, then 8 (i.e. 2^3) outcomes or possible destinations exist. If a binary tree has N equally probable outcomes then the set of instructions required to reach the correct destination is given by $h = (N)(1/N)\log_2(N) = \log_2(N)$. The *uncertainty* about reaching the correct destination will be

119 removed by providing $\log_2(N)$ *bits* of information. In other words, if entropy is h then 2^h states
120 are possible.

121 A second metric from Shannon (1948) which is also extensively used is now known as *mutual*
122 *information*. The mutual information between two signals x and y denoted by $I(X;Y)$ is (Cover
123 1999)

$$I = \sum_{j=1}^N \sum_{i=1}^N p_{ij} \log_2 \left(\frac{p_{ij}}{p_i p_j} \right), \quad (2)$$

124 where p_{ij} is joint probability of i^{th} outcome of x and j^{th} outcome of y . p_i and p_j is the marginal
125 probability of i^{th} and j^{th} outcome of x and y respectively. The addend within the summations can
126 be expanded to $p_{ij} (\log_2(p_{ij}) - \log_2(p_i) - \log_2(p_j))$. I can be interpreted as the extra information
127 in entropy of marginal distributions of x and y over joint distribution. Mutual information is
128 symmetric between x and y and is the measure of how much information they share. For example,
129 if the distributions are statistically independent, then $p_{ij} = p_i p_j$ and thus $I = 0$. If the two records
130 x and y are identical, then $p_{ij} = p_i = p_j$ and $I = H$. I is the average reduction in uncertainty in
131 x by knowing y or vice versa. It denotes how much information is transmitted between the two
132 variables.

133 In the context of ocean modeling (or in general climate modeling) entropy is used to measure
134 variability in a model output or available data. This is in tandem with interpretation of Shannon
135 entropy in physical sciences as given in Carcassi et al. (2019). When calculating the Shannon
136 entropy we are concerned about the possible states (for e.g. the various bins in a histogram) the
137 variable can (and does) go into while the variable value and its dimensions are of lesser importance.
138 Entropy metrics measure variability in *bits* (when logarithm is of base 2) and hence changing the
139 scale, for e.g. switching from Celsius to Fahrenheit for temperature, does not change the value
140 of variability (under equivalent binning). Mutual information and entropy are both dimensionally

agnostic. They are also not sensitive to outliers (due to the weighting prefactor) and can capture non linear interactions (Correa and Lindstrom 2013) and discontinuous or intermittent visited states. We will present the effect of correlation and outliers in section 3 a.

3. Proposed metric

Let $f(t)^i$ denote ensemble output as a function of time t and i is the index of ensemble members. For ocean model simulations, the output signal $f(t)^i$ consists of trend due to external forcing given by $g(t) = \langle f(t)^i \rangle$ where angled brackets represent mean over the ensemble members and the remaining part is the intrinsic variability due to chaos given by $\eta(t)^i$. Typically, g would be estimated as the mean of the ensemble, but in some cases might be a median or a known response to forcing—another advantage of the information theory approach is that it is not required that g be the mean or that $\eta(t)^i$ have no mean value. Thus they are related as $f(t)^i = g(t) + \eta(t)^i$, where $g(t)$ represents the forced component of the total variability and the model spread $\eta(t)^i$ can be attributed to the internal chaos (or model distinctions in multi-model ensembles) (e.g., Waldman et al. 2018; Llovel et al. 2018).

Usually we are limited in the number of ensemble members due to computational costs so we perform a *jugaad* with f and g in order to use all the members at once to sample widely and obtain an accurate probability distribution. Let f be a single array obtained by combining (concatenating) all members $f(t)^i, \forall i \in N$, where N is the total number of ensemble members. Let g be the array obtained after copying the model mean $g(t)$ N times. Hence if there are M data points in each member, f and g are of length $N \times M$ each. This makes them of equal length and the metric can be applied to the whole ensemble output. Using f and g , we propose the following metric γ

$$\gamma = 1 - \frac{I(f; g)}{H(f)}, \quad (3)$$

162 $H(f)$ is the Shannon entropy of f and $I(f;g)$ is mutual information between f and g . Note that
 163 g is due to extrinsic factors and f is obtained after internal chaos adds to g . $I(f;g)$ calculates
 164 the contribution of extrinsic signal g to the whole ensemble. $H(f)$ is the total variability in the
 165 ensemble output which is culmination of extrinsic and intrinsic factors. The metric γ gives the
 166 *ratio of intrinsic variability to total variability*.

167 $H(f)$ and $I(f;g)$ are related through conditional entropy by $H(f) = I(f;g) + H(f|g)$ (Cover
 168 1999). $H(f|g)$ is the conditional entropy¹, i.e., average uncertainty about the value of f after g
 169 is known. It is the uncertainty in f that is not attributed to g but is attributed to noise η . Hence
 170 $H(f) - I(f;g)$ determines variability due to intrinsic chaos.

171 Returning to the binary tree analogy, $I(f;g)$ would be the set of instructions sent by a source
 172 to reach one among $2^{H(f)}$ possible destinations in the presence of noise having $H(f|g)$ entropy.
 173 To capture the entropy in the noisy binary tree, to each of the $2^{I(f;g)}$ microstate possibilities noise
 174 $(2^{H(f|g)})$ gets multiplied and the relation becomes $2^{H(f)} = 2^{I(f;g)}2^{H(f|g)}$.

175 $I(f;g)$ takes into account any correlation or information shared between f and g . This is vital
 176 because even though the model spread η is being treated similarly to noise added to the mean
 177 signal, it might be that model spread depends on the mean signal. A simple example is if the model
 178 spread is relative (e.g., 10% of the mean signal), rather than absolute (e.g., 2 units), then there
 179 is information about the model spread contained in the mean signal. This situation is sometimes
 180 called multiplicative noise in contrast to additive noise. The nonlinear and chaotic nature of fluid
 181 mechanics often leads the mean flow to amplify the chaotic signal (e.g., eddies) and thereby result
 182 in altered variability statistics. When $f \rightarrow g$, then $I(f;g) \rightarrow H(f) = H(g)$ from (2). This makes
 183 $\gamma = 0$ when there is no intrinsic chaos. When intrinsic chaos fully dominates the ensemble output,

¹Conditional entropy $H(X|Y)$ is defined by $H(X|Y) = \sum p(x|y)\log_2 p(x|y)$ (Cover 1999). It is not necessary to calculate conditional entropy

to arrive at γ , but understanding is aided by the expected relation between entropy and mutual information.

