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**Power flow problem approached by geometric algebra**

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## **Abstract**

Analysing the electrical power systems' behaviour is significantly based on power flow analysis. This paper uses a geometric algebra (GA) mathematical framework to solve the power flow problem. It can combine and extend algebraic and geometric concepts in a unified and powerful way. While complex numbers are an extension of the real number field, geometric algebra builds on the ideas of linear algebra and geometry to provide a more complete and versatile mathematical framework. Additionally, GA enables handling multivectors through geometric functions, including wedge and geometric products. Thus, it allows a straightforward interpretation because of its ability to abstract the formulation. Therefore, by utilising GA, power flow analysis may be performed efficiently and precisely, resulting in improved design and operation of power systems. This paper presents the GA-based formulation and shows the comparative results between the conventional and the proposed technique.

***Keywords****: Complex numbers, Geometric algebra, Newton Raphson method, Power flow formulation, Vectors.*

1. **Introduction**

Providing energy to homes, companies, and industries is one of the essential functions of electrical power networks nowadays. However, because of the systems' behaviour complexity, considerable analysis is required before operation can be guaranteed to be dependable and effective. Understanding the electrical power systems' nature requires several tools, one of which is power flow analysis. Traditional related approaches represent the system using complex numbers and matrices. These approaches need a laborious and challenging formulation due to their complexity. Geometric algebra (GA) is a mathematical framework that provides a unified method to represent and handle complex numbers, vectors, and tensors. It also gives a more intuitive approach to power flow analysis; this becomes one of the main benefits of using GA.

Using multivariate vectors allows the representation of geometric objects in a higher dimensional space. This result in a more complete description of objects' geometric properties and enables them to be handled more intuitively. In addition, while complex numbers are limited to representing points in a plane, geometric algebra allows working with objects of higher complexity, such as hypersurfaces, hyperplanes and volumes in more elevated dimensional spaces.

Geometric algebra also provides tools to tackle problems more efficiently and elegantly. For example, it is possible to express algebraic and geometric operations in a single mathematical framework by introducing geometric products, such as the outer and inner products. This simplifies expressions and allows us to derive results more quickly. Furthermore, geometric algebra has applications in various branches of mathematics and physics, such as differential geometry, relativity theory and quantum mechanics, demonstrating its usefulness and versatility in broader contexts.

Furthermore, describing and evaluating power systems using GA becomes more straightforward since it enables a geometric comprehension of power flow equations. In this situation, GA proportionates benefits over conventional techniques in terms of accuracy, efficiency, and ease of interpretation, leading to improved design and operation. Therefore, this paper investigates the use of GA in power flow analysis and its advantages over conventional approaches.

Regarding the use of GA in engineering, the applications can be found in robotics, computer vision and control [1]-[4]. In the last decade, it has been finding applications in circuit analysis; the most notable work has been Menti [5], Castro [6,7] and a recent one from Montoya et al., [8]-[11]. All these works show an application of geometric algebra to studying an *RLC* circuit. Authors handle mathematical case studies of different harmonics in the excitation, and some transformation is provided for converting the time domain signal to a geometric variable. Likewise, instead of an imaginary '*j*', a geometric variable (*bivector*) is used. The analysis is focused initially on single-phase circuits, and in [12], it is shown how it can be used in three-phase scenarios and for the computation of multi-vectorial power and the harmonic interaction between different frequency components. A critical review of references [5]-[12] implies that the transformation and all the mathematical demonstrations are suitable for the case studies, including different harmonics. In this regard, when a comparison is drawn with other power theories, e.g. current's physical components (CPC) and others [18]-[21], some differences can be noted, especially the reactive power and distortion power, which put forward a question on the use of transformation and basis of geometric algebra in circuit analysis. Numerous contributions using complex algebra have advanced state-of-the-art power flow solutions; a few may be found in [13]–[17].

Although the inspiration for the present work has been drawn from [5]-[12], the main objective, which is intended precisely to be delivered, serves the following purposes:

* A solution based on geometric algebra for the power flow problem.
* Investigate the feasibility of geometric algebra as an alternative mathematical framework in the pragmatical scenario.
* Drawing a conclusion based on a comparative study of power flow employing GA and the conventional formulation in an IEEE benchmark system.
* As engineers and data scientists, it is crucial to be able to choose between different perspectives and select the most appropriate approach depending on the characteristics of the problem at hand.

