NEW APPROACH TO UNIFORMLY QUASI CIRCULAR MOTION OF QUASI VELOCITY BIHARMONIC MAGNETIC PARTICLES IN THE HEISENBERG SPACE

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Abstract. We firstly discuss the uniformly quasi circular motion (UQCM) with biharmonicity condition in the Heisenberg space. We define the energy of velocity magnetic particles and some Lorentz fields. Also, we construct the new relationship between the Fermi-Walker parallel transportation and the uniformly quasi circular motion in the Heisenberg space. In other words, we obtain the applied geometric characterization for the uniformly quasi circular motion of biharmonic velocity magnetic particles in the Heisenberg space. This concept also boosts to discover some physical and geometrical characterizations belonging to the particle such as the magnetic motion, the electrical energy functional, the torque, and the Poynting vector. Finally, we obtain electrical energy with respect to its electric field and energy flux density in the radial direction.

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1 Introduction

Magnetic particles gravitate to be a fragment confused. Lorentz field systems are flexible sequences of flows, gases, and solids, are full of intersections, model nonlinear counters, and are greatly influenced by thermal variations. These structures are too sufficient, enveloping biomaterials, colloidal terminations, polymers, foams, liquids, and crystallizes, and have a wide range of associated technologies. Progressing these technologies desires a complex consideration and characterization of the complexities of these extreme flux matter systems, [20,21].

Magnetic torque combinations are extensively operated as a magnetic moment component considering numerous varieties of machines like potential generators, turbines and activators. Magnetic hybrids are frequently embittered meanwhile application of these machines. Melting is a result of the destruction of electrical energy at the elastic magnetic torque. Accordingly, advancement in the capability of elastic magnetic torque is momentous to recover the electrical energy and to bypass to contaminate to bordering machines. Hence, preserving electrical energy benefits at preserving the earth's ordinary reserves and climates. The inferior amorphous-forming capability of hybrids combines cripples with its operational capability at power generators and turbines [22].

Electromagnetic cover, just as an appropriate cover approach to establish a point, has explicit operation forecasts in various areas, being biomedicine, essential sensibility, and act cover [16]. The ultimate significant utilization areas are essentially nosy surgery [9]. The electromagnetic cover system may be managed like a split of the navigation structure to establish a certain-future situation for the pharmaceutical apparatus in the inmate's frame, implement extreme advantage to scientists [7]. Correlated with alternative navigation designs as optical new navigation, the electromagnetic cover has no radioactivity cripple or no constraint of radiation procedure. Also, an electromagnetic traverse system found on abstains energy of some magnetic flux density field characterized by some azimuth intersection is recommended [20].

In the components, considered one of the significant dreams is to get magnetic particles connected to the magnetic field on some manifold. Subsequently, implicit geometrical highlights in this manifold can easily be utilized to decide the curvatures of the magnetic particles. These particles have been largely investigated, realizing that some scientists are classic fundamental with some perspective [1,4-7,11,12,21].

Differentiable function $\phi : N \longrightarrow M$ is labeled biharmonic if ϕ is a critical point for following functional

$$E_2(\phi) = \int_N \frac{1}{2} \left|\mathcal{T}(\phi)\right|^2 dv_h.$$

Here $\mathcal{T}(\phi) := \mathrm{tr}\nabla^{\phi} d\phi$ is tension, [10]. Also, bitension field of ϕ is described by means of

$$\mathcal{T}_2(\phi) = -\Delta_\phi \mathcal{T}(\phi) + \mathrm{tr} R\left(\mathcal{T}(\phi), d\phi\right) d\phi.$$

The biharmonic curve also plays important role in geometry. Also, a large number of experts have analyzed geometric biharmonic particles and surfaces conditions, [23,24]. On the other hand, the energy concept has been obtained with some characterizations, [2,3,13-15].

The construction of our article is as follows: Firstly, we construct the uniformly quasi circular motion (UQCM) with biharmonicity condition in the Heisenberg space. We define the energy of velocity magnetic particles and some Lorentz fields. Also, we construct the new relationship between the Fermi-Walker parallel transportation and the uniformly quasi circular motion in the Heisenberg space. In other words, we obtain the applied geometric characterization for an uniformly quasi circular motion of biharmonic velocity magnetic particles in the Heisenberg space. This concept also boosts to discover some physical and geometrical characterizations belonging to the particle such as the magnetic motion, the potential energy functional, the torque, and the Poynting vector. Finally, we obtain electrical energy with respect to its electric field and energy flux density in the radial direction.

2 Backround on Quasi Frame and Heisenberg Space

New Heisenberg metric is presented by

$$g_h = dx^2 + dy^2 + (dz - xdy)^2.$$

Basis fields Heisenberg space are given by

$$\boldsymbol{\nu}_1 = \frac{\partial}{\partial x}, \quad \boldsymbol{\nu}_2 = \frac{\partial}{\partial y} + x\frac{\partial}{\partial z}, \quad \boldsymbol{\nu}_3 = \frac{\partial}{\partial z}.$$

Let α be a regular particle in Heisenberg space. Also, $\mathbf{e}^{\varepsilon}_{(0)}$, $\mathbf{e}^{\varepsilon}_{(1)}$, and $\mathbf{e}^{\varepsilon}_{(2)}$ be tangent, principal normal, and secondary normal vector field, respectively. Thus, Frenet-Serret equations of a particle are presented by

$$\frac{d\alpha}{ds} = \mathbf{e}^{\varepsilon}_{(0)},$$

$$\begin{bmatrix} \nabla_{\mathbf{e}^{\varepsilon}_{(0)}} \mathbf{e}^{\varepsilon}_{(0)} \\ \nabla_{\mathbf{e}^{\varepsilon}_{(0)}} \mathbf{e}^{\varepsilon}_{(1)} \\ \nabla_{\mathbf{e}^{\varepsilon}_{(0)}} \mathbf{e}^{\varepsilon}_{(2)} \end{bmatrix} = \begin{bmatrix} & \kappa & \\ -\kappa & & \tau \\ & -\tau & \end{bmatrix} \begin{bmatrix} \mathbf{e}^{\varepsilon}_{(0)} \\ \mathbf{e}^{\varepsilon}_{(1)} \\ \mathbf{e}^{\varepsilon}_{(2)} \end{bmatrix},$$

where κ and τ are curvatures of the particle.

The quasi-frame of a regular particle α is given by [8],

$$\boldsymbol{\xi}^{\varepsilon}_{(0)}=\mathbf{e}^{\varepsilon}_{(0)},\boldsymbol{\xi}^{\varepsilon}_{(1)}=\frac{\mathbf{e}^{\varepsilon}_{(0)}\times\mathbf{k}}{\left\|\mathbf{e}^{\varepsilon}_{(0)}\times\mathbf{k}\right\|},\boldsymbol{\xi}^{\varepsilon}_{(2)}=\boldsymbol{\xi}^{\varepsilon}_{(0)}\wedge\boldsymbol{\xi}^{\varepsilon}_{(1)},$$

where projection vector is $k=(0,0,1)$.

New quasi frame equations are presented by

$$\begin{aligned}
\nabla_{\boldsymbol{\xi}^{\varepsilon}_{(0)}}\boldsymbol{\xi}^{\varepsilon}_{(0)} &= \varkappa_1\boldsymbol{\xi}^{\varepsilon}_{(1)}+\varkappa_2\boldsymbol{\xi}^{\varepsilon}_{(2)},\\
\nabla_{\boldsymbol{\xi}^{\varepsilon}_{(0)}}\boldsymbol{\xi}^{\varepsilon}_{(1)} &= -\varkappa_1\boldsymbol{\xi}^{\varepsilon}_{(0)}+\varkappa_3\boldsymbol{\xi}^{\varepsilon}_{(2)},\\
\nabla_{\boldsymbol{\xi}^{\varepsilon}_{(0)}}\boldsymbol{\xi}^{\varepsilon}_{(2)} &= -\varkappa_2\boldsymbol{\xi}^{\varepsilon}_{(0)}-\varkappa_3\boldsymbol{\xi}^{\varepsilon}_{(1)},
\end{aligned}$$

where the angle ψ is between $\mathbf{n_q}$ and $\mathbf{n}$.