184 i.e. f and g are fully decorrelated, then $I(f; g) = 0$ yielding $\gamma = 1$. We see that γ satisfies the
185 extremes of zero noise as well as total chaos.

186 Another analogy for a climate system component is a noisy channel as given in Leung and
187 North (1990), where the governing equations of ocean (atmosphere) modeling are represented
188 by a channel. The extrinsic forcings are inputs to the channel, the intrinsic chaos is the noise
189 created because of channel's inherent mechanisms while the outputs are the ensemble members.
190 A noiseless channel will give γ as zero and completely noisy channel where output is independent
191 of input will yield γ as 1.

192 A seemingly enticing and simpler alternative is $\gamma = 1 - \frac{H(g)}{H(f)}$, i.e. just the difference between
193 ensemble entropy and mean entropy as a ratio with the ensemble entropy. However, this formulation
194 is incorrect because $H(g)$ does not quantify the contribution of extrinsic factors to the variability
195 in the ensemble, it only quantifies the variability of the mean. Relatedly, $H(f) - H(g)$ does not
196 correctly manage mutual information between the ensemble members and their mean in estimating
197 the intrinsic variability.

198 *a. Idealized Gaussian Arrays*

199 We test our metric, γ , from (3) on synthetic data consisting of idealized arrays of Gaussian
200 data: $\mathcal{N}(0, 1)$. For a normal Gaussian distribution Shannon entropy depends² only on the standard
201 deviation σ i.e. $H = \log_2(2\pi e\sigma^2)$. The variability in a Gaussian distribution can be increased
202 or decreased by changing its standard deviation. To illustrate, we compare our metric γ with the

² $H = \log_2 2\pi e\sigma^2$ is the Shannon entropy of a Gaussian distribution when probability density is continuous with σ as standard deviation. The Shannon entropy of a discrete probability distribution differs, which is inconsequential here but the reader is encouraged to read Jaynes (1962). Consistently here discretely sampled and binned probability distributions are obtained directly from data without any further parameterization.

203 traditional metric of intrinsic versus extrinsic variability γ_{std} :

$$\gamma_{std} = \frac{\sigma_i}{(\sigma_i^2 + \sigma_g^2)^{1/2}}, \quad (4)$$

204 where subscript *std* stands for standard deviation, σ_i is the temporal mean of standard deviation
205 of model spread, and σ_g is the temporal standard deviation of the model mean (not a function of
206 time). Our goal is to compare γ and γ_{std} . We set out our numerical experiment as follows: we
207 create 10 arrays, each having 10,000 elements drawn from a Gaussian distribution. Any two arrays
208 from those 10 have a prescribed linear Pearson correlation coefficient from 0 to 1.

209 Thus, the 10 arrays covary linearly with a specified correlation coefficient. These 10 arrays
210 represent each of 10 ensemble members from climate simulations. The mean of 10 members
211 gives us the synthetic forced variability signal as would be determined from the model output;
212 averaging over the 10 ensemble members reduces the contribution from uncorrelated variability
213 and reaffirms the covarying component into the forced variability. As the individual members
214 have same correlation coefficient between them, every member also has a constant correlation
215 (different from correlation between any two ensemble members) with the mean member. We apply
216 γ and γ_{std} on this synthetic ensemble by varying the prescribed correlation coefficient from 0 to 1.
217 Figure 2 shows that as expected both metrics increase as the correlation decreases, i.e., as internal
218 variability dominates forced. Both metrics behave similarly when correlation decreases, i.e. noise
219 increases but γ is more sensitive as correlation tends to 1. This distinction is due to the logarithmic
220 nature of Shannon entropy for Gaussian distributions—in essence, information measured in bits
221 is not proportional to distance measured between distributions in terms of summed variance—in
222 the examples following the consequences of this distinction will become clearer. Critically both
223 functions are monotonic with correlation, however so relative comparisons (more intrinsic fraction
224 in this region vs. that region) are preserved.

A second related experiment was derived from the first is also shown in Figure 2: adding outliers outside of the Gaussian distribution. 50 out of 10000 elements of each individual member were artificially corrupted (values were set to a constant value of 5) to test the sensitivity of both the metrics. Figure 2 shows that γ is insensitive to outliers while γ_{std} is not. γ is not sensitive because outliers occur less frequently and hence do not affect the probability distribution much, especially with the prefactor in (1) and (2). Hence information theory metrics are robust in comparison to using standard deviation (or variance). If the outliers (extreme events) occur at higher frequencies, information metrics will naturally start sensing them even if they are discontinuous from the typical conditions (e.g., multimodal distributions). Increasing the number of ensemble members does not change the result qualitatively for both the experiments.

b. Regional coastal model output

In this section we show the results of applying γ and γ_{std} on realistic simulation data from the Ocean State Ocean Model, hereafter OSOM (Sane et al. 2020). OSOM uses the Regional Ocean Modeling System (ROMS) (Shchepetkin and McWilliams 2005) to model Narragansett Bay and surrounding coastal oceanic regions and waterways. OSOM’s primary purpose is for understanding and predictive modeling and forecasting of the estuarine state and climate of this Rhode Island body. Sane et al. (2020) gives more details about the model.