## **Preliminaries**

Geometric algebra (GA) is a mathematical framework that expands and consolidates various mathematical frameworks, such as vector algebra. Geometric algebra, also known as Clifford algebra, is a tool that extends the system of real numbers to vector calculus utilising a series of operations and properties. GA allows the concepts of geometry to be unified with algebra and trigonometry, facilitating their use in physics and engineering. One of the main Clifford algebra characteristics is that it allows the representation of entities of higher order with a compact symbology and to operate them linearly. Lines, planes or spheres are examples of entities represented as single elements of Clifford algebra. In addition, it employs a vector representation as multivectors encompassing scalars, vectors, bivectors (oriented planes), tri vectors (oriented volumes), and higher-dimensional analogues. The algebra of complex numbers is a sub-family of such geometric algebra.

In contrast, the normal vector representation solely comprises vectors and, occasionally, their components are conventionally employed to denote physical quantities like velocity or force. The conventional vector representation allows vector addition and scaling, but geometric algebra's operations surpass these basic functionalities. GA enables handling multivectors through geometric functions, including wedge and geometric products.

### Geometric product

Geometric algebra is used to be chosen for specific requirements. As in earlier works [5]-[12] the basis' *n*' in is selected based on the number of harmonics in the source. Geometric algebra , is a framework that extends the vectors' properties to higher dimensions, including oriented planes, volumes, and hyper-volumes. The notation speaks for a geometric algebra with *p* elements that square to 1 (e.g., the unit vectors), *q* elements that square to -1 (e.g., the bivectors), and *r* elements that are neither 1 nor -1 (e.g., tri vectors or higher-order multivectors).

The choice of *p*, *q*, and *r* hinges on the problem dimensionality being solved and the specific algebraic properties wished. The geometric algebra dimension is generally determined by the number of essential elements needed to represent the problem. For example, in three-dimensional space, a geometric algebra can represent vectors, while a can represent both vectors and planes.

Some common choices for *p, q*, and *r* become:

* for representing *n*-dimensional vectors
* for representing *n*-dimensional vectors and oriented planes
* for representing *n*-dimensional vectors, planes, and volumes
* for representing *n*-dimensional vectors, planes, volumes, and hyper-volumes

In general, the geometric algebra for *n* vectors is defined by,

(1)

where *I* represents the pseudo scalar or wedge product of the highest grade, and *ԑ* is a scalar and subsequently becomes the basis vector. In some other works, they are used as .

The geometric product between vectors **a** and **b** is defined as, Figs. 1(*a*) and (*b*),

(2)

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Descripción generada automáticamente

**Figure 1** (*a*): Scalar or dot product *a.b*; (*b*) Wedge or exterior product (grade 2 element)

The geometric product defined in (2) is anti-commutative and invertible. In GA, (2) generates a new quantity known as a multivector. Multivector addition and multiplication obey the associative and distributive laws. Next is a summary of essential GA [22],

* When two vectors are parallel, their geometric product results in a scalar quantity that is equivalent to the product of their magnitudes.
* The bivector resulting from the two vectors' multiplication is orthogonal to each other, known as the geometric product. This bivector represents the directed area that is formed by the two vectors.
* Parallel vectors exhibit commutative behaviour under the geometric product. Conversely, vectors that are orthogonal to each other demonstrate anti-commutative behaviour.
* The multiplication of vectors on their left results in a bivector which in geometric algebra means a rotation of 90◦ clockwise within the bivector plane. Multiplying a vector on its right results in its 90◦ anticlockwise rotation. This can be utilised to establish the orientation of the vectors . The scalar value of the square of the bivector area formed by the exterior product of is -1, expressed as.
* Based on its last two properties, the bivector is a potential candidate for the imaginary unit *j*. In two-dimensional contexts, it effectively serves as the imaginary unit *i*. Notice that the subalgebra comprised of even-grade elements, denoted as z = x + y, plays the role of complex numbers.

## **Geometrical algebra for power flow calculations**

Power flow calculation is critical in power system analysis, as it provides information about the voltages and power flow through the system. One commonly used method for power flow calculation is the Newton-Raphson method, which is traditionally applied using complex numbers. Instead, there has been growing interest in using GA as an alternative mathematical framework for power flow analysis. GA offers several advantages over complex numbers in power flow analysis, such as taking non-planar and nonlinear systems, dealing with multiple voltages and currents, and simplifying complex expressions.

