

Characterizing the role of non-linear interactions in the transition to submesoscale dynamics at a dense filament

Mara A. Freilich¹, Luc Lenain¹, and Sarah T. Gille²

¹Scripps Institution of Oceanography

²University of California, San Diego

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Abstract

Ocean dynamics at the submesoscale play a key role in mediating upper-ocean energy dissipation and dispersion of tracers. Observations of ocean currents from synoptic mesoscale surveys at submesoscale resolution (250 m–100 km) from a novel airborne instrument (MASS DoppVis) reveal that the kinetic energy spectrum in the California Current System is nearly continuous from 100 km to sub-kilometer scales, with a k^{-2} spectral slope. Although there is not a transition in the kinetic energy spectral slope, there is a transition in the dynamics to non-linear interactions at scales of $O(1 \text{ km})$. Barotropic kinetic energy transfer across spatial scales is enabled by interactions between the rotational and divergent components of the flow field at the submesoscale. Kinetic energy flux is intermittent but can be large, particularly at submesoscale fronts. Kinetic energy is transferred both downscale and upscale from 1 km in the observations of a cold filament.

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2 **transition to submesoscale dynamics at a dense**
3 **filament**

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6 **Key Points:**

- 7 • Remote sensing observations reveal a kinetic energy spectrum with a continuous
8 slope from 100 km to 1 km in an eastern boundary region.
9 • Between 1 and 10 km, ageostrophic non-linear interactions become dynamically
10 important
11 • Cross-scale kinetic energy transfers computed from 2D velocity observations are
12 associated with shear strain in the observed front.

Corresponding author: Mara Freilich, mfreilich@ucsd.edu

Abstract

Ocean dynamics at the submesoscale play a key role in mediating upper-ocean energy dissipation and dispersion of tracers. Observations of ocean currents from synoptic mesoscale surveys at submesoscale resolution (250 m–100 km) from a novel airborne instrument (MASS DoppVis) reveal that the kinetic energy spectrum in the California Current System is nearly continuous from 100 km to sub-kilometer scales, with a k^{-2} spectral slope. Although there is not a transition in the kinetic energy spectral slope, there is a transition in the dynamics to non-linear ageostrophic interactions at scales of $\mathcal{O}(1\text{ km})$. Kinetic energy transfer across spatial scales is enabled by interactions between the rotational and divergent components of the flow field at the submesoscale. Kinetic energy flux is patchy and localized at submesoscale fronts. Kinetic energy is transferred both downscale and upscale from 1 km in the observations of a cold filament.

Plain Language Summary

Ocean dynamics at scales of 100 m–10 km, called the submesoscale, are important because they are associated with large velocity gradients and non-linear interactions. Large gradients lead to vertical velocity, which facilitates ocean-atmosphere interactions and ocean biological processes. Velocity gradients and non-linear processes combine to transfer kinetic energy from the large-scale flow to small-scale perturbations. This can lead to instabilities that dissipate energy in the ocean surface layer (rather than the seafloor). Here we analyze novel observations that provide insight into ocean dynamics through the distributions of velocity gradients and energy transfer at 1 km scale. Dynamics at these scales have previously been modeled, but have not been observed directly. We observe a transition where non-linear dynamics become more important at scales of order 10 km. We also introduce new interpretations of spectral analysis (analysis of energy and correlations across scales). Moreover, we analyze covariance of velocity gradient quantities and flow energetics to demonstrate that energy flux is episodic and localized at fronts. Together, these observations demonstrate that fronts play an important role in boundary-layer kinetic energy processes and highlight the evolution of upwelling filaments.

1 Introduction

Ocean processes in the surface boundary layer play a critical role in mediating the influence of atmospheric and climate processes on the ocean. Heating, wind-driven momentum input, and gas exchange occur at the sea surface and are transmitted through the boundary layer and into the ocean interior. The boundary layer also resides in the photic zone, where there is enough light for photosynthesis.

Submesoscale dynamics, the dynamics that operate at the spatial scales between the nearly geostrophically balanced mesoscale eddies ($\sim 100\text{ km}$ scales in mid-latitudes) and three-dimensional turbulence ($< 100\text{ m}$ scales), are particularly important for these boundary layer processes (McWilliams, 2016). Submesoscales influence ocean biogeochemistry by modulating vertical transport (Mahadevan, 2016; Freilich et al., 2022) and influence air-sea interactions by modulating buoyancy and momentum transfer (Strobach et al., 2022). Submesoscale dynamics are hypothesized to facilitate a forward cascade of kinetic energy resulting in dissipation of eddy kinetic energy in the surface ocean (Müller et al., 2005; Capet et al., 2008b; Barkan et al., 2015; Srinivasan et al., 2023). However, submesoscale dynamics are also known to cascade energy upscale, strengthening mesoscale features (Schubert et al., 2020; Sandery & Sakov, 2017; Qiu et al., 2014). Determining the specifics of the dynamics in this transitional range of 100 m–100 km is essential for quantifying kinetic energy cycles in the ocean (Ferrari & Wunsch, 2009; McWilliams, 2016; Naveira Garabato et al., 2022).

