A Pythagorean Fuzzy Multiset Approach for Multi-Robot System

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Abstract: Most of the real life problems embroil uncertainties, imprecision and vagueness. Fuzzy multisets and Pythagorean fuzzy sets, initially suggested by Yager, are significant mathematical models to handle such real world problems. By combining these two notions, a new kind of hybrid mathematical model is exists: Pythagorean fuzzy multisets. In this study, Pythagorean fuzzy multisets approach is applied to a multi-robot system to achieve better results using less effort and less time.

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1. Introduction

It is aforementioned for an object to be a member of a set and not to be a member, in classical sets. According to this approach, an individual, element, or measurements within the work area that has the characteristic we want either belong to a defined set or not. In expressing such sets, characteristic functions are used. The characteristic function determines the set consisting of the elements defined on the universal set and having the characteristic we are interested in by assigning one of the values 1 and 0 to each element according to their membership status.

Contrary to classical set theory, the fuzzy set(FS) theory provides us with powerful and meaningful tools for measuring uncertainties in real life and also provides a significant expression of uncertain concepts in natural language. The membership(MS) transition of elements in a universal set containing FSs is gradual, in the FS theory. If an element is to belong to any set, the degree to which element belongs to the set is also considered.

Taking into account the uncertainty element, Zadeh [16] offered FSs in which a MS function is assigned to every member of the universe. Atanassov [2] presented intuitionistic fuzzy sets (IFSs) including MS and non-membership(NS) functions. Yager [13], [14] introduced the Pythagorean Fuzzy Sets (PFS) which is a generalization of IFS. Peng et al. [11], Guleria and Bajaj [8] and Kirisci [9] presented Pythagorean fuzzy soft sets (PFSSs).

Contrary to ordinary sets, multiset permits us to have multiple occurrences of the members. Blizzard [3], [4] introduced multiset theory as a generalization of the crisp set theory. After Blizzard, multisets were combined with different fields and new concepts as fuzzy multiset (FMS), intuitionistic fuzzy multiset (IFMS), Pythagorean fuzzy multiset(PFMS) emerged.

In multi-attribute group decision making (MAGDM) processes, PFMS is a vigorous mathematical model. While tackling real-world problems, IFMS cannot deal with the state if the sum of MS degree and NS degree of the parameter gets larger than one. This situation makes decision making demarcated, and influences the optimum decision. In this case, the decision-making process is demarcated and the optimum decision is affected. It makes decision making demarcated, and affects the optimum decision. PFMSs assist us in handling such situations.

The concept of FS theory is of paramount relevance to tackling the issues of uncertainties in real-life problems. In a quest to having a reasonable means of curbing imprecision, the idea of FSs had been generalized to IFSs, FMSs, PFSs among others. In IFMS theory, the sum of the degrees of MS and NS is less than or equals one at each level. Supposing the sum of the degrees of MS and NS is greater than or equal to one at any level, then the concept of PFMS is appropriate to handle such a scenario. In fact, PFMS is a PFS in the framework of a multiset.

Robots are machines that enable people to do their jobs more easily and effectively. There are two basic robot motions defined as point-to-point(PtP) and area sweeping(ASw). PtP is a motion from the starting location to the target location to implement particular tasks. ASw means to a motion to traverse a field such that the robot's sweeping appliance(sensor or actuator) will entirely upholster the area.

In its simplest sense, the robot can be described as follows: A robot is a programmable automated device that can construe info from the physical surroundings in order to transcribe its manner. It has the ability to interact with the surround and carry out diverse functions accordingly. Robots have three components as a control system, sensors and actuators.

If more than one robot is used to complete a task, this system is defined as a multi-robot system. In present work, the application of PFMS in robotics is investigated. The collaboration of robots was worked with PFSM. The scenario in this study is to explain the system created by robots the navigate and surveillance in a certain region through a central server, with PFSM.