Using OSOM, an ensemble of simulations have been performed using perturbed initial conditions for the months July - August of 2006. The data during the initial predictability window (20 days) has been ignored and the rest has been used to look at variability within the “climate projection” of the model beyond when forecasts sensitive to initial conditions are possible (see the related application of information theory to assess predictability in Sane et al. 2020). We examine whether the modeled temperature and salinity at each grid point follow normal distributions by evaluating

the skewness and kurtosis of the model mean at each grid point. Figure 3 shows skewness and kurtosis for sea surface salinity and temperature as well as bottom salinity and temperature for the Narragansett Bay region. The horizontal axis shows skewness and excess kurtosis, which are the third and fourth statistical moments respectively, normalized by powers of the standard deviation to dimensionless ratio and in the case of excess kurtosis a constant value of 3 is subtracted. For Gaussian distributions, skewness and excess kurtosis both should be close to zero. The vertical axis denotes the number of occurrences at a grid point. Observe that the majority of grid point values are away from zero. These variables are considerably non-Gaussian in OSOM. Thus, (4) is at a disadvantage, because the prevalence of higher statistical moments implies that the variance does not contain a complete description of the variability. The information theory metric (3) is suitable for such data as it takes into account higher moments and does not rely on Gaussian distributions.

Figure 4 shows the ratio of intrinsic variability to total variability applied on every grid point for OSOM. γ is displayed on left whereas γ_{std} is shown on right for comparison. The features highlighted by both metrics are qualitatively different. The contribution of intrinsic chaos to total variability is more uniform using the γ metric than using γ_{std} . The intrinsic chaos displayed using γ_{std} might be misleading because the probability distributions are non-Gaussian. Furthermore, where the γ metric highlights internal variability tends to agree in similar dynamical locations—all river mouths show high surface salinity intrinsic variability, while surface temperature intrinsic variability is higher in more open regions of the Bay where eddies form from topography intermittently. Also note that the ranges are quite different between γ and γ_{std} , but this is to be expected from the different rate of increase with correlation seen in Figure 2.

269 *c. Community Earth System Model large ensemble*

270 A complementary experiment was performed by using γ to evaluate internal vs. forced variability
271 in the global climate simulation output for climate change scenario RCP8.5 using the (randomly
272 selected among the models compared) GFDL CM3 model (Deser et al. 2020). Variability of sea
273 surface temperature (Figures 5) as well as sea surface salinity (6) were estimated using both γ and
274 γ_{std} (upper left and upper right). The skewness and excess kurtosis of the model mean were also
275 plotted to find the deviation of variables away from Gaussian distributions (lower). Regions shaded
276 in purple have low values of excess kurtosis and skewness and might be considered Gaussian.

277 Note in particular the Arctic sea surface temperatures, which have a highly skewed and excessive
278 kurtosis distribution due to the freezing point of seawater. The standard metric deems this region
279 to be among the most intrinsically variable in the world, while the information theory metric has
280 it as a low intrinsic variability region. It is clear that a Gaussian metric should not be applied to
281 this region, and the inference is opposite using the two metrics. In the equatorial Pacific where
282 Gaussian statistics are more reliable, the two metrics agree that internal variability is high.

283 A less drastic failure occurs from the modest excess kurtosis in extra-tropical temperatures and
284 in a few isolated regions in surface salinity. These regions are also non-Gaussian, but also are
285 not heavily skewed (i.e., they are more long-tailed and intermittent than Gaussian). These regions
286 differ in relative estimation of intrinsic versus total variability. It is also the case that the γ metric
287 is closer to one in most regions than γ_{std} , which is to be expected when the correlation coefficients
288 are low from Figure 2.

289 **4. Quantifying extrinsic factors**

290 In section 3 we proposed our metric and showcased its applications. Here instead of us-
291 ing the new metric γ , we use its components: Shannon entropy and mutual information in-

dividually to compare variability between different simulations. Quantifying differences be-
cause of changes in the extrinsic forcings may be required for coastal applications where sys-
tems vary predominantly due to external forcings (note that intrinsic variability is less than
half by both metrics in Figure 4). For these forcing significance experiments, OSOM was
run after modifying the external forcings (Table 1). OSOM is forced by tides, river runoff,
atmospheric winds and air-sea fluxes, etc. (Full details of the model can be found in Sane
et al. 2020). For this comparison, we quantify the effects of altering forcing on 4 modeled
variables: sea surface temperature and salinity, and bottom temperature and salinity. Four al-
tered forcing sets were utilized, beyond set (1) Full set of atmospheric forcings using the North
American Mesoscale (NAM) analyses, a data-assimilating, high resolution (12 km) meteorologi-
cal simulation ([https://www.ncei.noaa.gov/data/north-american-mesoscale-model/](https://www.ncei.noaa.gov/data/north-american-mesoscale-model/access/historical/analysis)
[access/historical/analysis](https://www.ncei.noaa.gov/data/north-american-mesoscale-model/access/historical/analysis)) denoted as FF. FF stands for full forcing. (2) Full set of at-
mospheric forcings but using the Northeast Coastal Ocean Forecast System (NECOFS) winds
(Beardsley and Chen 2014) instead of NAM, denoted as NECOFS. (3) River flows are replaced
with their monthly-averaged flow, other forcing as in FF (4) River flows set to zero, other forcing
as in FF. (5) Wind forcing set to zero, other forcing as in FF. These forcings have been tabulated in
Table 1. The aim is to quantify the effect on total variability by removing or altering one of many
processes which might contribute.

We show results in Figures 7 and 8. Entropy has been plotted with respect to time. In Figure 7,
Shannon entropy is plotted for spatial quantities. For example, for surface salinity, all the surface
values have been considered to find Shannon entropy. Figure 8 displays mutual information.
Mutual information has been calculated in the following way: for surface temperature comparison
between FF and MR, surface grid point values for FF simulation were ordered and put in a single
row array. We compared this array with a similar array made of surface grid point values of MR

surface temperature. We consistently followed the same method for all the forcings and variables. It is user's choice to choose the type of domain, here we have chosen the same domain of OSOM as shown in figure 4. If Shannon entropy is more or less equal for two forcings, it implies they similarly affect variability. Mutual information should be compared for two pairs of forcings. Greater mutual information implies the two pairs share more *bits* of information, suggesting one of the forcing in that pair can be replaced with the other without significantly affecting variability.

