A joint weighted power detector for Willie in two-hop covert communication system

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In this letter, a joint weighted power detector (JWPD) based on maximum a posterior probability (MAP) criteria is proposed for Willie aiming at two-hop covert communication scenario, which is a near optimal detector. Instead of only supervising one single phase, Willie combines the observations of two phases to make joint decision in the proposed scheme. The proposed scheme achieves lower probability of detection error (PDE) than the existing single-phase-detector (SPD) scheme and adding-power-directly-detector (APDD) scheme due to sufficient utilization of the two-phases observations. Numerical results demonstrate the benefit of our proposed scheme.

Introduction: Covert communication, also known as Low probability of detection (LPD) communication, is regarded as the first barrier to defend privacy security in wireless networks. The information-theoretic fundamental limits in additive white Gaussian noise (AWGN) channels is derived in [1], which also illustrates the benefits can be achieved by introducing uncertainty at Willie [2]. As reported in [3], the transmitted signal is transformed into Gaussian noise to confuse Willie's detector. In [4], an optimal detector for Willie under AWGN and block fading channels is proposed in single-hop system, [4] also reveals that covert communication can be achieved by varying the transmit power at Alice. Moreover, the randomness of fading channel and nodes like Jammer and Relay can be also treated as interference for Willie's detection [5, 6]. The authors in [6, 7] employ the single-phase-detection (SPD) scheme to analyze the covertness of two-hop scenario that Willie detects the two phases independently. Nevertheless, the joint power detector for two-hop system has rarely been studied.

In this letter, to make Willie's detection more efficient and practical, a joint weighted power detector (JWPD) based on two-phase observation is proposed. Different form the adding-power-directly-detector (APDD) scheme that adding Willie's total received power in two phases straightforwardly, the proposed JWPD scheme combines two-phase observations efficiently based on maximum a posterior probability (MAP) criterion to achieve a near optimal performance. Numerical results coincide with the analysis and show that JWPD scheme achieves the lowest PDE compared to SPD and APDD schemes. Therefore, the covertness of two-hop system can be further analyzed with tighter covert constraint with the proposed JWPD scheme.

System Model: Considering a typical two-hop covert communication system consisting of a legitimate transmitter Alice (A), a legitimate receiver Bob (B), a friendly Jammer (J) and a cooperative decode-and-forward (DF) Relay (R) assisting Alice for covert communication. A malicious Willie (W) is monitoring whether covert communication between A and B exists. Let us assume that the direct link between A and B does not exist due to the environment factors. It is assumed that each nodes in the system are equipped with single antenna and work in half-duplex (HF) mode. As shown in Fig. 2, a quasi-static Rayleigh block fading channel environment is considered, where the channel coefficients between X and Y are constants within one slot but varies independently in different slots. $h_{xy}(x, y \in \{a, r, j, w\})$ denotes the channel coefficient between X and Y $(X, Y \in \{A, R, J, W\})$, which is independent, circularly symmetric complex Gaussian random variable with distribution $h_{xy} \sim CN(0, 1), (xy \in \{aw, rw, jw\})$. $d_{x,y}$ denotes the distance between X and Y.

Alice and Relay employ independent generated Gaussian codebook to map their message to $\boldsymbol{x_a}$ and $\boldsymbol{x_r}$ i.e., $x_l(i) \sim \mathcal{CN}(0, 1)(l = a, r)$, respectively. Then Alice transmits $\boldsymbol{x_a}$ and Relay forwards $\boldsymbol{x_r}$ with power P_a and P_r, respectively. Assuming that Alice randomly transmits the covert message with prior probabilities $\theta = 0.5$. The covert communication process can be separated into two phases with the same time length in one slot. In phase 1, Alice transmits covert signal to Relay, then the Relay decodes and forwards covert signal to Bob simultaneously in phase 2. In this work, Jammer first maps his/her Jamming symbols $\boldsymbol{x_j}$ with Gaussian codebook, i.e., $x_j(i) \sim \mathcal{CN}(0, 1)$ and transmits them in each complete slot with Jamming power P_j that follows from uniform distribution $P_j \sim U[0, P_{jmax}]$, where P_j keeps invariant within one slot but changes independently in different slots similar to h_{xy}. It is assumed that Willie have the knowledge of the value h_{aw} , h_{rw} and h_{jw} in each slot excluding the value of P_j. Then the received signal at Willie in Phase 1 and 2 are given as follows:

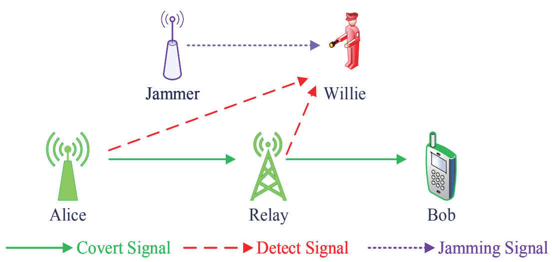

Fig 1 *Typical System link model of two-hop covert communication.*

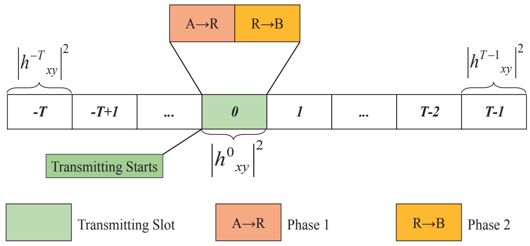

Fig 2 *Time slot diagram.*

***Phase*1 :**

$$z_{w1}(i) = \begin{cases} \sqrt{P_a} h_{aw} d_{a,w}^{-\alpha/2} x_a(i) + \\ \sqrt{P_j} h_{jw} d_{j,w}^{-\alpha/2} x_j(i) + n_w(i), H_1 \\ \sqrt{P_j} h_{jw} d_{j,w}^{-\alpha/2} x_j(i) + n_w(i), H_0 \end{cases} \tag{1}$$

***Phase*2 :**

$$z_{w2}(i) = \begin{cases} \sqrt{P_r} h_{rw} d_{r,w}^{-\alpha/2} x_r(i) + \\ \sqrt{P_j} h_{jw} d_{j,w}^{-\alpha/2} x_j(i) + n_w(i), H_1 \\ \sqrt{P_j} h_{jw} d_{j,w}^{-\alpha/2} x_j(i) + n_w(i), H_0 \end{cases} \tag{2}$$

where $i = 1, 2, \ldots, N$ and $\boldsymbol{z_{wt}}$ denotes Willie's received signal in phase $t\,(t = 1, 2)$, N is the codeword length; $\boldsymbol{n_w}$ represents the received noise at Willie following $n_w(i) \sim \mathcal{CN}(0, \sigma_w^2)$; Moreover, $\alpha(\alpha \geq 2)$ is the path loss exponent; H_0 represents the null hypothesis that indicates non-existence of covert communication, while H_1 denotes the alternative hypothesis that covert communication is in progress.

Optimal detector at Willie:

Theorem 1. *Based on the system model discussed above, the optimal detector at Willie is a joint weighted power detector (JWPD).*

$$T(\mathbf{z}) = \frac{\sigma_a^2}{\sigma_a^2 + \sigma^2} \sigma_1^2 + \frac{\sigma_r^2}{\sigma_r^2 + \sigma^2} \sigma_2^2 \underset{D_0}{\overset{D_1}{\gtrless}} \gamma.$$

where $\sigma_a^2 = E\left[P_a |h_{aw}|^2 d_{a,w}^{-\alpha}\right]$, $\sigma_r^2 = E\left[P_r |h_{rw}|^2 d_{r,w}^{-\alpha}\right]$ is the expectation of Willie's received signal power from Alice and Relay under H_1, respectively. $\sigma^2 = E\left[P_{j\,\max} |h_{jw}|^2 d_{j,w}^{-\alpha} + \sigma_w^2\right]$ is the expected power of Willie's received signal under H_0. Moreover, σ_1^2 and

σ_2^2 are power of Willie's observation in Phase 1 and Phase 2, respectively:

$$\sigma_1^2 = \begin{cases} P_a |h_{aw}|^2 d_{a,w}^{-\alpha} + P_j \left|h_{jw}\right|^2 d_{j,w}^{-\alpha} + \sigma_w^2, & H_1 \\ P_j \left|h_{jw}\right|^2 d_{j,w}^{-\alpha} + \sigma_w^2, & H_0 \end{cases}$$
$$\sigma_2^2 = \begin{cases} P_r |h_{rw}|^2 d_{r,w}^{-\alpha} + P_j \left|h_{jw}\right|^2 d_{j,w}^{-\alpha} + \sigma_w^2, & H_1 \\ P_j \left|h_{jw}\right|^2 d_{j,w}^{-\alpha} + \sigma_w^2, & H_0 \end{cases},$$

Furthermore, D_0 represents covert communication can not be detected by Willie, whereas D_1 indicates the covert communication is found by Willie.