Also, quasi curvatures are obtained by

$$\varkappa_1=\kappa\cos\psi,\quad \varkappa_2=-\kappa\sin\psi,\quad \varkappa_3=\psi'+\tau,$$

and

$$\boldsymbol{\xi}^{\varepsilon}_{(1)}\times\boldsymbol{\xi}^{\varepsilon}_{(2)}=\boldsymbol{\xi}^{\varepsilon}_{(0)},\ \boldsymbol{\xi}^{\varepsilon}_{(2)}\times\boldsymbol{\xi}^{\varepsilon}_{(0)}=\boldsymbol{\xi}^{\varepsilon}_{(1)},\ \boldsymbol{\xi}^{\varepsilon}_{(0)}\times\boldsymbol{\xi}^{\varepsilon}_{(1)}=\boldsymbol{\xi}^{\varepsilon}_{(2)}.$$

3 Uniformly Quasi Circular Motion of Velocity Magnetic Biharmonic Particles in Heisenberg Space

In this section, we construct the uniformly quasi circular motion (UQCM) of moving charged velocity magnetic biharmonic particle in Heisenberg space. We obtain necessary and sufficient conditions that have to be satisfied by the particle in terms of the Frenet scalars of the worldline of magnetic curves. We present the following definition of the uniformly quasi accelerated motion [14-19].

Definition 1. *Let α be a regular particle and $\mathcal{B}$ be a magnetic field in Heisenberg space. We call the curve α as a quasi-velocity magnetic particle if the quasi tangent field of the particle satisfies the following Lorentz equation.*

$$\nabla_s\boldsymbol{\xi}^{\varepsilon}_{(0)}=\phi(\boldsymbol{\xi}^{\varepsilon}_{(0)})=\mathcal{B}\times\boldsymbol{\xi}^{\varepsilon}_{(0)}.$$

Let $\nabla_s^{\mathcal{FW}}$ be the covariant derivative corresponding to Levi-Civita connection ∇ of h. Thus, we obtain

$$\nabla_s^{\mathcal{FW}}\mathcal{R} = \nabla_s\mathcal{R} - h\left(\mathbf{T},\mathcal{R}\right)\nabla_s\mathbf{T} + h(\nabla_s\mathbf{T},\mathcal{R})\mathbf{T},$$

where $\mathcal{R}$ is a vector field on particle [20] *.*

Definition 2. *The field $\mathbf{X}$ observes uniformly quasi circular motion iff*

$$\nabla_s^{\mathcal{FW}}(|\nabla_s\mathbf{X}|^{-1}\nabla_s\mathbf{X}) = 0.$$

Theorem 1. *Fermi Walker derivatives of $\phi\left(\boldsymbol{\xi}_{(0)}^{\varepsilon}\right), \phi\left(\boldsymbol{\xi}_{(1)}^{\varepsilon}\right), \phi\left(\boldsymbol{\xi}_{(2)}^{\varepsilon}\right)$ are*

$$\begin{aligned}
\nabla_s^{\mathcal{FW}}\phi(\boldsymbol{\xi}_{(0)}^{\varepsilon}) &= ((\varkappa_2' + \varkappa_1\varkappa_3)\cos\varphi\sin\left[\chi_1 s + \chi_2\right] + (\varkappa_1' - \varkappa_2\varkappa_3)\cos\left[\chi_1 s + \chi_2\right])\boldsymbol{\nu}_1 \\
&\quad + ((\varkappa_2' + \varkappa_1\varkappa_3)\cos\varphi\cos\left[\chi_1 s + \chi_2\right] - (\varkappa_1' - \varkappa_2\varkappa_3)\sin\left[\chi_1 s + \chi_2\right])\boldsymbol{\nu}_2 \\
&\quad - (\varkappa_2' + \varkappa_1\varkappa_3)\sin\varphi\boldsymbol{\nu}_3,
\end{aligned}$$

$$\begin{aligned}
\nabla_s^{\mathcal{FW}}\phi(\boldsymbol{\xi}_{(1)}^{\varepsilon}) &= (\varpi'\cos\varphi\sin\left[\chi_1 s + \chi_2\right] - \varkappa_1'\sin\varphi\sin\left[\chi_1 s + \chi_2\right] - \varkappa_3\varpi\cos\left[\chi_1 s + \chi_2\right])\boldsymbol{\nu}_1 \\
&\quad + (\varpi'\cos\varphi\cos\left[\chi_1 s + \chi_2\right] - \varkappa_1'\sin\varphi\cos\left[\chi_1 s + \chi_2\right] + \varkappa_3\varpi\sin\left[\chi_1 s + \chi_2\right])\boldsymbol{\nu}_2 \\
&\quad - (\varpi'\sin\varphi + \varkappa_1'\cos\varphi)\boldsymbol{\nu}_3,
\end{aligned}$$

$$\begin{aligned}
\nabla_s^{\mathcal{FW}}\phi(\boldsymbol{\xi}_{(2)}^{\varepsilon}) &= -(\varpi\varkappa_3\cos\varphi\sin\left[\chi_1 s + \chi_2\right] + \varkappa_2'\sin\varphi\sin\left[\chi_1 s + \chi_2\right] + \varpi'\cos\left[\chi_1 s + \chi_2\right])\boldsymbol{\nu}_1 \\
&\quad - (\varpi\varkappa_3\cos\varphi\cos\left[\chi_1 s + \chi_2\right] + \varkappa_2'\sin\varphi\cos\left[\chi_1 s + \chi_2\right] - \varpi'\sin\left[\chi_1 s + \chi_2\right])\boldsymbol{\nu}_2 \\
&\quad + (\varpi\varkappa_3\sin\varphi - \varkappa_2'\cos\varphi)\boldsymbol{\nu}_3.
\end{aligned}$$

Theorem 2. $\phi\left(\boldsymbol{\xi}_{(0)}^{\varepsilon}\right), \phi\left(\boldsymbol{\xi}_{(1)}^{\varepsilon}\right), \phi\left(\boldsymbol{\xi}_{(2)}^{\varepsilon}\right)$ *are Fermi Walker parallel iff*

$$\begin{gathered}
((\varkappa_2' + \varkappa_1\varkappa_3)\cos\varphi\sin\left[\chi_1 s + \chi_2\right] + (\varkappa_1' - \varkappa_2\varkappa_3)\cos\left[\chi_1 s + \chi_2\right]) = 0, \\
((\varkappa_2' + \varkappa_1\varkappa_3)\cos\varphi\cos\left[\chi_1 s + \chi_2\right] - (\varkappa_1' - \varkappa_2\varkappa_3)\sin\left[\chi_1 s + \chi_2\right]) = 0, \\
-(\varkappa_2' + \varkappa_1\varkappa_3)\sin\varphi = 0,
\end{gathered}$$

$$\begin{gathered}
(\varpi'\cos\varphi\sin\left[\chi_1 s + \chi_2\right] - \varkappa_1'\sin\varphi\sin\left[\chi_1 s + \chi_2\right] - \varkappa_3\varpi\cos\left[\chi_1 s + \chi_2\right]) = 0, \\
(\varpi'\cos\varphi\cos\left[\chi_1 s + \chi_2\right] - \varkappa_1'\sin\varphi\cos\left[\chi_1 s + \chi_2\right] + \varkappa_3\varpi\sin\left[\chi_1 s + \chi_2\right]) = 0, \\
(\varpi'\sin\varphi + \varkappa_1'\cos\varphi) = 0,
\end{gathered}$$

$$(\varpi\varkappa_3\cos\varphi\sin[\chi_1 s+\chi_2]+\varkappa_2'\sin\varphi\sin[\chi_1 s+\chi_2]+\varpi'\cos[\chi_1 s+\chi_2])=0,$$
$$(\varpi\varkappa_3\cos\varphi\cos[\chi_1 s+\chi_2]+\varkappa_2'\sin\varphi\cos[\chi_1 s+\chi_2]-\varpi'\sin[\chi_1 s+\chi_2])=0,$$
$$(\varpi\varkappa_3\sin\varphi-\varkappa_2'\cos\varphi)=0.$$

4 Unchanged quasi direction motion (UDQM)

In this section, we will characterize the uniformly quasi circular motion (UQCM) in the Heisenberg space.