5. Discussion

Our numerical experiments performed using γ on idealized Gaussian arrays show that γ is monotonic and decreases as the linear Pearson correlation coefficient increases. Thus aside from the qualitative differences the new metric finds when the data are non-Gaussian, the ranges of intrinsic versus total variability are quite different between γ and γ_{std} . This is to be expected from the different rates of increase with correlation seen in Figure 2. Approximately, the traditional metric falls approximately linearly as the correlation coefficient increases, so that a correlation coefficient of 0.5 gives a γ_{std} just above 0.5. The new metric γ agrees with γ_{std} that correlation of 0 implies $\gamma = 1$, and correlation of 1 implies $\gamma = 0$, but correlation of 0.5 is closer to $\gamma = 0.9$. Only very near correlation coefficients of 1 does γ fall below 0.5. If roughly linear dependence on correlation coefficient is desired, γ can be raised to a power— γ^3 resembles γ_{std} and γ^6 resembles the correlation coefficient. These higher powers do not lose the ability to apply to non-Gaussian data nor become non-monotonic, but they will lose their interpretation as a ratio of bits of information entropy, and instead reflect ratios of bits cubed of information entropy, etc. An alternative is to take γ_{std} raised to a different power: $\gamma_{std}^{1/3}$ is roughly similar to γ .

As can be seen in Figures 4, 5, and 6, information theory metrics show different patterns as compared to variance. Information theory metrics, especially mutual information, account for

339 *all* non-linear shared information between the ensemble members and the mean including linear
340 correlation, and this is one reason for the differences. We have argued that non-Gaussian statistics
341 are another (which is not wholly independent of non-linear shared relationships). There are likely
342 other aspects of differences between these metrics, but the management of these two expected
343 aspects of geophysical fluids—nonlinear relationships and non-Gaussian distributions—justify the
344 introduction of the new metric.

345 For the regional coastal model OSOM, forcings differ as to how they affect different variables.
346 As might be expected, river runoff is more important for salinity than for temperature. How-
347 ever, replacing rivers with the monthly-mean river flow gives nearly the same result (in terms of
348 variability) as fully time-varying rivers. For the duration considered (July-August), averaging the
349 river runoff gives similar effect for salinity as compared to giving the observed river runoff in the
350 simulations, see Figure 7. Temperature is less sensitive to any of the forcing alterations, because
351 although temperature and salinity are passive tracers they have different sources and sinks. Switch-
352 ing the wind product from NAM to NECOFS does not have any significant effect on the sources
353 or sinks of temperature or salinity, but switching the wind off definitely affects the parameters by
354 eliminating wind-driven mixing altogether. Figure 8 shows that zero wind (ZW) simulations are
355 markedly different than the rest in terms of *mutual information* (i.e., they do not covary), although
356 very similar in terms of amount of variability (Shannon entropy, Figure 7), because even without
357 winds tides, fluxes, and rivers still vary. The zero river case tends to eliminate both variability and
358 mutual information (ZR). Please note that our simulations are for July-August, and results might
359 be different for different season.

360 If we were to prioritize improvements based on Shannon entropy and mutual information, note
361 that the two highest mutual information cases are where NAM is substituted with NECOFS and
362 where mean rivers are substituted for varying rivers. The first observation is important from a

forecast perspective, because it means that we can not easily tell the difference between different wind products, although something rather than zero winds should be used if the estuary is forecast out to the full 20 day predictability range (weather forecasts only good out about 7 days in this region). Similarly, knowing that substituting the mean of the rivers for the fully varying rivers has little impact implies that rivers can be fixed in time for forecasts beyond where they might be predicted based on expected weather and precipitation. Finally, despite the fact that Narragansett Bay is a dominantly tidally-mixed estuary, among the sources of overall variability (i.e., sources of information entropy) considered, preserving an inflow of fresh water is key, even though that inflow can be steady. Winds do not appreciably increase information entropy of the Bay, but they are an important source of forced co-variation, and so are important for predictions but do not raise the overall level of variability.

6. Conclusion

We have proposed an information theory metric to determine contribution of intrinsic chaos and external variability to total variability in ensemble model simulations. Our metric uses Shannon entropy and mutual information and has several advantages over using only standard deviation (or variance). We have applied our metric on idealized Gaussian arrays as well as realistic coastal ocean and global climate model. We conclude that:

1. The new information theory metric is more reliable when outliers are present, because outliers get assigned less probability and because Gaussian distributions have a difficult time approximating long-tailed (i.e., outlier prone) distributions.
2. The new information theory metric is more reliable when variability is non-Gaussian because it is based on non-parametric measures of the probability distributions.

3. The new information theory metric varies monotonically with ensemble member to ensemble mean correlation, but is quantified in fraction of bits required to capture internal variability versus bits required to capture of total variability.
4. The use of the information theory metric in a coastal ocean model ensemble and a climate model ensemble qualitatively changes the focus to regions that were previously erroneously labeled as having high or low internal variability.
5. In this case, the coastal ensemble had a much smaller intrinsic (chaotic) proportion of its total variability in comparison to the climate ensemble had more intrinsic (weather, climate oscillations, etc.) as a proportion of its total. Importantly, the resolution of the models helps determine the proportion of intrinsic variability, so such comparisons are model-specific: a higher resolution coastal model might well have a larger intrinsic fraction than a coarser climate model.

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Data availability statement. All the data and the codes used to plot results can be downloaded via Brown University's digital archive DOI: [urlplaceholder](#).

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Forcing Set	Wind forcing	River forcing
FF	NAM	As Observed
NECOFS	NECOFS	As Observed
MR	NAM	Time-averaged
ZR	NAM	Zero river input
ZW	Zero winds	As Observed

TABLE 1. Different types of forcing combinations employed to test their effect on variability. FF stands for full forcing: winds, tides, rivers, etc. For more details see Sane et al. (2020). MR: mean rives; ZR: zero rivers; ZW: zero wind.