Proof: Willies optimal detector is a threshold test on the received power in a slot [4]. Let us consider Willie's joint observation vector $\mathbf{z}$ in two Phases:

$$\mathbf{z} = (\mathbf{z_{w1}}, \mathbf{z_{w2}})^{\mathbf{T}}, \tag{3}$$

where $\mathbf{z}$ is the Two-dimensional Gaussian Process with zero mean and variance matrix $\mathbf{C_z}$, i.e., $\mathbf{z} \sim \begin{cases} \mathcal{CN}\left(0, \left(\mathbf{C_z} + \sigma^2\mathbf{I}\right), \right. & H_1 \\ \mathcal{CN}\left(0, \sigma^2\mathbf{I}\right), & H_0 \end{cases}$.

$$\begin{aligned} \mathbf{z_{w1}} &= \left(z_{w1}(1), z_{w1}(2), \ldots, z_{w1}(i), \ldots, z_{w1}(N)\right), \\ \mathbf{z_{w2}} &= \left(z_{w2}(1), z_{w2}(2), \ldots, z_{w2}(i), \ldots, z_{w2}(N)\right), \end{aligned} \tag{4}$$

combining (1), (2) and (4), $\mathbf{z}$ under the condition H_1 and H_0 can be given by

$$\mathbf{z}(i) \begin{cases} \begin{bmatrix} z_{w1}(i) \\ z_{w2}(i) \end{bmatrix} = \begin{bmatrix} \sqrt{P_a} h_{aw} d_{a,w}^{-\alpha/2} x_a(i) + \sqrt{P_j} h_{jw} d_{j,w}^{-\alpha/2} x_j(i) + n_w(i) \\ \sqrt{P_r} h_{rw} d_{r,w}^{-\alpha/2} x_r(i) + \sqrt{P_j} h_{jw} d_{j,w}^{-\alpha/2} x_j(i) + n_w(i) \end{bmatrix}, & H_1 \\ \begin{bmatrix} z_{w1}(i) \\ z_{w2}(i) \end{bmatrix} = \begin{bmatrix} \sqrt{P_j} h_{jw} d_{j,w}^{-\alpha/2} x_j(i) + n_w(i) \\ \sqrt{P_j} h_{jw} d_{r,w}^{-\alpha/2} x_j(i) + n_w(i) \end{bmatrix}, & H_0 \end{cases}. \tag{5}$$

Note that MAP criterion is equivalent to maximum likelihood (ML) when legitimate transmitters have the same probability of whether transmitting covert communication or not. By applying the Neyman-Pearson (NP) criterion, the likelihood ratio test (LRT) can be derived as follows

$$L(\mathbf{z}) = \frac{\frac{1}{(2\pi)^{\frac{N}{2}} \det^{\frac{1}{2}}(\mathbf{C_z} + \sigma^2\mathbf{I})} \exp\left[-\frac{1}{2}\mathbf{z}^T\left(\mathbf{C_z} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{z}\right]}{\frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left[-\frac{1}{2\sigma^2}\mathbf{z}^T\mathbf{z}\right]} \underset{D_0}{\overset{D_1}{\gtrless}} \gamma_0, \tag{6}$$

where $\mathbf{C_z} = \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}$ is the variance matrix of $\mathbf{z}$. Note that Willie is only interested in the parts related to observations. After some manipulations for (6), the detector can be rewrite as

$$T'(\mathbf{z}) = -\frac{1}{2}\mathbf{z}^T\left[\left(\mathbf{C_z} + \sigma^2\mathbf{I}\right)^{-1} - \frac{1}{\sigma^2}\mathbf{I}\right]\mathbf{z} \underset{D_0}{\overset{D_1}{\gtrless}} \gamma_1, \tag{7}$$

where $\gamma_1 = \gamma_0 + \frac{1}{2}\log\left(\det\left(\mathbf{C_z} + \sigma^2\mathbf{I}\right)\right) - \frac{N}{2}\log\left(\sigma^2\right)$. After using the matrix inversion theorem [8], we have:

$$(\mathbf{A} + \mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{DA}^{-1}\mathbf{B} + \mathbf{C}^{-1}\right)^{-1}\mathbf{DA}^{-1}, \tag{8}$$