In the power flow studies, the bus voltages play the role of the state to be determined, and they are typically specified in terms of phasors. In this paper, it is converted to the geometric context. In *G3,0,0* geometric algebra (which is defined for vectors specifically), a phasor can be expressed as a three-dimensional vector, which is a combination of the basis vectors , , and *e3*Each basis vector corresponds to a particular physical quantity (voltage, current, or power). The phasor vector is then given by a linear combination of these basis vectors, with the coefficients representing the phasor's magnitude and phase. For example, a phasor can be described as,

(3)

In (3), represent the cosine and sine of the phase angle , and , , and are the basis vectors. Thus, it is possible to convert the phasors into geometric variables. With this interpretation, any bus variable may be converted into a geometric entity or multivectors without worrying about geometric algebra's dimension. Combining this with the bivector representation of impedances as proposed in earlier work [7, 8, 9], it is pretty feasible to perform the power flow study using any mathematical tool, whether Gauss-Seidel, Newton-Raphson (NR), or another similar method. Therefore, the geometric algebra will be used throughout the process for computing power flow. A flowchart for implementing geometric algebra using the Newton-Raphson (NR) method for power flow studies is depicted in Fig .2. This method has also been verified for the Gauss-Seidel method. For brevity, the NR method is presented.

![Diagram

Description automatically generated]()

**Figure 2**: Power flow flowchart using NR method employing GA.

Thus, the GA-based Newton-Raphson method for power flow calculation involves representing the system variables as multivectors: GA elements representing vectors, bivectors, tri vectors, etc. The process then proceeds by solving a set of nonlinear equations iteratively, with the solution being obtained when a specific convergence criterion is met.

Compared to the conventional complex number-based Newton-Raphson method, the GA-based approach offers several characteristics, including similar precision, increased computational efficiency, and enhanced ability to handle complex and nonlinear systems. Through the case study, such aspects have been demonstrated in the following section. Additionally, the GA-based approach offers a more intuitive and visual representation of the system variables, which can aid in understanding and interpreting the results. Table 1 summarises the algorithm for power flow calculation using GA as a mathematical tool.

**Table 1**: Power flow calculation algorithm using GA

|  |
| --- |
| Defining System parameters (G, B, Y, Z) of the power network (*definitions in appendix*)  Voltage magnitude vector (|V|) and angle vector (θ) at each bus  Power injections (P, Q) at each bus   1. Set the maximum number of iterations (max\_iter) and convergence tolerance (tol). 2. Initialise the iteration counter (iter) to 0 and set the convergence flag (converged) to False. 3. While iter < max\_iter and not converged:    1. Increment the iteration counter iter. 4. Initialise the basis blades for the Euclidean space.   Represent the voltage magnitude vector |V| and angle vector θ using basis blades.   * 1. *|V| = |v1| \* e1 + |v2| \* e2 + |v3| \* e3 + ... + |vn| \* en*   2. *θ = θ1 \* e1 + θ2 \* e2 + θ3 \* e3 + ... + θn \* en*  1. Calculate the real and reactive power injections using basis blades.   For each bus i:  i). While iter < max\_iter and not converged:  Increment the iteration counter iter.   * 1. Calculate the complex voltage phasor Vi using the magnitude |Vi| and angle θi in GA   2. Calculate the active power injection Pi and reactive power injection Qi using the geometric algebra expressions derived from the power flow equations in GA        * 1. Calculate the power injections (P\_calc, Q\_calc) using the complex voltage phasors (V) and the power flow equations in GA   2. Calculate the mismatch vectors (ΔPQ) as the differences between the specified power injections (P, Q) and the calculated power injections (P\_calc, Q\_calc):   *ΔP = P - P\_calc*  *ΔQ = Q - Q\_calc*   * 1. Calculate the Jacobian matrix (J) using geometric algebra:  1. Jacobian calculation    1. Calculate the partial derivatives (∂P\_i/∂θ\_j) and (∂Q\_i/∂θ\_j) for each angle θ\_j using the geometric algebra expressions derived from the power flow equations.    2. Assign the calculated partial derivatives to the corresponding elements of the Jacobian matrix J.   Solve the linear system J \* Δθ = [ΔP, ΔQ] for the voltage angle correction Δθ using geometric algebra.   * 1. Update the voltage angle vector (θ) by adding the angle correction Δθ:   *θ = θ + Δθ*   * 1. Check for convergence by calculating the maximum absolute value of the mismatch vector (max(|ΔPQ|)):   2. If max(|ΔPQ|) < tol, the set converged to True.   ii). Return the updated voltage angle vector (θ). |