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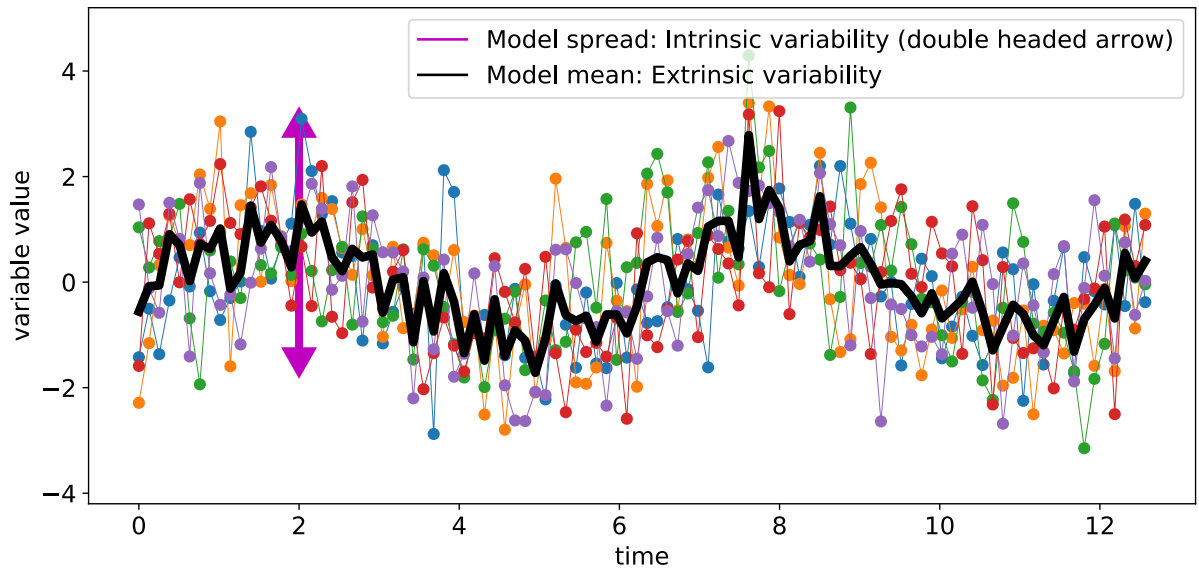


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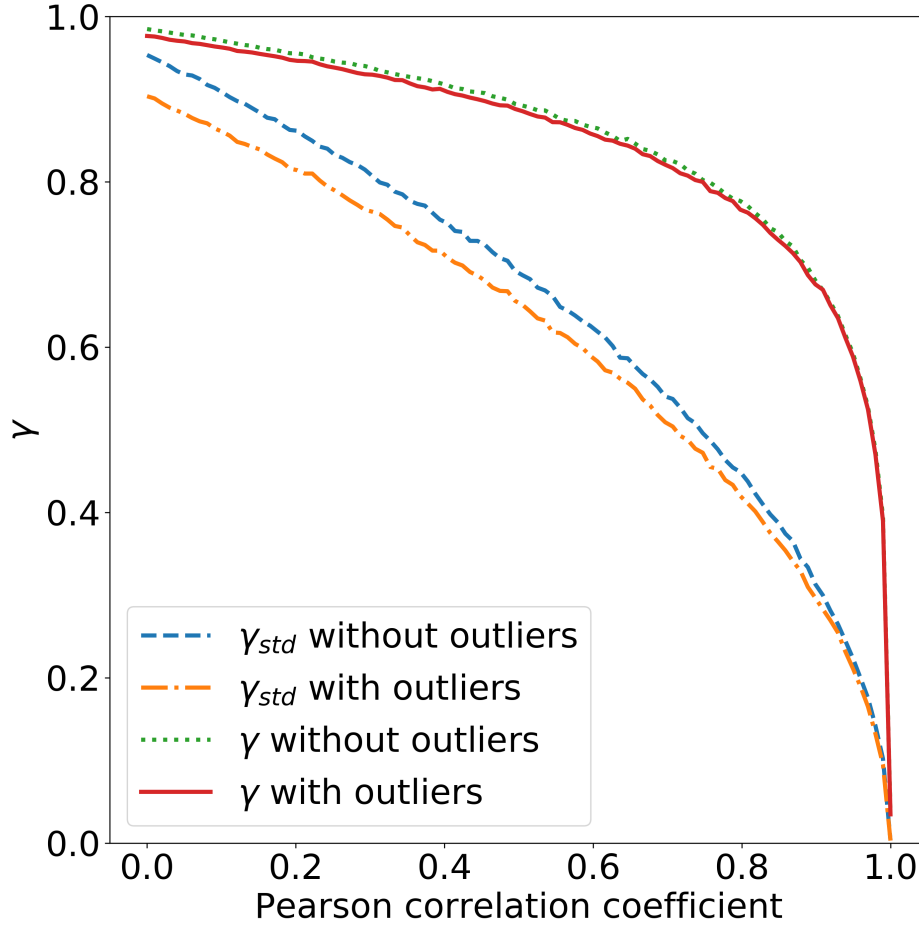


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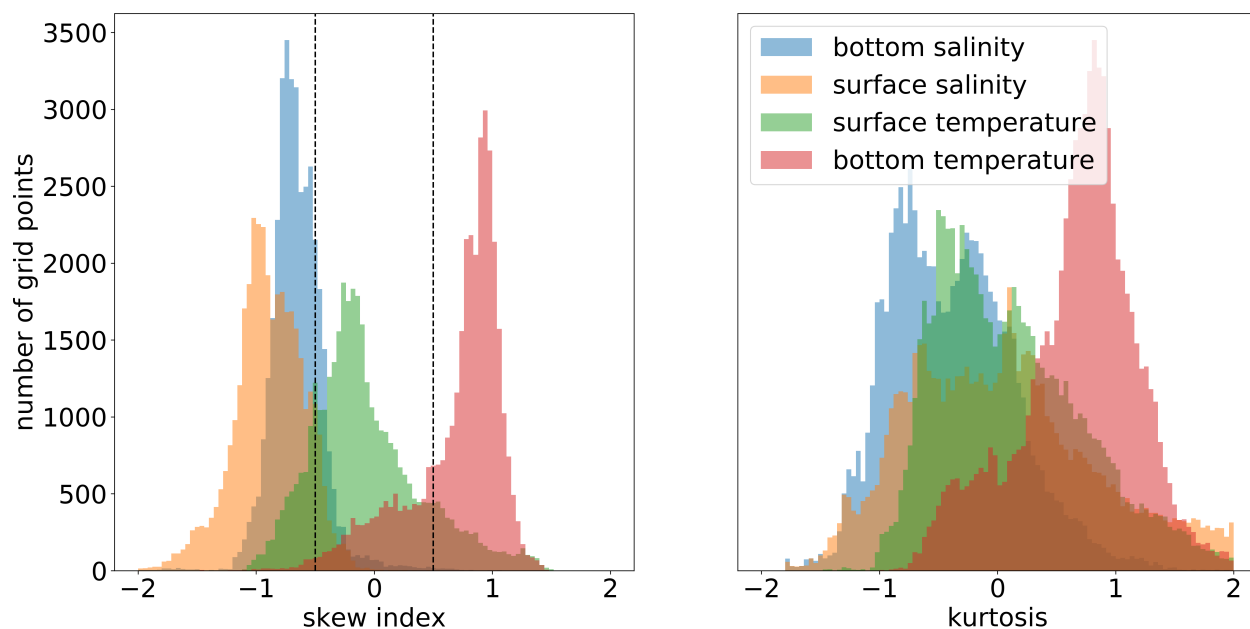