Uniformly quasi circular motion (UQCM) is defined following condition:

$$\nabla_s^{\mathcal{FW}}(|\nabla_s\mathbf{X}|^{-1}\nabla_s\mathbf{X})=0.$$

From quasi frame, we get

$$\begin{aligned}
&\nabla_s^{\mathcal{FW}}(\left|\nabla_s\phi(\boldsymbol{\xi}_{(0)}^{\varepsilon})\right|^{-1}\nabla_s\phi(\boldsymbol{\xi}_{(0)}^{\varepsilon}))=[\psi(((\varkappa_2'+\varkappa_1\varkappa_3)'+\varkappa_3(\varkappa_1'-\varkappa_2\varkappa_3))\cos\varphi\sin[\chi_1 s+\chi_2]\\
&-((\varkappa_1^2+\varkappa_2^2)')\sin\varphi\sin[\chi_1 s+\chi_2]+((\varkappa_1'-\varkappa_2\varkappa_3)'-\varkappa_3(\varkappa_2'+\varkappa_1\varkappa_3))\cos[\chi_1 s+\chi_2])+\psi'((\varkappa_2'\\
&+\varkappa_1\varkappa_3)\cos\varphi\sin[\chi_1 s+\chi_2]-(\varkappa_1^2+\varkappa_2^2)\sin\varphi\sin[\chi_1 s+\chi_2]+(\varkappa_1'-\varkappa_2\varkappa_3)\cos[\chi_1 s+\chi_2])]\boldsymbol{\nu}_1\\
&+[\psi(((\varkappa_2'+\varkappa_1\varkappa_3)'+\varkappa_3(\varkappa_1'-\varkappa_2\varkappa_3))\cos\varphi\cos[\chi_1 s+\chi_2]-((\varkappa_1^2+\varkappa_2^2)')\sin\varphi\cos[\chi_1 s+\chi_2]\\
&-((\varkappa_1'-\varkappa_2\varkappa_3)'-\varkappa_3(\varkappa_2'+\varkappa_1\varkappa_3))\sin[\chi_1 s+\chi_2])+\psi'((\varkappa_2'+\varkappa_1\varkappa_3)\cos\varphi\cos[\chi_1 s+\chi_2]\\
&-(\varkappa_1^2+\varkappa_2^2)\sin\varphi\cos[\chi_1 s+\chi_2]-(\varkappa_1'-\varkappa_2\varkappa_3)\sin[\chi_1 s+\chi_2])]\boldsymbol{\nu}_2-[\psi(((\varkappa_2'+\varkappa_1\varkappa_3)'\\
&+\varkappa_3(\varkappa_1'-\varkappa_2\varkappa_3))\sin\varphi+((\varkappa_1^2+\varkappa_2^2)')\cos\varphi)+\psi'((\varkappa_2'+\varkappa_1\varkappa_3)\sin\varphi+(\varkappa_1^2+\varkappa_2^2)\cos\varphi)]\boldsymbol{\nu}_3,
\end{aligned}$$

where $\psi=\left|\nabla_s\phi(\boldsymbol{\xi}_{(0)}^{\varepsilon})\right|^{-1}$.

♠*The $\phi(\boldsymbol{\xi}_{(0)}^{\varepsilon})$ obeys uniformly quasi circular motion iff*

$$\begin{aligned}
&[\psi(((\varkappa_2'+\varkappa_1\varkappa_3)'+\varkappa_3(\varkappa_1'-\varkappa_2\varkappa_3))\cos\varphi\sin[\chi_1 s+\chi_2]-((\varkappa_1^2+\varkappa_2^2)')\sin\varphi\sin[\chi_1 s+\chi_2]\\
&+((\varkappa_1'-\varkappa_2\varkappa_3)'-\varkappa_3(\varkappa_2'+\varkappa_1\varkappa_3))\cos[\chi_1 s+\chi_2])+\psi'((\varkappa_2'+\varkappa_1\varkappa_3)\cos\varphi\sin[\chi_1 s+\chi_2]\\
&\quad-(\varkappa_1^2+\varkappa_2^2)\sin\varphi\sin[\chi_1 s+\chi_2]+(\varkappa_1'-\varkappa_2\varkappa_3)\cos[\chi_1 s+\chi_2])]=0,\\
&+[\psi(((\varkappa_2'+\varkappa_1\varkappa_3)'+\varkappa_3(\varkappa_1'-\varkappa_2\varkappa_3))\cos\varphi\cos[\chi_1 s+\chi_2]-((\varkappa_1^2+\varkappa_2^2)')\sin\varphi\cos[\chi_1 s+\chi_2]\\
&-((\varkappa_1'-\varkappa_2\varkappa_3)'-\varkappa_3(\varkappa_2'+\varkappa_1\varkappa_3))\sin[\chi_1 s+\chi_2])+\psi'((\varkappa_2'+\varkappa_1\varkappa_3)\cos\varphi\cos[\chi_1 s+\chi_2]\\
&\quad-(\varkappa_1^2+\varkappa_2^2)\sin\varphi\cos[\chi_1 s+\chi_2]-(\varkappa_1'-\varkappa_2\varkappa_3)\sin[\chi_1 s+\chi_2])]=0,\\
&\quad-[\psi(((\varkappa_2'+\varkappa_1\varkappa_3)'+\varkappa_3(\varkappa_1'-\varkappa_2\varkappa_3))\sin\varphi+((\varkappa_1^2+\varkappa_2^2)')\cos\varphi)\\
&\qquad+\psi'((\varkappa_2'+\varkappa_1\varkappa_3)\sin\varphi+(\varkappa_1^2+\varkappa_2^2)\cos\varphi)]=0,
\end{aligned}$$

where $\psi = \left|\nabla_s \phi(\boldsymbol{\xi}^{\varepsilon}_{(0)})\right|^{-1}$.

Also,

$$\nabla_s^{\mathcal{FW}}(\left|\nabla_s \phi(\boldsymbol{\xi}^{\varepsilon}_{(1)})\right|^{-1} \nabla_s \phi(\boldsymbol{\xi}^{\varepsilon}_{(1)})) = [\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\cos\varphi \sin[\chi_1 s + \chi_2]$$
$$-((\varkappa_1' + \varkappa_2\varpi)')\sin\varphi\sin[\chi_1 s + \chi_2] - ((\varkappa_1^2 + \varkappa_3\varpi)' + \varkappa_3(\varpi' - \varkappa_1\varkappa_2))\cos[\chi_1 s + \chi_2])$$
$$+\delta'((\varpi' - \varkappa_1\varkappa_2)\cos\varphi\sin[\chi_1 s + \chi_2] - (\varkappa_1' + \varkappa_2\varpi)\sin\varphi\sin[\chi_1 s + \chi_2] - (\varkappa_1^2$$
$$+\varkappa_3\varpi)\cos[\chi_1 s + \chi_2])]\boldsymbol{\nu}_1 + [\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\cos\varphi\cos[\chi_1 s + \chi_2]$$
$$-((\varkappa_1' + \varkappa_2\varpi)')\sin\varphi\cos[\chi_1 s + \chi_2] + ((\varkappa_1^2 + \varkappa_3\varpi)' + \varkappa_3(\varpi' - \varkappa_1\varkappa_2))\sin[\chi_1 s + \chi_2])$$
$$+\delta'((\varpi' - \varkappa_1\varkappa_2)\cos\varphi\cos[\chi_1 s + \chi_2] - (\varkappa_1' + \varkappa_2\varpi)\sin\varphi\cos[\chi_1 s + \chi_2]$$
$$+(\varkappa_1^2 + \varkappa_3\varpi)\sin[\chi_1 s + \chi_2])]\boldsymbol{\nu}_2 - [\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\sin\varphi$$
$$+((\varkappa_1' + \varkappa_2\varpi)')\cos\varphi) + \delta'((\varpi' - \varkappa_1\varkappa_2)\sin\varphi + (\varkappa_1' + \varkappa_2\varpi)\cos\varphi)]\boldsymbol{\nu}_3.$$