2. Preliminaries

Consider X is a non-empty set. If the function $\alpha : X \to \mathbb{N}$ is a function on an underlying crisp set X, then the pair $< X, \alpha >$ is called a multiset(MTS). A MS M is given by

$$M =< X, \alpha >= \left[\frac{\alpha(\varrho_i)}{\varrho}_i : i = 1, 2, \cdots, n \right]$$

where $\alpha(\varrho_i)$ is the duplicity of $\varrho_i \in X$ [4]. For example, if $X = \{\varrho_i : i = 1, 2, 3\}$ then

$$M = \{\varrho_1, \varrho_1, \varrho_2, \varrho_3, \varrho_3, \varrho_3, \varrho_3\} = \left[\frac{2}{\varrho_1}, \frac{1}{\varrho_2}, \frac{4}{\varrho_3} \right]$$

is a MTS over X.

Other synonyms used in literature for MTS are bag, list, bunch, heap, sample, weighted set, occurrence set, and fireset.

A fuzzy multiset(FMS) A is typified by a mapping $\alpha_A : X \to M$ (count MS, CM), where M is the collection of all MTSs extracted from the interval $[0, 1]$. For every $\varrho \in X$, the affiliation sequence (MS function) is defined as decreasingly organized sequence of elements in $\alpha_A(\varrho)$ and is characterized as $\left(\zeta_A^{(1)}(\varrho), \zeta_A^{(2)}(\varrho), \cdots, \zeta_A^{(n)}(\varrho)\right)$ with the constraint that $\zeta_A^{(i)}(\varrho) \geq \zeta_A^{(i+1)}(\varrho)$.

A intuitionistic fuzzy multiset(IFMS) A over the underlying non-empty set X is portrayed by two mappings $\zeta_A : X \to M$ (CM) and $\xi_A : X \to M$ (count NS, CN), where M is the collection of all MTSs extracted from the unit closed interval. For every $\varrho \in X$, the MS sequence is defined as decrasingly arrayed progression of elements in $\zeta_A(\varrho)$ represented as $\left(\zeta_A^{(1)}(\varrho), \zeta_A^{(2)}(\varrho), \cdots, \zeta_A^{(n)}(\varrho)\right)$ bearing the constraint $\zeta_A^{(i)}(\varrho) \geq \zeta_A^{(i+1)}(\varrho)$. The corresponding NS sequence is represented as $\left(\xi_A^{(1)}(\varrho), \xi_A^{(2)}(\varrho), \cdots, \xi_A^{(n)}(\varrho)\right)$. Further, $0 \leq \zeta_A^{(i)}(\varrho) + \xi_A^{(i)}(\varrho) \leq 1$, for all i [5].

An IFMS may expressed in set-builder notation as

$$A = \{< \varrho : \left(\zeta_A^{(1)}(\varrho), \zeta_A^{(2)}(\varrho), \cdots, \zeta_A^{(n)}(\varrho)\right), \left(\xi_A^{(1)}(\varrho), \xi_A^{(2)}(\varrho), \cdots, \xi_A^{(n)}(\varrho)\right) >: \varrho \in X\}.$$

It is remarkable to note that the NS sequence need not to be in ascending or descending order, contrary to the MS sequence.

A Pythagorean fuzzy multiset (PFMS) P over a non-empty underlying set X is described by two mappings $\zeta_P : X \to M$ (CM) ad $\xi_P : X \to M$ (CN), where M is the collection of all MTSs drawn from the $[0,1]$. The MS sequence is defined as descending ordered progression of members in $\zeta_P(\varrho)$ represented as $\left(\zeta_P^{(1)}(\varrho), \zeta_P^{(2)}(\varrho), \cdots, \zeta_P^{(n)}(\varrho)\right)$ where $\zeta_P^{(i)}(\varrho) \geq \zeta_P^{(i+1)}(\varrho)$, for every $\varrho \in X$. The corresponding NS sequence is represented as $\left(\xi_P^{(1)}(\varrho), \xi_P^{(2)}(\varrho), \cdots, \xi_P^{(n)}(\varrho)\right)$. Further, $0 \leq \left(\zeta_A^{(i)}(\varrho)\right)^2 + \left(\xi_A^{(i)}(\varrho)\right)^2 \leq 1$, for all i [6]. Supposing $\left(\zeta_A^{(i)}(\varrho)\right)^2 + \left(\xi_A^{(i)}(\varrho)\right)^2 \leq 1$, then there is a degree of indeterminacy of $\varrho \in X$ to A defined by $\eta_P^i(\varrho) = \sqrt{1 - \left[\left(\zeta_A^{(i)}(\varrho)\right)^2 + \left(\xi_A^{(i)}(\varrho)\right)^2\right]}$ and $\eta_P^i(\varrho) \in [0,1]$. In what follws, $\left(\zeta_A^{(i)}(\varrho)\right)^2 + \left(\xi_A^{(i)}(\varrho)\right)^2 + \left(\eta_P^i(\varrho)\right)^2 = 1$.