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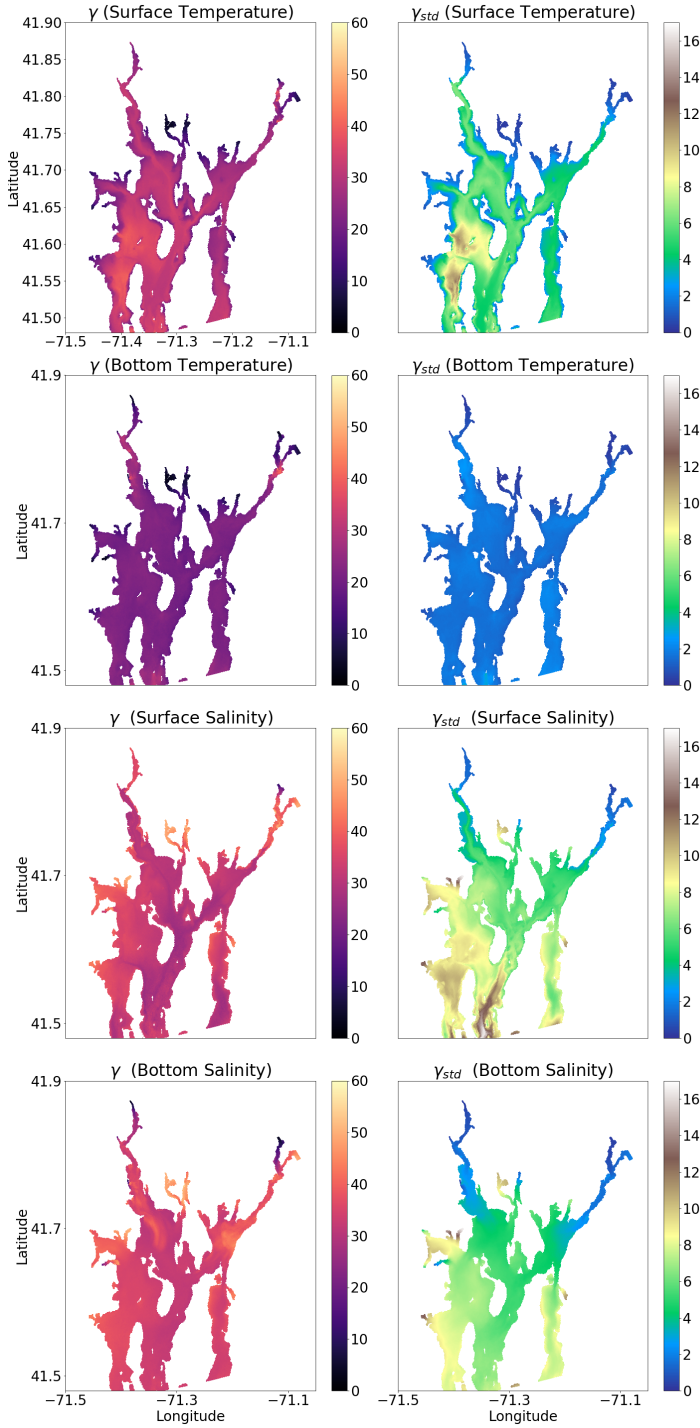
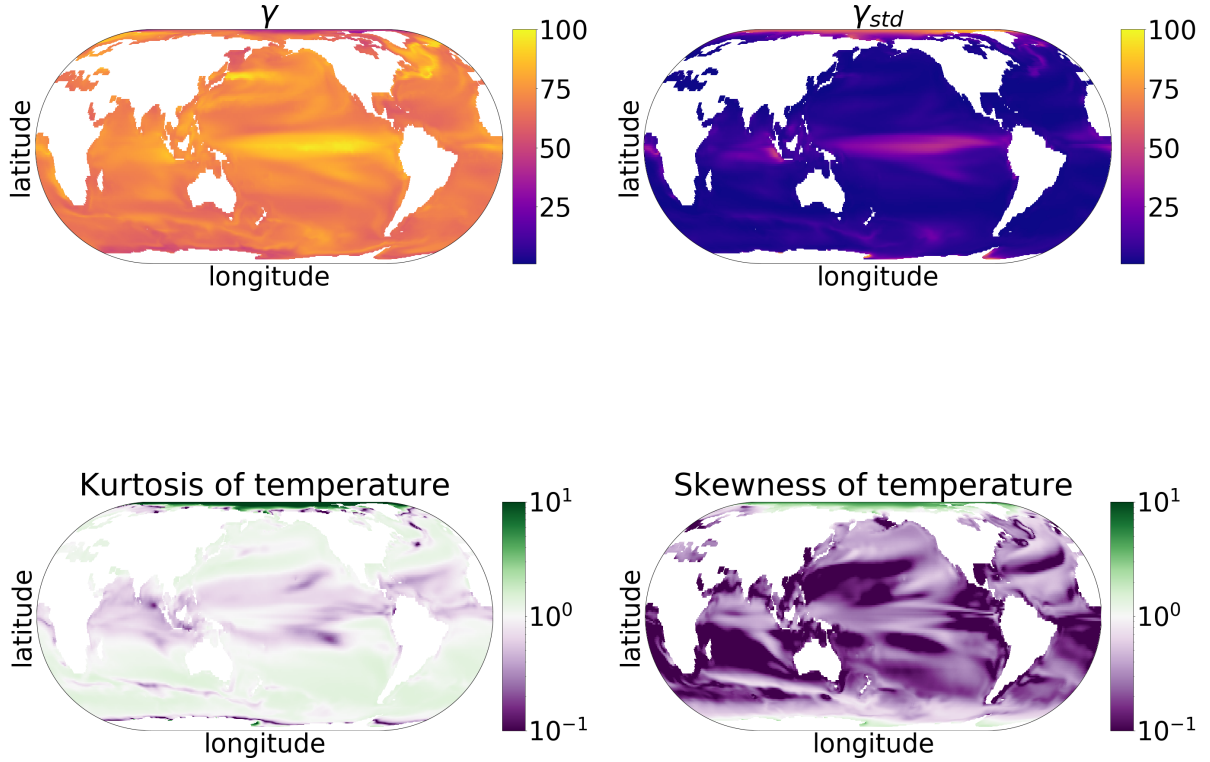