Substituting (8) into (7) after defining $\mathbf{A} = \sigma^2$, $\mathbf{C} = \mathbf{C_z}$ and $\mathbf{B} = \mathbf{D} = \mathbf{I}$, (7) can be further simplified

$$\begin{aligned} T''(\mathbf{z}) &= \mathbf{z}^T\left[\frac{1}{\sigma^2}\left(\frac{1}{\sigma^2}\mathbf{I} + \mathbf{C_z}^{-1}\right)^{-1}\right]\mathbf{z} \\ &\overset{a}{=} \mathbf{z}^T\widehat{\mathbf{s}} \underset{D_0}{\overset{D_1}{\gtrless}} \gamma_2 \\ \Leftrightarrow T''(\mathbf{z}) &= \sum_{i=1}^{N}\mathbf{z}(i)\widehat{\mathbf{s}}(i) \\ &= \sum_{i=1}^{N}\left(\frac{\sigma_a^2}{\sigma_a^2 + \sigma^2} z_{w1}{}^2(i) + \frac{\sigma_r^2}{\sigma_r^2 + \sigma^2} z_{w2}{}^2(i)\right) \underset{D_0}{\overset{D_1}{\gtrless}} \gamma_2, \end{aligned} \tag{9}$$

where (a) is follows from the substitution $\widehat{\mathbf{s}} = \mathbf{C_z}\left(\mathbf{C_z} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{z}$ and $\gamma_2 = \sigma^2\gamma_1$. Moreover, according to the Fisher-Neyman Factorization Theorem, the weighted sum $T''(\mathbf{z})$ of observation in (9) is a sufficient statistic for Willies test. For practical analysis, let $T(\mathbf{z}) = \frac{1}{N}T''(\mathbf{z})$, and thus the power detector can be written by

$$\begin{aligned} T(\mathbf{z}) &= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma^2} z_{w1}{}^2(i) + \frac{\sigma_r^2}{\sigma_r^2+\sigma^2} z_{w2}{}^2(i)\right) \\ &= \frac{\sigma_a^2}{\sigma_a^2+\sigma^2}\sum_{i=1}^{N}\frac{z_{w1}{}^2(i)}{N} + \frac{\sigma_r^2}{\sigma_r^2+\sigma^2}\sum_{i=1}^{N}\frac{z_{w2}{}^2(i)}{N} \underset{D_0}{\overset{D_1}{\gtrless}} \gamma, \end{aligned} \tag{10}$$

where $\gamma = \gamma_2/N$. Furthermore, $\sum\limits_{i=1}^{N} z_{w1}{}^2(i) = \sigma_1^2\chi_{2N}^2$ and $\sum\limits_{i=1}^{N} z_{w2}{}^2(i) = \sigma_2^2\chi_{2N}^2$, where χ_{2N}^2 denotes a chi-squared random variable with $2N$ degrees of freedom. By the weak law of large numbers, $\frac{\chi_{2N}^2}{N}$ converges in probability to 1 when $N \to \infty$

$$\begin{aligned} \sum_{i=1}^{N}\frac{z_{w1}{}^2(i)}{N} &= \frac{\sigma_1^2\chi_{2N}^2}{N} \xrightarrow{N\to\infty} \sigma_1^2, \\ \sum_{i=1}^{N}\frac{z_{w2}{}^2(i)}{N} &= \frac{\sigma_2^2\chi_{2N}^2}{N} \xrightarrow{N\to\infty} \sigma_2^2, \end{aligned} \tag{11}$$

After combining the results in (10) and (11), the proof of Theorem 1 is completed. ■

Probability of detection error (PDE) is also considered in this letter to evaluate the performance of Willie's detection. Let ξ represents the PDE, then PDE of Willie can be derived by

$$\xi = \frac{1}{2}p_{MD} + \frac{1}{2}p_{FA}, \tag{12}$$

where

$$\begin{aligned} p_{MD} &= \Pr\left\{D_0 \left|H_1\right.\right\} \\ &= \Pr\left\{\frac{\sigma_a^2}{\sigma_a^2+\sigma^2}\left(P_a|h_{aw}|^2 d_{a,w}^{-\alpha} + P_j\left|h_{jw}\right|^2 d_{j,w}^{-\alpha} + \sigma_w^2\right)\right. \\ &\quad \left. + \frac{\sigma_r^2}{\sigma_r^2+\sigma^2}\left(P_r|h_{rw}|^2 d_{r,w}^{-\alpha} + P_j\left|h_{jw}\right|^2 d_{j,w}^{-\alpha} + \sigma_w^2\right) < \gamma\right\}, \end{aligned} \tag{13}$$