In the following equations (4)-(5), the voltage is expressed as a linear combination of the basis vectors and , which are the standard basis vectors in 2D Euclidean space. This means that each voltage vector can be written as,

, (4)

In (4), *a* and *b* are scalars that represent the components of the vector along each basis vector. In contrast, complex numbers are typically used to describe voltage phasors in electrical engineering. A complex voltage phasor has the form,

(5)

In (5), stands the phasor magnitude, and **Θ** is the phase angle. This representation is proper because it allows for wielding voltage phasors using complex arithmetic. Thus, geometric algebra provides a clear and concise way to represent the voltage vectors in their component directions. With a similar interpretation, current and power can also be expressed in Euclidean space. This can be useful for analysing power systems and other electrical networks, where it is essential to understand the flow of power and energy between different components.

## **Case studies**

#### 4.1: Two buses system

In this case, the system is assumed to have two buses with subscripts 1 and 2 denoting their bus number. '*G*' denotes the generator, '*S*' means the load, and '*Z*' represents the impedance. All used values are in p.u.

1∠0 ; 1∠unknown generators' voltages

; loads (6)

(impedance connecting buses 1-2) line impedance

First, the bus voltages are converted into geometric entities using (5). Therefore, (4) gets transformed into (7),

1\* ; cosθ\*

; (7)

By (7), the computation may be performed for the power flow between the two buses. Here, the results in geometric representation are given with vector and bivector notation. Table 2 summarises the comparative results using a conventional NR method.

**Table 2** Voltages at the end of the iterative process

|  |  |
| --- | --- |
| Geometric algebra results | Results by complex algebra |
| V1=1.0317 \*(0.7979 + 0.3273\*e1) | V1=1.0317 ∠ 6.7715 |
| V2=1.0174\* (0.8919 - 0.1829\*e1) | V2=1.0174 ∠ -2.0446 |
| S1=1.3894 + 0.9444\*e1 | S1=1.3894 + 0.9444j |
| S2=1.4326 + 0.6242\*e1 | S2=1.4326 + 0.6242j |

In Table 2, V1 and V2 are the bus voltages, and S1 and S2 are complex power at buses 1 and 2. The geometric variables may be converted into complex notation using (3). Then, it is demonstrated that both results are the same. A notorious difference in both computations becomes the number of iterations. For the calculations based on geometric algebra, the error function is kept in a threshold, which results in an increased number of iterations. However, how close is our guess for the unknown state variable, and a slight increase in error function can reduce the number of iterations.

#### 4.2 : Three-buses system

1∠0 ; unknown; 0.988∠unknown

; ; (8)

; ; (impedances connecting buses)

Complex Algebra approach

Table 3 presents the defined system, and Table 4 summarises the load flow by Newton Raphson using complex and geometric algebra; all information is in p.u. Table 4 represents the results obtained by the Newton-Raphson method using complex algebra and geometric algebra to show the comparative results. In contrast, Table 5 represents the state variables during the iterative process by geometric algebra for the three-buses system.

**Table 3** General information three-buses system

|  |  |
| --- | --- |
| Number of buses | **3** |
| Number of lines | **3** |
| Number of generators | **3** |
| Slack bus | **Bus1** |

**Table 4** Power flow results by Newton-Raphson

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Complex Algebra | | | Geometric Algebra | | |
|  |  | Generation | Load |  | **Generation** | **Load** |
| Bus | Volts (V∠θ) | Powers (PG+jQG) | Powers (PL+jQL) | Volts (V∠θ) | Powers (PG+jQG) | Powers (PL+jQL) |
| 1.00 | 1.02∠0 | 2.505+j1.1827 | 1.500+j0.800 | 1.02∠0 | 2.460+j1.1827 | 1.500+j0.800 |
| 2.00 | 1.00∠-3.3238 | 1.00+j0.685 | 1.500+j0.700 | 0.9755∠-3.342 | 1.00+j0.485 | 1.500+j0.700 |
| 3.00 | 0.988∠-3.2820 | 0.00+j0.00 | 0.500+j0.300 | 0.988∠-3.327 | 0.00+j0.00 | 0.500+j0.300 |

The convergence criteria defined for unknown voltage angles is =, and the iteration-wise result is as in Table 5, retrieved using geometric algebra. This convergence threshold is elected for illustrative purposes only.