A PFMS may expressed in set-builder notation as

$$\begin{aligned} P &= \{< \varrho : \left(\zeta_P^{(1)}(\varrho), \zeta_P^{(2)}(\varrho), \cdots, \zeta_P^{(n)}(\varrho)\right), \left(\xi_P^{(1)}(\varrho), \xi_P^{(2)}(\varrho), \cdots, \xi_P^{(n)}(\varrho)\right) >: \varrho \in X\} \\ &= \{< \varrho : \left(\left\{\zeta_P^{(i)}(\varrho)\right\}_{i=1}^n\right), \left(\left\{\xi_P^{(i)}(\varrho)\right\}_{i=1}^n\right) >: \varrho \in X\}. \end{aligned}$$

It is remarkable to note that the non-membership sequence need not to be in ascending or descending order, contrary to the membership sequence.

Choose two PFSs A and B. The Euclidean distance is defined by [7]

$$d_{PFS}(A,B)_E = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[(\zeta_A^i(\varrho) - \zeta_B^i(\varrho))^2 + (\xi_A^i(\varrho) - \xi_B^i(\varrho))^2 + (\eta_A^i(\varrho) - \eta_B^i(\varrho))^2\right]}.$$

The cardinality of $\zeta_P(\varrho)$ (or $\xi_P(\varrho)$) in a PFMS P is called length of the element $\varrho \in P$ and is designated as $L(\varrho : P)$ i.e. $L(\varrho : P) = \#(\zeta_P(\varrho)) = \#(\xi_P(\varrho))$, where $\#(\zeta_P(\varrho))$ denotes the cardinality of the membership sequence $\zeta_P(\varrho)$ and $\#(\xi_P(\varrho))$ that of non-membership sequence $\xi_P(\varrho)$.

If P_1 and P_2 are two PFMSs in X, then $L(\varrho : P_1, P_2) = \max\{L(\varrho : P_1, L(\varrho : P_2))\}$. Two PFMSs are called equivalent ($P_1 \sim P_2$) if and only if $L(\varrho : P_1) = L(\varrho : P_2)$.

A is contained in B ($A \subseteq B$), if $\zeta_A(x) \leq \zeta_B(x)$ and $\xi_A(x) \geq \xi_B(x)$, for all $\varrho \in X$ [6].

Some operations [6]:

$$\begin{aligned} &A^c = \{< \varrho, \xi_A(\varrho), \zeta_A(\varrho) >: \varrho \in X\} \\ &A \cup B = \{< \varrho, (\max(\zeta_A(\varrho), \zeta_B(\varrho))), (\min(\xi_A(\varrho), \xi_B(\varrho))) >: \varrho \in X\} \\ &A \cap B = \{< \varrho, (\min(\zeta_A(\varrho), \zeta_B(\varrho))), (\max(\xi_A(\varrho), \xi_B(\varrho))) >: \varrho \in X\} \\ &A \oplus B = \{< \varrho, \sqrt{(\zeta_A(\varrho))^2 + (\zeta_B(\varrho))^2 - (\zeta_A(\varrho))^2.(\zeta_B(\varrho))^2}, \xi_A(\varrho)\xi_B(\varrho) >: \varrho \in X\} \\ &A \otimes B = \{< \varrho, \zeta_A(\varrho)\zeta_B(\varrho), \sqrt{(\xi_A(\varrho))^2 + (\xi_B(\varrho))^2 - (\xi_A(\varrho))^2.(\xi_B(\varrho))^2} >: \varrho \in X\} \end{aligned}$$

3. Multi Robot System

Algorithm:

Step 1: Describe the each sensor reading with values of MS, NS and hesitancy margin.