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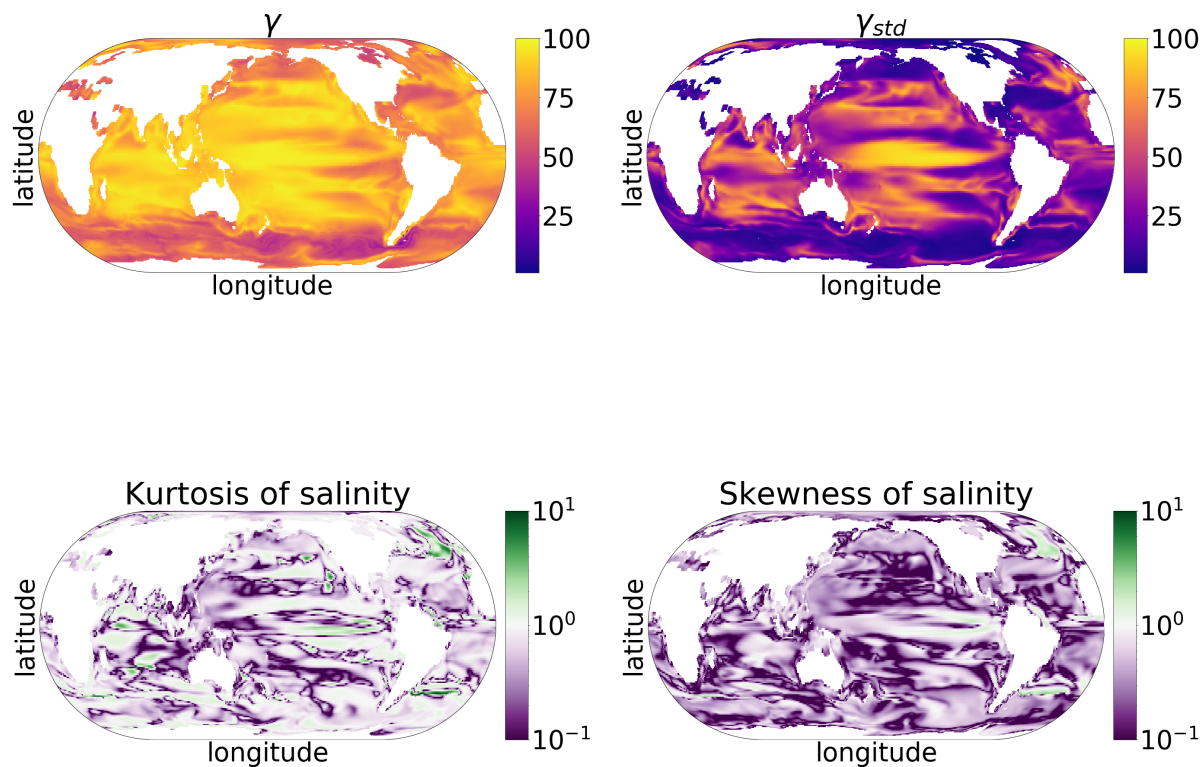