and

$$\begin{aligned} p_{FA} &= \Pr\left\{D_1 \left|H_0\right.\right\} \\ &= \Pr\left\{\frac{\sigma_a^2}{\sigma_a^2+\sigma^2}\left(P_j\left|h_{jw}\right|^2 d_{j,w}^{-\alpha} + \sigma_w^2\right)\right. \\ &\quad \left. + \frac{\sigma_r^2}{\sigma_r^2+\sigma^2}\left(P_j\left|h_{jw}\right|^2 d_{j,w}^{-\alpha} + \sigma_w^2\right) > \gamma\right\}, \end{aligned} \tag{14}$$

where p_{MD} denotes the probability of miss detection (MD), while p_{FA} is the probability of false alarm (FA). Note that in practice, Willie does not know the value of the channel coefficient h_{aw} and h_{rw} in each slot. As such, the achievability result for covert communication can be analyzed under tighter constraints, provided that Willie has extra knowledge. In practical scenario, Willie aims to minimize the PDE by setting a reasonable threshold γ^*, and thus there exists an optimization problem.

$$\gamma^* = \arg\min_{\gamma} \xi. \tag{15}$$

Note that the optimization problem can be solved by multiple optimization algorithms such as numerical search or analytic solutions, which depends on the specific problem constructions. These are not the focus of this letter.

Furthermore, considering a special case that $\sigma_a^2 \gg \sigma^2$ and $\sigma_r^2 \gg \sigma^2$, the weight factors in Theorem 1 approaches to 1. On the other hand, when $\sigma_a^2 \ll \sigma^2$ and $\sigma_r^2 \ll \sigma^2$, the weight factors in Theorem 1 approaches to 0. It can be observed that the weight factors in Theorem 1 intend to be equal, allowing the JWPD degenerates to an adding-power-directly-detector (APDD) T_a:

$$T_a(\mathbf{z}) = \sigma_1^2 + \sigma_2^2 \underset{D_0}{\overset{D_1}{\gtrless}} \gamma_3. \tag{16}$$

Based on the above analysis, it can be seen that the APDD combines two phases together but neglects the difference between two-phase received signal, which is a suboptimal power detector.

Numerical Results: In this section, numerical simulations are presented to verify the analysis of proposed detection scheme. In the simulations, we set $P_a = 20$ W and $P_r = 80$ W, respectively. Furthermore, it is assumed $d_{x,y} = 100$ m, $(xy \in \{ar, aw, rw\})$, $d_{j,w} = 80$ m, and$\alpha = 2.2$. Moreover, the noise power at Willie is $\sigma_w^2 = 0$ dB.

Fig. 3 compares PDE versus P_{jmax} of existing detection schemes with the scheme proposed in this letter. As shown in the figure, on the one hand, PDE of JWPD is about 16% lower than PDE of SPD and is about 5% lower than PDE of APDD when $P_{jmax} = 1000W$, which validates that JWPD is more efficient than SPD and APDD. On the other hand, higher P_{jmax} further worsens the $SINR$ at Willie, thus, PDE of each schemes increase monotonically with the increase of P_{jmax} and eventually approach to 0.5. Moreover, our scheme achieves lower PDE than other schemes with the increase of P_{jmax}, which demonstrates the benefit of our proposed scheme.

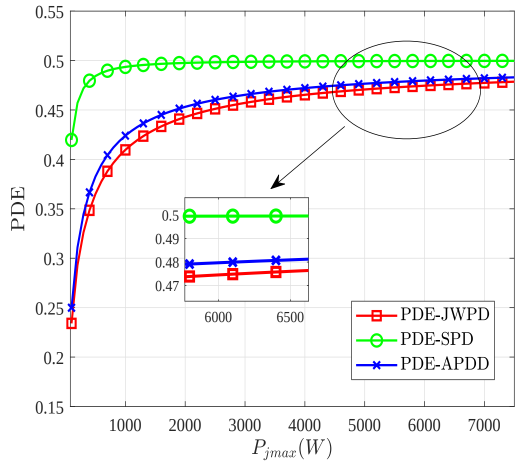

Fig 3 *PDE versus P_{jmax} for different detection schemes.*

Conclusion: In this letter, a near optimal power detector at Willie for two-hop covert communication system is proposed, which is termed as JWPD. The proposed scheme has the lowest PDE than other existing SPD and APDD schemes. Hence, the covertness of two-hop system can be further analyzed with tighter covert constraints by employing JWPD, whose result is more practical and accurate. In addition, the proposed JWPD can be extended to multi-hops covert communication scenario, which will be studied in our future work.

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