♠*The* $\phi(\boldsymbol{\xi}^{\varepsilon}_{(1)})$ *obeys uniformly quasi circular motion iff*

$$[\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\cos\varphi\sin[\chi_1 s + \chi_2] - ((\varkappa_1' + \varkappa_2\varpi)')\sin\varphi\sin[\chi_1 s + \chi_2]$$
$$-((\varkappa_1^2 + \varkappa_3\varpi)' + \varkappa_3(\varpi' - \varkappa_1\varkappa_2))\cos[\chi_1 s + \chi_2]) + \delta'((\varpi' - \varkappa_1\varkappa_2)\cos\varphi\sin[\chi_1 s + \chi_2]$$
$$-(\varkappa_1' + \varkappa_2\varpi)\sin\varphi\sin[\chi_1 s + \chi_2] - (\varkappa_1^2 + \varkappa_3\varpi)\cos[\chi_1 s + \chi_2])] = 0,$$
$$[\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\cos\varphi\cos[\chi_1 s + \chi_2] - ((\varkappa_1' + \varkappa_2\varpi)')\sin\varphi\cos[\chi_1 s + \chi_2]$$
$$+((\varkappa_1^2 + \varkappa_3\varpi)' + \varkappa_3(\varpi' - \varkappa_1\varkappa_2))\sin[\chi_1 s + \chi_2]) + \delta'((\varpi' - \varkappa_1\varkappa_2)\cos\varphi\cos[\chi_1 s + \chi_2]$$
$$-(\varkappa_1' + \varkappa_2\varpi)\sin\varphi\cos[\chi_1 s + \chi_2] + (\varkappa_1^2 + \varkappa_3\varpi)\sin[\chi_1 s + \chi_2])] = 0,$$
$$[\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\sin\varphi + ((\varkappa_1' + \varkappa_2\varpi)')\cos\varphi)$$
$$+\delta'((\varpi' - \varkappa_1\varkappa_2)\sin\varphi + (\varkappa_1' + \varkappa_2\varpi)\cos\varphi)] = 0,$$

where $\delta = \left|\nabla_s \phi(\boldsymbol{\xi}^{\varepsilon}_{(1)})\right|^{-1}$.

In a similar way, we get

$$\nabla_s^{\mathcal{FW}}(\left|\nabla_s\phi(\boldsymbol{\xi}^{\varepsilon}_{(2)})\right|^{-1}\nabla_s\phi(\boldsymbol{\xi}^{\varepsilon}_{(2)}))=[\xi((-(\varpi'+\varkappa_2\varkappa_1)\varkappa_3-(\varkappa_2^2+\varpi\varkappa_3)')\cos\varphi\sin[\chi_1 s+\chi_2]$$
$$+(\varpi\varkappa_1-\varkappa_2')'\sin\varphi\sin[\chi_1 s+\chi_2]+(-(\varpi'+\varkappa_2\varkappa_1)'+\varkappa_3(\varkappa_2^2+\varpi\varkappa_3))\cos[\chi_1 s+\chi_2])$$
$$+\xi'((\varpi\varkappa_1-\varkappa_2')\sin\varphi\sin[\chi_1 s+\chi_2]-(\varpi'+\varkappa_2\varkappa_1)\cos[\chi_1 s+\chi_2]-(\varkappa_2^2+\varpi\varkappa_3)\cos\varphi$$
$$\sin[\chi_1 s+\chi_2])]\boldsymbol{\nu}_1+[\xi((-(\varpi'+\varkappa_2\varkappa_1)\varkappa_3-(\varkappa_2^2+\varpi\varkappa_3)')\cos\varphi\cos[\chi_1 s+\chi_2]$$
$$+(\varpi\varkappa_1-\varkappa_2')'\sin\varphi\cos[\chi_1 s+\chi_2]-(-(\varpi'+\varkappa_2\varkappa_1)'+\varkappa_3(\varkappa_2^2+\varpi\varkappa_3))\sin[\chi_1 s+\chi_2])$$
$$+\xi'((\varpi\varkappa_1-\varkappa_2')\sin\varphi\cos[\chi_1 s+\chi_2]+(\varpi'+\varkappa_2\varkappa_1)\sin[\chi_1 s+\chi_2]-(\varkappa_2^2$$
$$+\varpi\varkappa_3)\cos\varphi\cos[\chi_1 s+\chi_2])]\boldsymbol{\nu}_2+[\xi((\varpi\varkappa_1-\varkappa_2')'\cos\varphi-(-(\varpi'+\varkappa_2\varkappa_1)\varkappa_3$$
$$-(\varkappa_2^2+\varpi\varkappa_3)')\sin\varphi)+\xi'((\varpi\varkappa_1-\varkappa_2')\cos\varphi+(\varkappa_2^2+\varpi\varkappa_3)\sin\varphi)]\boldsymbol{\nu}_3.$$

♠*The* $\phi(\boldsymbol{\xi}^{\varepsilon}_{(2)})$ *obeys uniformly quasi circular motion iff*

$$[\xi((-(\varpi'+\varkappa_2\varkappa_1)\varkappa_3-(\varkappa_2^2+\varpi\varkappa_3)')\cos\varphi\sin[\chi_1 s+\chi_2]+(\varpi\varkappa_1-\varkappa_2')'\sin\varphi\sin[\chi_1 s+\chi_2]$$
$$+(-(\varpi'+\varkappa_2\varkappa_1)'+\varkappa_3(\varkappa_2^2+\varpi\varkappa_3))\cos[\chi_1 s+\chi_2])+\xi'((\varpi\varkappa_1-\varkappa_2')\sin\varphi\sin[\chi_1 s+\chi_2]$$
$$-(\varpi'+\varkappa_2\varkappa_1)\cos[\chi_1 s+\chi_2]-(\varkappa_2^2+\varpi\varkappa_3)\cos\varphi\sin[\chi_1 s+\chi_2])]=0,$$
$$[\xi((-(\varpi'+\varkappa_2\varkappa_1)\varkappa_3-(\varkappa_2^2+\varpi\varkappa_3)')\cos\varphi\cos[\chi_1 s+\chi_2]+(\varpi\varkappa_1-\varkappa_2')'\sin\varphi\cos[\chi_1 s+\chi_2]$$
$$-(-(\varpi'+\varkappa_2\varkappa_1)'+\varkappa_3(\varkappa_2^2+\varpi\varkappa_3))\sin[\chi_1 s+\chi_2])+\xi'((\varpi\varkappa_1-\varkappa_2')\sin\varphi\cos[\chi_1 s+\chi_2]$$
$$+(\varpi'+\varkappa_2\varkappa_1)\sin[\chi_1 s+\chi_2]-(\varkappa_2^2+\varpi\varkappa_3)\cos\varphi\cos[\chi_1 s+\chi_2])]=0,$$
$$[\xi((\varpi\varkappa_1-\varkappa_2')'\cos\varphi-(-(\varpi'+\varkappa_2\varkappa_1)\varkappa_3-(\varkappa_2^2+\varpi\varkappa_3)')\sin\varphi)$$
$$+\xi'((\varpi\varkappa_1-\varkappa_2')\cos\varphi+(\varkappa_2^2+\varpi\varkappa_3)\sin\varphi)]=0,$$

where $\xi=\left|\nabla_s\phi(\boldsymbol{\xi}^{\varepsilon}_{(2)})\right|^{-1}$.