Step 2: Construct the table related to the robots and the suitable MS functions to the sensor values. The goal is to make an appropriate decision for each Robot. Therefore the readings are observed for a specific interval time (3 minutes).

Step 3: Establish the table related to the sensor readings observed for 3 minutes, one reading per minute.

Step 4: Calculate the distance of every robot to the location considered. So, by using this distance function, PFMS can show the accurate position of each Robot.

Step 5: Choose the lowest distance point in Robots.

Case Study:

In this study, multi-robot scenarios in papers [1] and [12] were used. In [1], continuous ASw is expressed by two sub-problems. First, it is ensured that a single robot in a sub-region can autonomously apply a continuous ASw assignment. Secondly, dividing the entire area among the multiple robots. When the area is split among robots, each one of them sweeps its own part of the environment using the single robot ASw method. In the work of Ahmadi and Stone [1], particularly, the area partitioning is focused.

Shinnoj and Sunil's [12] paper clarifies how the notion of IFMS can be operative in Robotics. To clarify the notion of IFMS, a multi Robot scenario is aforethought occurring of a central server and a group of mobile Robots patrolling a given area for surveillance implementation. Using the distance function, the sensor readings were agreeably explicated for appropriate recognition of the problem faced by the Robot.

In this study, a Nao H25 model humanoid robot (Figure 1) was used [10], [15] .

FIGURE 1. Nao H25 model robot

The sets

$R = \{R1, R2, R3, R4\}$

$C = \{$*Bump(B), Cliff(C), Fire(F), Obstacle(O), Vibration(V)*$\}$

$S = \{$*accelerometer sensor(AS), bump sensor(BS), cliff sensor(CS), ultrasonic sensor(US), Temperature sensor(TS),* $\}$

show Robots, Cases and Sensors, respectively. Different MS and NS values can be assigned to a single robot for five distinct sensor readings.

Step 1: Each sensor reading is given in Table 1 as numbers of MS ζ, NS ξ, and hesitation margin η of a triple PFSM.

Table 1

	F	O	B	C	V
TS	(0.8, 0.3, 0.52)	(0.2, 0.9, 0.39)	(0.4, 0.7, 0.59)	(0.6, 0.5, 0.62)	(0.5, 0.8, 0.33)
US	(0.7, 0.4, 0.59)	(0.9, 0.3, 0.31)	(0.8, 0.6, 0.0)	(0.5, 0.7, 0.50)	(0.3, 0.7, 0.065)
BS	(0.1, 0.9, 0.42)	(0.5, 0.5, 0.70)	(0.9, 0.2, 0.39)	(0.6, 0.7, 0.38)	(0.7, 0.5, 0.50)
CS	(0.3, 0.9, 0.31)	(0.5, 0.8, 0.33)	(0.6, 0.8, 0.0)	(0.8, 0.4, 0.44)	(0.4, 0.9, 0.17)
AS	(0.4, 0.6, 0.69)	(0.2, 0.8, 0.38)	(0.3, 0.8, 0.52)	(0.6, 0.6, 0.53)	(0.7, 0.7, 0.14)

Table 2

	TS	US	BS	CS	AS
R_1	(0.8, 0.3, 0.52)	(0.8, 0.3, 0.52)	(0.1, 0.9, 0.42)	(0.2, 0.8, 0.56)	(0.3, 0.9, 0.31)
R_2	(0.4, 0.7, 0.59)	(0.2, 0.8, 0.56)	(0.4, 0.8, 0.44)	(0.3, 0.9, 0.31)	(0.9, 0.2, 0.38)
R_3	(0.1, 0.9, 0.42)	(0.2, 0.9, 0.38)	(0.8, 0.4, 0.44)	(0.4, 0.7, 0.59)	(0.2, 0.8, 0.56)
R_4	(0.2, 0.9, 0.38)	(0.4, 0.8, 0.44)	(0.3, 0.8, 0.52)	(0.9, 0.3, 0.31)	(0.1, 0.9, 0.42)