**Table 5** Three-buses system (GA method: state variables during the iterations)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Unknown state variable | Initial value | Ist iteration | IInd iteration | IIIrd iteration |
|  | 0.0 | -1.966 | -3.319 | -3.319 |
|  | 0.0 | -1.964 | -3.281 | -3.281 |
|  | 1 | 0.871 | 0.988 | 0.988 |

Notice that, through the results, there is no discrepancy in results for the three-bus system, Table 4, and geometric algebra can be used for power flow studies.

Figure 3 shows the geometric interpretation of voltages at each bus. It shows geometrically how the voltage magnitude and phase angle changes dictate the system's active and reactive power flow. From the figure, it can be understood that a higher voltage magnitude governs the active power flow. The coloured number represents the bus number, and the colour bar provides information on active and reactive power at a particular bus number. Similarly, the phase angle relationship holds with reactive power.

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(*a*)

A picture containing text, screenshot, number, plot

Description automatically generated

(*b*)

**Figure 3** (*a*) Voltages and angles after convergence; (*b*) active and reactive power

#### 4.3: Nine-buses power system [23]

This is the standard IEEE-9 bus benchmark system. A comparative study is also done to analyse the power flow using geometric and complex algebra.

Newton-Raphson approach

Table 6 summarises the information attained in such a system using the conventional Newton-Raphson after convergence (A tolerance ε = 0.001 is used.) using complex algebra and GA.

**Table 6** General information 9-buses power system [18]

|  |  |
| --- | --- |
| Number of buses | **9** |
| Number of lines | **9** |
| Number of generators | **9** |
| Slack bus | **Bus1** |

**Table 7.** Power flow results by Newton-Raphson (nine-buses)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Complex Algebra** | | | **Geometric Algebra** | | |
|  |  | Generation | Load |  | **Generation** | **Load** |
| Bus | Volts (V∠θ) | Powers (PG+jQG) | Powers (PL+jQL) | Volts (V∠θ) | Powers (PG+jQG) | Powers (PL+jQL) |
| 1 | 1.00∠0 | 72.29+j34.72 | 0+j0 | 1.0000∠0 | 0.074+j0.0346 | 0.00+j0.00 |
| 2 | 1.00∠19.51 | 163+j45.50 | 0+j0 | 0.9831∠-2.639 | 0.163+j0.045 | 0.8182+j0.3064 |
| 3 | 1.00∠11.48 | 85+j17.47 | 0+j0 | 0.9713∠-4.312 | 0.085+j0.017 | 0.199+j0.0782 |
| 4 | 0.9627∠-5.10 | 0+j0 | 0+j0 | 0.9421∠-7.686 | 0+j0 | 0.00+j0.00 |
| 5 | 0.9285∠-7.14 | 0+j0 | 125+j50 | 0.9541∠-6.40 | 0+j0 | 0.12+j0.0500 |
| 6 | 0.95∠-6.78 | 0+j0 | 90+j30 | 0.9601∠-5.75 | 0+j0 | 0.09+j0.0300 |
| 7 | 0.9640∠1.55 | 0+j0 | 0+j0 | 0.9813∠-3.47 | 0+j0 | 0.0+j0.00 |
| 8 | 0.9560∠-1.85 | 0+j0 | 100+j35 | 0.9681∠-4.81 | 0+j0 | 0.10+j0.0350 |
| 9 | 0.9795∠-0.52 | 0+j0 | 0+j0 | 0.9647∠-5.13 | 0+j0 | 0.00+j0.00 |

Likewise, Table 7 also exhibits the corresponding results employing geometric and complex algebra, Fig. 4. One point to be noticed here is all results of geometric algebra for power flow have been converted back into complex notation for better comprehension. For example, the basis vectors *e1* and *e2* can be recovered into complex notation using (4) and (5).