5 Energy Flux Density

The force acting on an electron in the cold plasma is given by

$$\mathbf{F}=q(\mathcal{E}+\mathbf{v}\times\mathcal{B}),\ \frac{m}{q}=\sigma,$$

where $\mathbf{v}$ is a velocity of electrons, m is a mass, q is an electric charge, $\mathcal{B}$ is a magnetic field and $\mathcal{E}$ is an electric field [18,19].

From the force equation, the electric field is given by

$$\begin{aligned}\mathcal{E} &= ((\sigma\varkappa_2+\varkappa_1)\cos\varphi\sin[\chi_1 s+\chi_2]+(\sigma\varkappa_1+\varkappa_2)\cos[\chi_1 s+\chi_2])\boldsymbol{\nu}_1+((\sigma\varkappa_2\\ &\quad+\varkappa_1)\cos\varphi\cos[\chi_1 s+\chi_2]-(\sigma\varkappa_1+\varkappa_2)\sin[\chi_1 s+\chi_2])\boldsymbol{\nu}_2-(\sigma\varkappa_2+\varkappa_1)\sin\varphi\boldsymbol{\nu}_3.\end{aligned}$$

The last relation gives

$$\begin{gathered}\nabla_s^{\mathcal{FW}}(|\nabla_s\mathcal{E}|^{-1}\nabla_s\mathcal{E})=[((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))'+\varkappa_3((\sigma\varkappa_1+\varkappa_2)'\\ -\varkappa_3(\sigma\varkappa_2+\varkappa_1)))\cos\varphi\sin[\chi_1 s+\chi_2]+(((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))'\\ -\varkappa_3((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2)))\cos[\chi_1 s+\chi_2]-((\varkappa_1(\sigma\varkappa_1+\varkappa_2)\\ +\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\sin\varphi\sin[\chi_1 s+\chi_2])+\vartheta'(((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1\\ +\varkappa_2))\cos\varphi\sin[\chi_1 s+\chi_2]-(\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\sin\varphi\\ \times\sin[\chi_1 s+\chi_2]+((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))\cos[\chi_1 s+\chi_2])]\boldsymbol{\nu}_1\\ +[((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))'+\varkappa_3((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1)))\\ \times\cos\varphi\cos[\chi_1 s+\chi_2]-((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\sin\varphi\cos[\chi_1 s+\chi_2]\\ -(((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))'-\varkappa_3((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2)))\\ \times\sin[\chi_1 s+\chi_2])+\vartheta'(((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))\cos\varphi\cos[\chi_1 s+\chi_2]\\ -(\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\sin\varphi\cos[\chi_1 s+\chi_2]-((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2\\ +\varkappa_1))\sin[\chi_1 s+\chi_2])]\boldsymbol{\nu}_2-[((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))'+\varkappa_3((\sigma\varkappa_1+\varkappa_2)'\\ -\varkappa_3(\sigma\varkappa_2+\varkappa_1)))\sin\varphi+((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\cos\varphi)+\vartheta'(((\sigma\varkappa_2+\varkappa_1)'\\ +\varkappa_3(\sigma\varkappa_1+\varkappa_2))\sin\varphi+(\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\cos\varphi)]\boldsymbol{\nu}_3.\end{gathered}$$

♠*The $\mathcal{E}$ obeys uniformly quasi circular motion iff*

$$\begin{gathered}[((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))'+\varkappa_3((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1)))\cos\varphi\\ \times\sin[\chi_1 s+\chi_2]+(((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))'-\varkappa_3((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1\\ +\varkappa_2)))\cos[\chi_1 s+\chi_2]-((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\sin\varphi\sin[\chi_1 s+\chi_2])\\ +\vartheta'(((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))\cos\varphi\sin[\chi_1 s+\chi_2]-(\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2\\ +\varkappa_1))\sin\varphi\sin[\chi_1 s+\chi_2]+((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))\cos[\chi_1 s+\chi_2])]=0,\\ [((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))'+\varkappa_3((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1)))\cos\varphi\\ \cos[\chi_1 s+\chi_2]-((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\sin\varphi\cos[\chi_1 s+\chi_2]-(((\sigma\varkappa_1\\ +\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))'-\varkappa_3((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2)))\sin[\chi_1 s+\chi_2])\\ +\vartheta'(((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))\cos\varphi\cos[\chi_1 s+\chi_2]-(\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2\\ +\varkappa_1))\sin\varphi\cos[\chi_1 s+\chi_2]-((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))\sin[\chi_1 s+\chi_2])]=0,\\ [((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))'+\varkappa_3((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1)))\sin\varphi\\ +((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\cos\varphi)+\vartheta'(((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1\\ +\varkappa_2))\sin\varphi+(\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\cos\varphi)]=0,\end{gathered}$$

where $\vartheta = |\nabla_s \mathcal{E}|^{-1}$.

By magnetic field field, we get

$$
\begin{gathered}
\nabla_s^{\mathcal{FW}}(|\nabla_s \mathcal{B}|^{-1}\,\nabla_s \mathcal{B}) = [\eta(\varpi'' \sin\varphi \sin[\chi_1 s+\chi_2] + ((\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2')' \\
-\varkappa_3(\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1'))\cos[\chi_1 s+\chi_2] + ((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 \\
-\varkappa_1\varkappa_3 - \varkappa_2'))\cos\varphi\sin[\chi_1 s+\chi_2]) + \eta'(\varpi'\sin\varphi\sin[\chi_1 s+\chi_2] + (\varpi\varkappa_1 - \varkappa_1\varkappa_3 \\
-\varkappa_2')\cos[\chi_1 s+\chi_2] + (\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')\cos\varphi\sin[\chi_1 s+\chi_2])]\boldsymbol{\nu}_1 \\
+[\eta(\varpi''\sin\varphi\cos[\chi_1 s+\chi_2] - ((\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2')' - \varkappa_3(\varpi\varkappa_2 - \varkappa_2\varkappa_3 \\
+\varkappa_1'))\sin[\chi_1 s+\chi_2] + ((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 - \varkappa_1\varkappa_3 \\
-\varkappa_2'))\cos\varphi\cos[\chi_1 s+\chi_2]) + \eta'(\varpi'\sin\varphi\cos[\chi_1 s+\chi_2] - (\varpi\varkappa_1 \\
-\varkappa_1\varkappa_3 - \varkappa_2')\sin[\chi_1 s+\chi_2] + (\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')\cos\varphi\cos[\chi_1 s+\chi_2])]\boldsymbol{\nu}_2 \\
+[\eta(\varpi''\cos\varphi - ((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2'))\sin\varphi) \\
+\eta'(\varpi'\cos\varphi - (\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')\sin\varphi)]\boldsymbol{\nu}_3.
\end{gathered}
$$

♠*The $\mathcal{B}$ obeys uniformly quasi circular motion iff*

$$
\begin{gathered}
[\eta(\varpi''\sin\varphi\sin[\chi_1 s+\chi_2] + ((\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2')' - \varkappa_3(\varpi\varkappa_2 - \varkappa_2\varkappa_3 \\
+\varkappa_1'))\cos[\chi_1 s+\chi_2] + ((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 - \varkappa_1\varkappa_3 \\
-\varkappa_2'))\cos\varphi\sin[\chi_1 s+\chi_2]) + \eta'(\varpi'\sin\varphi\sin[\chi_1 s+\chi_2] + (\varpi\varkappa_1 - \varkappa_1\varkappa_3 \\
-\varkappa_2')\cos[\chi_1 s+\chi_2] + (\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')\cos\varphi\sin[\chi_1 s+\chi_2])] = 0, \\
[\eta(\varpi''\sin\varphi\cos[\chi_1 s+\chi_2] - ((\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2')' - \varkappa_3(\varpi\varkappa_2 - \varkappa_2\varkappa_3 \\
+\varkappa_1'))\sin[\chi_1 s+\chi_2] + ((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 - \varkappa_1\varkappa_3 \\
-\varkappa_2'))\cos\varphi\cos[\chi_1 s+\chi_2]) + \eta'(\varpi'\sin\varphi\cos[\chi_1 s+\chi_2] - (\varpi\varkappa_1 - \varkappa_1\varkappa_3 \\
-\varkappa_2')\sin[\chi_1 s+\chi_2] + (\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')\cos\varphi\cos[\chi_1 s+\chi_2])] = 0, \\
[\eta(\varpi''\cos\varphi - ((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2'))\sin\varphi) \\
+\eta'(\varpi'\cos\varphi - (\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')\sin\varphi)] = 0,
\end{gathered}
$$

where $\eta = |\nabla_s \mathcal{B}|^{-1}$.