Table 3

	TS	US	BS	CS	AS
R_1	(0.64, 0.41, 0.32)	(0.26, 0.33, 0.29)	(0.44, 0.44, 0.35)	(0.2, 0.28, 0.56)	(0.3, 0.29, 0.31)
	(0.28, 0.37, 0.2)	(0.27, 0.66, 0.28)	(0.43, 0.4, 0.44)	(0.05, 0.2, 0.46)	(0.5, 0.46, 0.48)
	(0.36, 0.16, 0.23)	(0.58, 0.34, 0.4)	(0.23, 0.6, 0.34)	(0.67, 0.32, 0.1)	(0.54, 0.3, 0.42)
R_2	(0.51, 0.62, 0.4)	(0.74, 0.36, 0.88)	(0.28, 0.45, 0.13)	(0.14, 0.35, 0.39)	(0.19, 0.46, 0.25)
	(0.55, 0.6, 0.32)	(0.25, 0.37, 0.25)	(0.48, 0.45, 0.14)	(0.62, 0.3, 0.41)	(0.57, 0.35, 0.44)
	(0.59, 0.6, 0.38)	(0.6, 0.4, 0.78)	(0.36, 0.47, 0.18)	(0.55, 0.38, 0.4)	(0.52, 0.41, 0.39)
R_3	(0.38, 0.41, 0.73)	(0.22, 0.23, 0.49)	(0.76, 0.77, 0.54)	(0.26, 0.28, 0.39)	(0.27, 0.44, 0.48)
	(0.07, 0.24, 0.29)	(0.09, 0.22, 0.17)	(0.32, 0.29, 0.24)	(0.31, 0.28, 0.25)	(0.47, 0.44, 0.42)
	(0.11, 0.21, 0.42)	(0.18, 0.12, 0.4)	(0.54, 0.48, 0.46)	(0.18, 0.59, 0.3)	(0.22, 0.58, 0.5)
R_4	(0.78, 0.9, 0.85)	(0.22, 0.15, 0.4)	(0.67, 0.45, 0.36)	(0.53, 0.48, 0.34)	(0.32, 0.6, 0.41)
	(0.49, 0.19, 0.34)	(0.26, 0.29, 0.22)	(0.44, 0.48, 0.81)	(0.56, 0.27, 0.19)	(0.22, 0.58, 0.29)
	(0.57, 0.91, 0.76)	(0.84, 0.75, 0.25)	(0.12, 0.25, 0.09)	(0.65, 0.47, 0.64)	(0.38, 0.25, 0.77)

Table 4

	F	O	B	C	V
R_1	0.9764	0.7483	0.6899	0.836	0.7935
R_2	0.8106	0.6738	0.6496	0.7954	0.8201
R_3	09015	0.713	0.7456	0.5924	0.6972
R_4	0.6158	0.7944	0.6278	0.9245	0.2458

Step 2: Table 2 demonstrates the Robots and their MS functions corresponding to the sensor values. The aim is to make a suitable decision for each Robot. Therefore the readings are observed for a specific interval time (3 minutes).

Step 3: The sensor readings observed for 3 minutes including one reading per minute can be seen in Table 3.

Step 4: Table 4 indicates the distances of every Robot from the location considered.

Information will be obtained by recording the readings of the robots during their tasks for three minutes.

The values obtained are given in Table 4, and the accuracy position of every robot gives the lowest distance point. According to this situation, R_1, R_2, R_3, R_4 are near an Fire, Bump, Cliff and Vibration, respectively.

4. Conclusion

In this study, diverse primary properties of PFSM and its applications in robotics were debated. An algorithm is proposed to model uncertainties in the MAGDM by using PFMSs. In the offered method, the distance of each robot to each situation was measured, taking into account the sensor readings. The notion of multiplicity is combined by taking the samples from the same Robot for a certain period time. Then, the accuracy of the multi-robot system is discussed using PFSM.

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