A picture containing diagram, circle, line, text

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*(a)*

A picture containing text, screenshot, number, diagram

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*(b)*

**Figure 4** (*a*) Voltages and angle; (*b*) active and reactive power

The power flow analysis results were compared using the Newton-Raphson algorithm with geometric algebra and complex numbers. Notice that the bus voltages and angles are similar, with slight variations in the third decimal place. However, there are some differences in the active and reactive power values, particularly for buses 2 and 3, Table 6 (all values are in p.u for active and reactive power at a base MVA of 1000). These differences may be attributed to how the two methods handle complex power calculations. Nevertheless, overall there is a slight advantage in processing time with geometric algebra, Table 8.

**Table 8** CPU times (nine-buses)

|  |  |
| --- | --- |
| Complex algebra | 0.385076s |
| Geometric algebra | *0.368520s* |

Although the 9-bus system in Table 8 demonstrates a computing time advantage. This must be verified by analysing other IEEE benchmark systems. Overall, geometric algebra and complex numbers can be used for power flow analysis, but the choice of method may depend on the specific requirements of the research. For example, geometric algebra offers advantages in terms of ease of interpretation and computational efficiency. At the same time, complex numbers may be more familiar to engineers and may have better support in commercial software packages. Ultimately, the choice between the two methods should be based on careful consideration of the specific needs of the power flow analysis.

Thus, in summary, geometric algebra is a mathematical framework that extends and enriches simple algebra by incorporating geometric concepts into algebraic operations. It uses geometric objects such as vectors, matrices and linear transformations to represent and solve problems. This allows for a more complete and accurate representation of spatial information, which is particularly valuable. One of the key advantages of a geometric algebra-based algorithm is its ability to approach problems more intuitively. By working with geometric objects, we can take advantage of the richness of geometry to better visualise and understand the underlying data and relationships. This can facilitate reasoning and decision-making, as we can tap into human spatial intuition. Such is the case of the application exposed to solve the power flow problem.

Finally, a relevant aspect is computational efficiency. Algorithms based on geometric algebra can take advantage of the algebraic properties of geometric objects to optimise computations and reduce computational complexity. This can lead to faster and more efficient algorithms than their simple algebra-based counterparts. The authors will address this aspect. Thus, using the geometric algebra-based algorithm offers several advantages over others. The ability to capture spatial information more accurately, the intuitive approach and the improved computational efficiency are key factors that support its attributes in specific contexts. As engineers and data scientists, it is crucial to be able to choose between different perspectives and select the most appropriate approach depending on the characteristics of the problem at hand.

## **Conclusions**

Geometric algebra provides a broader and more powerful perspective for tackling algebraic and geometric problems. Its ability to represent objects in higher dimensional spaces and unify algebraic and geometric concepts make it an invaluable tool in various fields of mathematics and physics. Studying and understanding geometric algebra enriches our mathematical vision and gives us the tools needed to explore and understand more complex phenomena in the world around us.

The findings derived from the power flow analysis conducted utilising both complex numbers and geometric algebra indicate that using geometric algebra offers a natural and intuitive approach to expressing and handling complex quantities in power systems, especially a retrospection of (4) and (7). Furthermore, using geometric products and other algebraic operations simplifies the mathematical expressions involved in power flow analysis, leading to more efficient computation and straightforward implementation. Also, it provides compactness in representation.

Using geometric algebra representation facilitates the seamless execution of vector and matrix operations on phasors, a prerequisite for conducting power flow analysis. Moreover, it offers a means to visually represent phasors and their interconnections more comprehensibly than solely relying on complex numbers.

Moreover, geometric algebra offers a cohesive structure for employing actual and imaginary numbers, enabling seamless handling of both entities. This technique holds significant potential in power systems, where real and complex quantities are simultaneously involved. The utilisation of GA's inherent geometric operations enables more efficient and concise calculations, leading to potentially faster convergence and reduced computational time for large-scale power systems.

Hence, the utilisation of geometric algebra in power flow analysis holds potential as a viable method that could potentially provide benefits over the conventional application of complex numbers. In the presented cases, there is agreement on results but further investigation and practical implementations are required to comprehensively assess the advantages of employing geometric algebra in analysing power systems.

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**Appendix**

*GA* : geometric algebra

: geometric algebra for Euclidean or pseudo- euclidean metric

: 3-D Euclidean geometric algebra

*V*: voltage vector

**Θ:** phase angle

*P:* Active power

*Q:* Reactive power

: Inverse of Y

*X\*Y*: scalar or dot product

: Wedge product or bivector part

*G* : line conductance

*B* : line Susceptance

*Z*: line Impedance