By using quasi-velocity magnetic fields, we have

$$
\begin{aligned}
\phi(\mathcal{E}) \quad = \quad & -((\varkappa_1(\sigma\varkappa_1 + \varkappa_2) + \varkappa_2(\sigma\varkappa_2 + \varkappa_1))\sin\varphi\sin[\chi_1 s+\chi_2] + \varpi(\sigma\varkappa_2 \\
& +\varkappa_1)\cos[\chi_1 s+\chi_2] - \varpi(\sigma\varkappa_1 + \varkappa_2)\cos\varphi\sin[\chi_1 s+\chi_2])\boldsymbol{\nu}_1 \\
& -((\varkappa_1(\sigma\varkappa_1 + \varkappa_2) + \varkappa_2(\sigma\varkappa_2 + \varkappa_1))\sin\varphi\cos[\chi_1 s+\chi_2] - \varpi(\sigma\varkappa_2 \\
& +\varkappa_1)\sin[\chi_1 s+\chi_2] - \varpi(\sigma\varkappa_1 + \varkappa_2)\cos\varphi\cos[\chi_1 s+\chi_2])\boldsymbol{\nu}_2 \\
& -((\varkappa_1(\sigma\varkappa_1 + \varkappa_2) + \varkappa_2(\sigma\varkappa_2 + \varkappa_1))\cos\varphi + \varpi(\sigma\varkappa_1 + \varkappa_2)\sin\varphi)\boldsymbol{\nu}_3.
\end{aligned}
$$

By above tensor field, energy flux density is presented by

$$\begin{aligned}\mathcal{S} &= ((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\sin\varphi\sin[\chi_1 s+\chi_2]+\varpi(\sigma\varkappa_2+\varkappa_1)\cos[\chi_1 s+\chi_2] \\ &\quad -\varpi(\sigma\varkappa_1+\varkappa_2)\cos\varphi\sin[\chi_1 s+\chi_2])\boldsymbol{\nu}_1+((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\sin\varphi \\ &\quad \times\cos[\chi_1 s+\chi_2]-\varpi(\sigma\varkappa_2+\varkappa_1)\sin[\chi_1 s+\chi_2]-\varpi(\sigma\varkappa_1+\varkappa_2)\cos\varphi\cos[\chi_1 s+\chi_2])\boldsymbol{\nu}_2 \\ &\quad +((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\cos\varphi+\varpi(\sigma\varkappa_1+\varkappa_2)\sin\varphi)\boldsymbol{\nu}_3.\end{aligned}$$

6 Application to uniformly quasi circular Potential Electric Energy

In this section, we obtain the polar plot for the time variation of potential electrical energy with respect to its electric field and energy flux density in the radial direction.

Uniformly quasi circular potential electric energy of field $\mathbf{X}$ in the electric field $\mathcal{E}$ is defined by

$$\mathcal{L}^{\mathcal{FW}}\mathbf{X}=\nabla_s^{\mathcal{FW}}(|\nabla_s\mathbf{X}|^{-1}\nabla_s\mathbf{X})\cdot\mathcal{E}$$

• Uniformly quasi circular potential electric energy of $\phi(\boldsymbol{\xi}^{\varepsilon}_{(0)})$ in the electric field $\mathcal{E}$

$$\begin{aligned}&\mathcal{L}^{\mathcal{FW}}\phi(\boldsymbol{\xi}^{\varepsilon}_{(0)})=((\sigma\varkappa_2+\varkappa_1)\cos\varphi\sin[\chi_1 s+\chi_2]+(\sigma\varkappa_1+\varkappa_2)\cos[\chi_1 s+\chi_2])[\psi(((\varkappa_2' \\ &+\varkappa_1\varkappa_3)'+\varkappa_3(\varkappa_1'-\varkappa_2\varkappa_3))\cos\varphi\sin[\chi_1 s+\chi_2]-((\varkappa_1^2+\varkappa_2^2)')\sin\varphi\sin[\chi_1 s+\chi_2] \\ &+((\varkappa_1'-\varkappa_2\varkappa_3)'-\varkappa_3(\varkappa_2'+\varkappa_1\varkappa_3))\cos[\chi_1 s+\chi_2])+\psi'((\varkappa_2'+\varkappa_1\varkappa_3)\cos\varphi\sin[\chi_1 s+\chi_2]-(\varkappa_1^2 \\ &+\varkappa_2^2)\sin\varphi\sin[\chi_1 s+\chi_2]+(\varkappa_1'-\varkappa_2\varkappa_3)\cos[\chi_1 s+\chi_2])]+((\sigma\varkappa_2+\varkappa_1)\cos\varphi\cos[\chi_1 s+\chi_2] \\ &-(\sigma\varkappa_1+\varkappa_2)\sin[\chi_1 s+\chi_2])[\psi(((\varkappa_2'+\varkappa_1\varkappa_3)'+\varkappa_3(\varkappa_1'-\varkappa_2\varkappa_3))\cos\varphi\cos[\chi_1 s+\chi_2] \\ &-((\varkappa_1^2+\varkappa_2^2)')\sin\varphi\cos[\chi_1 s+\chi_2]-((\varkappa_1'-\varkappa_2\varkappa_3)'-\varkappa_3(\varkappa_2'+\varkappa_1\varkappa_3))\sin[\chi_1 s+\chi_2]) \\ &+\psi'((\varkappa_2'+\varkappa_1\varkappa_3)\cos\varphi\cos[\chi_1 s+\chi_2]-(\varkappa_1^2+\varkappa_2^2)\sin\varphi\cos[\chi_1 s+\chi_2]-(\varkappa_1' \\ &\quad -\varkappa_2\varkappa_3)\sin[\chi_1 s+\chi_2])]+(\sigma\varkappa_2+\varkappa_1)\sin\varphi[\psi(((\varkappa_2'+\varkappa_1\varkappa_3)'+\varkappa_3(\varkappa_1' \\ &-\varkappa_2\varkappa_3))\sin\varphi+((\varkappa_1^2+\varkappa_2^2)')\cos\varphi)+\psi'((\varkappa_2'+\varkappa_1\varkappa_3)\sin\varphi+(\varkappa_1^2+\varkappa_2^2)\cos\varphi)].\end{aligned}$$

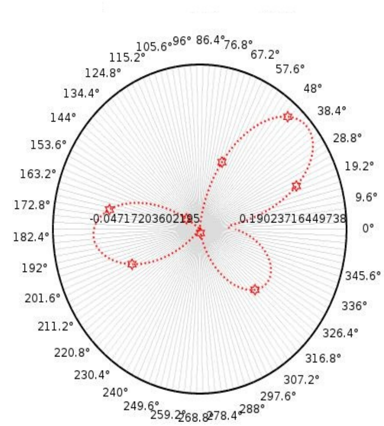

Fig.1. Unchanged quasi potential electric energy of $\phi(\mathring{\mathbf{e}}^{\varepsilon}_{(0)})$ in the electric field $\mathcal{E}$

Figure 1 shows the polar modeling for the time variation of the unchanged quasi potential electric energy of of $\phi(\mathring{\mathbf{e}}^{\varepsilon}_{(0)})$ corresponding to its base in a radial direction.