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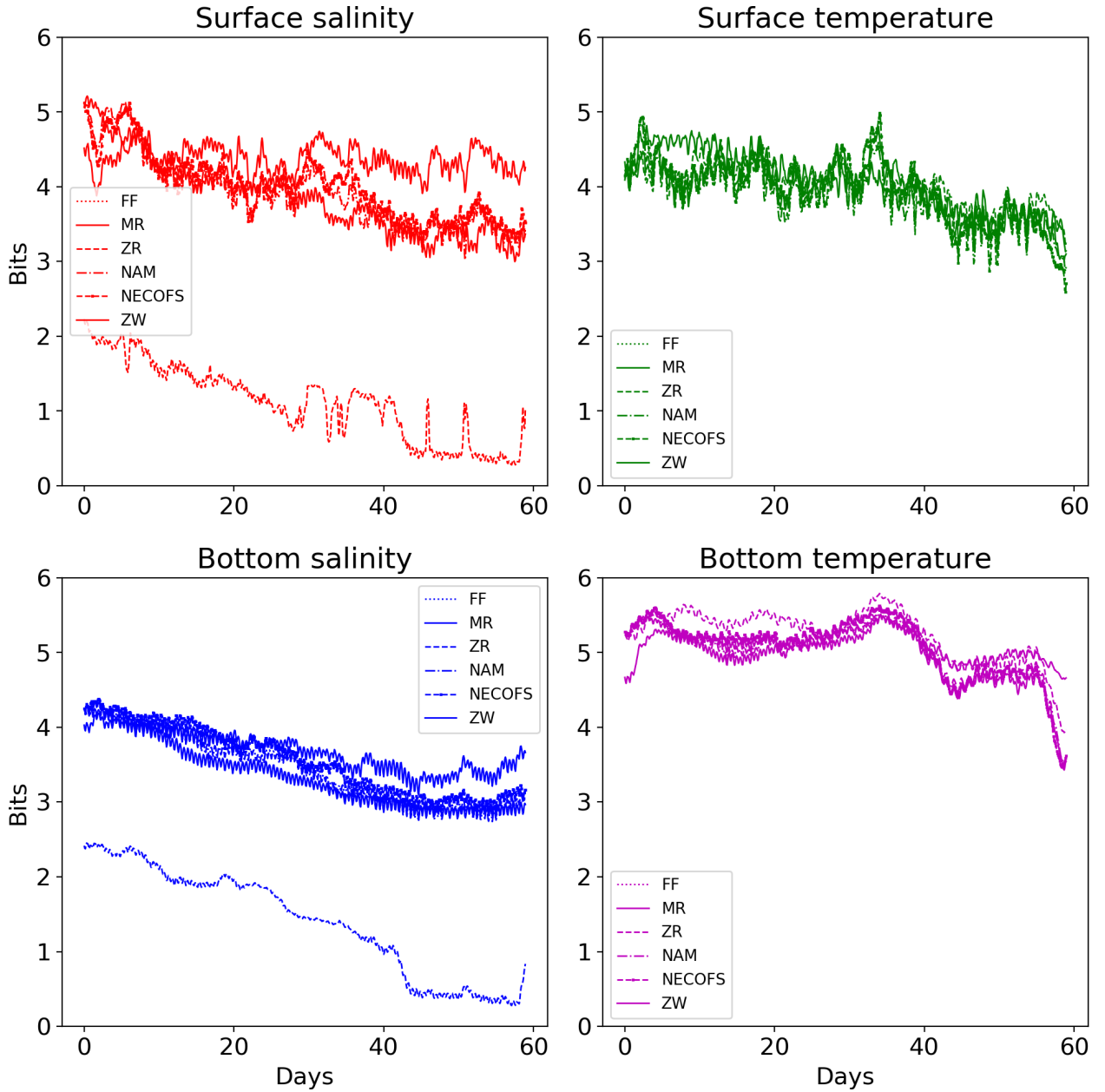


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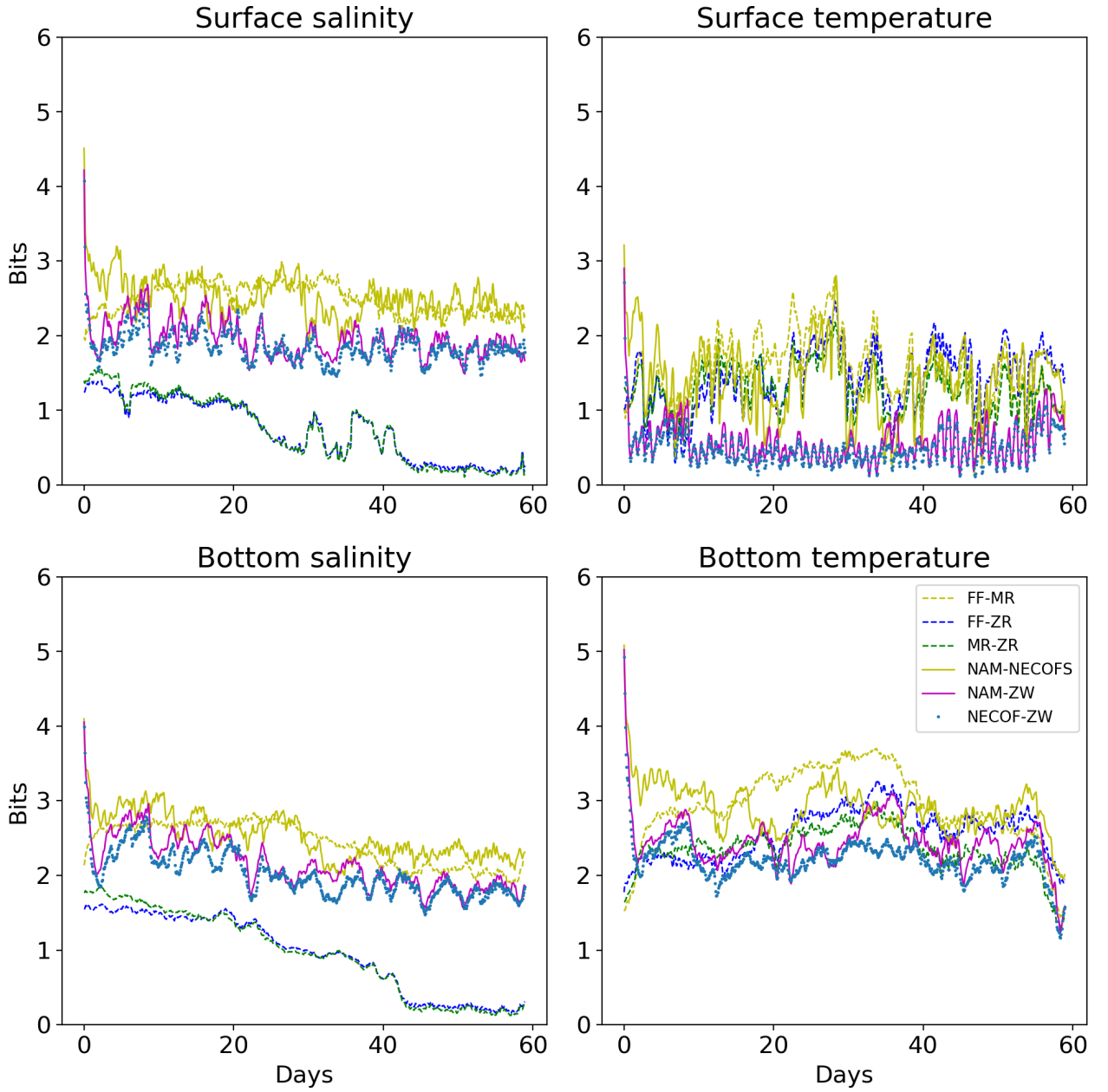


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