• Uniformly quasi circular potential electric energy of $\phi(\boldsymbol{\xi}^{\varepsilon}_{(1)})$ in the electric field $\mathcal{E}$

$$
\begin{aligned}
\mathcal{L}^{\mathcal{FW}}\phi(\boldsymbol{\xi}^{\varepsilon}_{(1)}) &= [\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\cos\varphi\sin[\chi_1 s + \chi_2] - ((\varkappa_1' \\
&+\varkappa_2\varpi)')\sin\varphi\sin[\chi_1 s + \chi_2] - ((\varkappa_1^2 + \varkappa_3\varpi)' + \varkappa_3(\varpi' - \varkappa_1\varkappa_2))\cos[\chi_1 s + \chi_2]) \\
&+\delta'((\varpi' - \varkappa_1\varkappa_2)\cos\varphi\sin[\chi_1 s + \chi_2] - (\varkappa_1' + \varkappa_2\varpi)\sin\varphi\sin[\chi_1 s + \chi_2] \\
&-(\varkappa_1^2 + \varkappa_3\varpi)\cos[\chi_1 s + \chi_2])][((\sigma\varkappa_2 + \varkappa_1)\cos\varphi\sin[\chi_1 s + \chi_2] + (\sigma\varkappa_1 \\
&+\varkappa_2)\cos[\chi_1 s + \chi_2]) + [\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\cos\varphi\cos[\chi_1 s + \chi_2] \\
&-((\varkappa_1' + \varkappa_2\varpi)')\sin\varphi\cos[\chi_1 s + \chi_2] + ((\varkappa_1^2 + \varkappa_3\varpi)' + \varkappa_3(\varpi' - \varkappa_1\varkappa_2))\sin[\chi_1 s + \chi_2]) \\
&+\delta'((\varpi' - \varkappa_1\varkappa_2)\cos\varphi\cos[\chi_1 s + \chi_2] - (\varkappa_1' + \varkappa_2\varpi)\sin\varphi\cos[\chi_1 s + \chi_2] \\
&+(\varkappa_1^2 + \varkappa_3\varpi)\sin[\chi_1 s + \chi_2])][((\sigma\varkappa_2 + \varkappa_1)\cos\varphi\cos[\chi_1 s + \chi_2] - (\sigma\varkappa_1 + \varkappa_2) \\
&\times\sin[\chi_1 s + \chi_2]) + [\delta(((\varpi' - \varkappa_1\varkappa_2)' - (\varkappa_1^2 + \varkappa_3\varpi)\varkappa_3)\sin\varphi + ((\varkappa_1' + \varkappa_2\varpi)')\cos\varphi) \\
&+\delta'((\varpi' - \varkappa_1\varkappa_2)\sin\varphi + (\varkappa_1' + \varkappa_2\varpi)\cos\varphi)](\sigma\varkappa_2 + \varkappa_1)\sin\varphi.
\end{aligned}
$$

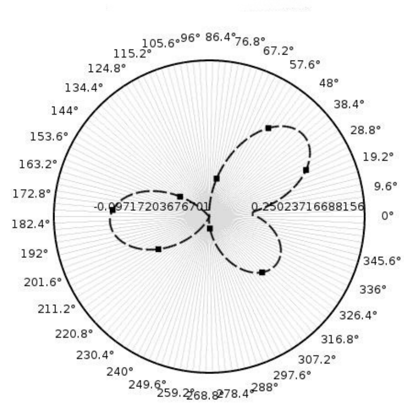

Fig.2. Unchanged quasi potential electric energy of $\phi(\boldsymbol{\xi}^{\varepsilon}_{(1)})$ in the electric field $\mathcal{E}$

Figure 2 shows the polar modeling for the time variation of the unchanged quasi potential electric energy of $\phi(\mathring{\mathbf{e}}^{\varepsilon}_{(1)})$ corresponding to its base in a radial direction.

• Uniformly quasi circular potential electric energy of $\phi(\boldsymbol{\xi}^{\varepsilon}_{(2)})$ in the electric field $\mathcal{E}$

$$
\begin{aligned}
\mathcal{L}^{\mathcal{FW}}\phi(\boldsymbol{\xi}^{\varepsilon}_{(2)}) &= [\xi((-(\varpi' + \varkappa_2\varkappa_1)\varkappa_3 - (\varkappa_2^2 + \varpi\varkappa_3)')\cos\varphi\sin[\chi_1 s + \chi_2] + (\varpi\varkappa_1 \\
&-\varkappa_2')'\sin\varphi\sin[\chi_1 s + \chi_2] + (-(\varpi' + \varkappa_2\varkappa_1)' + \varkappa_3(\varkappa_2^2 + \varpi\varkappa_3))\cos[\chi_1 s + \chi_2]) \\
&+\xi'((\varpi\varkappa_1 - \varkappa_2')\sin\varphi\sin[\chi_1 s + \chi_2] - (\varpi' + \varkappa_2\varkappa_1)\cos[\chi_1 s + \chi_2] - (\varkappa_2^2 \\
&+\varpi\varkappa_3)\cos\varphi\sin[\chi_1 s + \chi_2]])]((\sigma\varkappa_2 + \varkappa_1)\cos\varphi\sin[\chi_1 s + \chi_2] + (\sigma\varkappa_1 + \varkappa_2) \\
&\times\cos[\chi_1 s + \chi_2]) + [\xi((-(\varpi' + \varkappa_2\varkappa_1)\varkappa_3 - (\varkappa_2^2 + \varpi\varkappa_3)')\cos\varphi\cos[\chi_1 s + \chi_2] \\
&+(\varpi\varkappa_1 - \varkappa_2')'\sin\varphi\cos[\chi_1 s + \chi_2] - (-(\varpi' + \varkappa_2\varkappa_1)' + \varkappa_3(\varkappa_2^2 + \varpi\varkappa_3))\sin[\chi_1 s + \chi_2]) \\
&+\xi'((\varpi\varkappa_1 - \varkappa_2')\sin\varphi\cos[\chi_1 s + \chi_2] + (\varpi' + \varkappa_2\varkappa_1)\sin[\chi_1 s + \chi_2] - (\varkappa_2^2 \\
&+\varpi\varkappa_3)\cos\varphi\cos[\chi_1 s + \chi_2]])]((\sigma\varkappa_2 + \varkappa_1)\cos\varphi\cos[\chi_1 s + \chi_2] - (\sigma\varkappa_1 + \varkappa_2)\sin[\chi_1 s + \chi_2]) \\
&-[\xi((\varpi\varkappa_1 - \varkappa_2')'\cos\varphi - (-(\varpi' + \varkappa_2\varkappa_1)\varkappa_3 - (\varkappa_2^2 + \varpi\varkappa_3)')\sin\varphi) + \xi'((\varpi\varkappa_1 - \varkappa_2')\cos\varphi \\
&+(\varkappa_2^2 + \varpi\varkappa_3)\sin\varphi)](\sigma\varkappa_2 + \varkappa_1)\sin\varphi.
\end{aligned}
$$

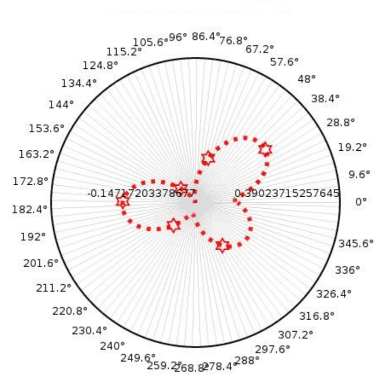

Fig.3. Unchanged quasi potential electric energy of $\phi(\boldsymbol{\xi}^{\varepsilon}_{(2)})$ in the electric field $\mathcal{E}$

Figure 3 shows the polar modeling for the time variation of the unchanged quasi potential electric energy of $\phi(\boldsymbol{\xi}^{\varepsilon}_{(2)})$ corresponding to its base in a radial direction.

• Uniformly quasi circular potential electric energy of $\mathcal{E}$ in the electric field $\mathcal{E}$

$$
\begin{aligned}
&\mathcal{L}^{\mathcal{FW}}\mathcal{E} = ((\sigma\varkappa_2+\varkappa_1)\cos\varphi\sin[\chi_1 s+\chi_2] + (\sigma\varkappa_1+\varkappa_2)\cos[\chi_1 s+\chi_2])[((((\sigma\varkappa_2\\
&+\varkappa_1)' + \varkappa_3(\sigma\varkappa_1+\varkappa_2))' + \varkappa_3((\sigma\varkappa_1+\varkappa_2)' - \varkappa_3(\sigma\varkappa_2+\varkappa_1)))\cos\varphi\sin[\chi_1 s+\chi_2]\\
&+(((\sigma\varkappa_1+\varkappa_2)' - \varkappa_3(\sigma\varkappa_2+\varkappa_1))' - \varkappa_3((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2)))\cos[\chi_1 s+\chi_2]\\
&-((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\sin\varphi\sin[\chi_1 s+\chi_2]) + \vartheta'(((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1\\
&+\varkappa_2))\cos\varphi\sin[\chi_1 s+\chi_2] - (\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\sin\varphi\sin[\chi_1 s+\chi_2] + ((\sigma\varkappa_1\\
&+\varkappa_2)' - \varkappa_3(\sigma\varkappa_2+\varkappa_1))\cos[\chi_1 s+\chi_2])]\boldsymbol{\nu}_1 + ((\sigma\varkappa_2+\varkappa_1)\cos\varphi\cos[\chi_1 s+\chi_2] - (\sigma\varkappa_1\\
&+\varkappa_2)\sin[\chi_1 s+\chi_2])[((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))' + \varkappa_3((\sigma\varkappa_1+\varkappa_2)' - \varkappa_3(\sigma\varkappa_2\\
&+\varkappa_1)))\cos\varphi\cos[\chi_1 s+\chi_2] - ((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\sin\varphi\cos[\chi_1 s+\chi_2]\\
&-(((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))' - \varkappa_3((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2)))\sin[\chi_1 s+\chi_2])\\
&+\vartheta'((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))\cos\varphi\cos[\chi_1 s+\chi_2] - (\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2\\
&+\varkappa_1))\sin\varphi\cos[\chi_1 s+\chi_2] - ((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2+\varkappa_1))\sin[\chi_1 s+\chi_2])]\boldsymbol{\nu}_2\\
&+(\sigma\varkappa_2+\varkappa_1)\sin\varphi[((((\sigma\varkappa_2+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))'+\varkappa_3((\sigma\varkappa_1+\varkappa_2)'-\varkappa_3(\sigma\varkappa_2\\
&\quad+\varkappa_1)))\sin\varphi + ((\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))')\cos\varphi) + \vartheta'(((\sigma\varkappa_2\\
&\;+\varkappa_1)'+\varkappa_3(\sigma\varkappa_1+\varkappa_2))\sin\varphi + (\varkappa_1(\sigma\varkappa_1+\varkappa_2)+\varkappa_2(\sigma\varkappa_2+\varkappa_1))\cos\varphi)]\boldsymbol{\nu}_3.
\end{aligned}
$$

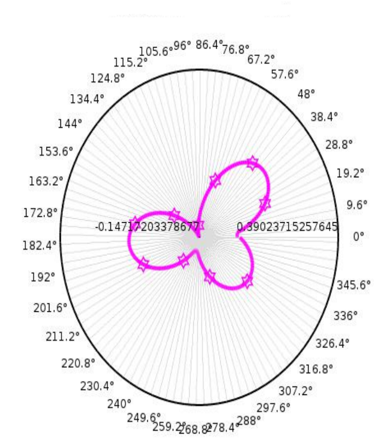

Fig.4. Unchanged quasi potential electric energy of $\mathcal{E}$ in the electric field $\mathcal{E}$

Figure 4 shows the polar modelling for the time variation of the unchanged quasi potential electric energy of $\mathcal{E}$ corresponding its base in radial direction.

• Uniformly quasi circular potential electric energy of $\mathcal{B}$ in the electric field $\mathcal{E}$

$$
\begin{aligned}
\mathcal{L}^{\mathcal{FW}}\mathcal{B} = {} & [\eta(\varpi'' \sin\varphi \sin[\chi_1 s + \chi_2] + ((\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2')' - \varkappa_3(\varpi\varkappa_2 \\
& -\varkappa_2\varkappa_3 + \varkappa_1')) \cos[\chi_1 s + \chi_2] + ((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 \\
& -\varkappa_1\varkappa_3 - \varkappa_2')) \cos\varphi \sin[\chi_1 s + \chi_2]) + \eta'(\varpi' \sin\varphi \sin[\chi_1 s + \chi_2] + (\varpi\varkappa_1 \\
& -\varkappa_1\varkappa_3 - \varkappa_2') \cos[\chi_1 s + \chi_2] + (\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1') \cos\varphi \sin[\chi_1 s + \chi_2])]((\sigma\varkappa_2 \\
& +\varkappa_1) \cos\varphi \sin[\chi_1 s + \chi_2] + (\sigma\varkappa_1 + \varkappa_2) \cos[\chi_1 s + \chi_2]) + [\eta(\varpi'' \sin\varphi \\
& \cos[\chi_1 s + \chi_2] - ((\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2')' - \varkappa_3(\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')) \sin[\chi_1 s + \chi_2] \\
& +((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2')) \cos\varphi \cos[\chi_1 s + \chi_2]) \\
& +\eta'(\varpi' \sin\varphi \cos[\chi_1 s + \chi_2] - (\varpi\varkappa_1 - \varkappa_1\varkappa_3 - \varkappa_2') \sin[\chi_1 s + \chi_2] + (\varpi\varkappa_2 - \varkappa_2\varkappa_3 \\
& +\varkappa_1') \cos\varphi \cos[\chi_1 s + \chi_2])]((\sigma\varkappa_2\varkappa_1) \cos\varphi \cos[\chi_1 s + \chi_2] - (\sigma\varkappa_1 \\
& ++\varkappa_2) \sin[\chi_1 s + \chi_2]) + [\eta(\varpi'' \cos\varphi - ((\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1')' + \varkappa_3(\varpi\varkappa_1 \\
& -\varkappa_1\varkappa_3 - \varkappa_2')) \sin\varphi) + \eta'(\varpi' \cos\varphi - (\varpi\varkappa_2 - \varkappa_2\varkappa_3 + \varkappa_1') \sin\varphi)](\sigma\varkappa_2 + \varkappa_1) \sin\varphi.
\end{aligned}
$$

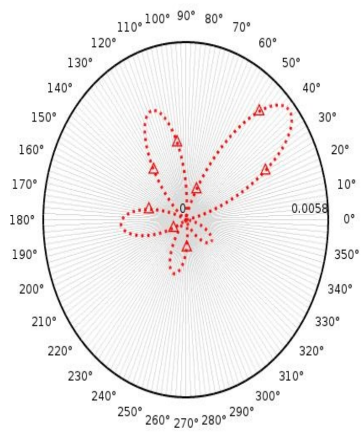

Fig.5. Unchanged quasi potential electric energy of $\mathcal{B}$ in the electric field $\mathcal{E}$

Figure 5 shows the polar modeling for the time variation of the unchanged quasi potential electric energy of $\mathcal{B}$ corresponding to its base in a radial direction.

7 Conclusion

Geometrical models that describe a relativistic magnetic particle may be constructed using the geometrical scalars associated with the embedding of the particle worldline in Heisenberg spacetime as building blocks for the action. In this paper, we construct the unchanged quasi direction motion with biharmonicity condition in the Heisenberg space. We define the energy of velocity magnetic particles and some Lorentz fields. Also, we construct the new relationship between the Fermi-Walker parallel transportation and the unchanged quasi direction motion in the Heisenberg space. In other words, we obtain the applied geometric characterization for an uniformly quasi accelerated motion of biharmonic velocity magnetic particles in the Heisenberg space. This concept also boosts to discover some physical and geometrical characterizations belonging to the particle such as the magnetic motion, the potential energy functional, the torque, and the Poynting vector. Finally, we obtain electrical energy with respect to its electric field and energy flux density in the radial direction.

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