The submesoscale is defined dynamically as the regime where the Rossby number, a non-dimensional parameter defined as $\text{Ro} = U/(fL)$, is order 1 with velocity U , hor-

63 izontal length scale L , and Coriolis parameter f . While geostrophic dynamics are thought
 64 to predominate at the mesoscale and larger, geostrophic balance can begin to break down
 65 at the submesoscale. At the larger end of the submesoscale, the surface quasigeostrophy
 66 framework presupposes that surface density fronts modify geostrophic balance (Klein &
 67 Lapeyre, 2009) while other theoretical results emphasize the role of non-linear advection
 68 in submesoscale dynamics (Barkan et al., 2019).

69 In this work we characterize the transition to submesoscale dynamics at scales smaller
 70 than 10 km and provide observational analysis of the kinetic energy cascade that has been
 71 hypothesized from models and theory. We observe submesoscale ocean surface velocity
 72 using remote sensing from airplanes during the submesoscale ocean dynamics experiment
 73 (S-MODE) field campaign (Farrar et al., 2020). We find substantial kinetic energy at
 74 the submesoscale, with a kinetic energy spectral slope that is nearly continuous from 100
 75 to 1 km spatial scales. The dynamics that result in the spatial distribution of kinetic en-
 76 ergy at the submesoscale are diagnosed through analysis of velocity cross spectra. These
 77 reveal that non-linear interactions between balanced and unbalanced dynamics contribute
 78 to submesoscale energy and illuminate the dynamics influencing upper-ocean velocity
 79 gradient distributions.

80 2 Methods

81 2.1 Remote sensing

82 The observations used in this study were collected by the DoppVis instrument (Lenain
 83 et al., 2023), a new sensor that is part of the Modular Aerial Sensing System (MASS;
 84 Melville et al., 2016), that infers currents from optical observations of the spatio-temporal
 85 evolution, i.e. dispersion relationship, of surface waves. This method infers the depth-
 86 resolved Lagrangian current in the upper ocean. Here, we use the depth averaged cur-
 87 rent over the upper 2 meters. Details about the DoppVis instrument are available in Lenain
 88 et al. (2023). The instrument package was installed on a Twin Otter DH-6 aircraft, fly-
 89 ing at constant altitude above mean sea level (hereafter, altitude), with a flight profile
 90 consisting of repeated reciprocal straight tracks. Consistency between the reciprocal passes
 91 is used to validate velocity measurements. Velocity observations are binned to 256 m or
 92 500 m prior to analysis.

93 Sea surface temperature observations are collected with a Flir SC6700SLS long-
 94 wave IR camera (1 m resolution) and Heitronics KT19.85 II infrared thermometer (50 m
 95 resolution) (see Melville et al., 2016; Lenain et al., 2023, for details).

96 Observations from two field campaigns are considered in this study. The first field
 97 campaign sampled across a cold filament approximately 70 nautical miles offshore of Cal-
 98 ifornia, as part of the NASA S-MODE program (Farrar et al., 2020). This region is sub-
 99 sequently referred to as the “filament region” and is the focus of this study. These ob-
 100 servations occurred on November 3, 2021 from 18:23 to 23:33 UTC while flying at ap-
 101 proximately 500 m altitude and on November 5, 2021 from 22:40 to 23:00 UTC while
 102 flying at 940 m altitude (Figure 1A-C).

103 The higher altitude flight on November 5 enables collection of multiple data points
 104 in the cross-swath direction, resulting in a 1.5 km wide swath that is used to compute
 105 velocity gradients using central differences. The observations from November 5 are binned
 106 at 256 m prior to analysis.

107 The second field campaign collected observations across two counter-rotating ed-
 108 dies approximately 45 nautical miles offshore of San Diego on May 19, 2021 from 20:56
 109 to 23:26 UTC (Lenain et al., 2023). This region is referred to as the “eddy region”. Ob-
 110 servations using a vessel mounted ADCP were collected under the long northwest-to-southeast
 111 leg of the DoppVis observations from May 19, 2021 10:00 to May 20, 2021 13:00 UTC.

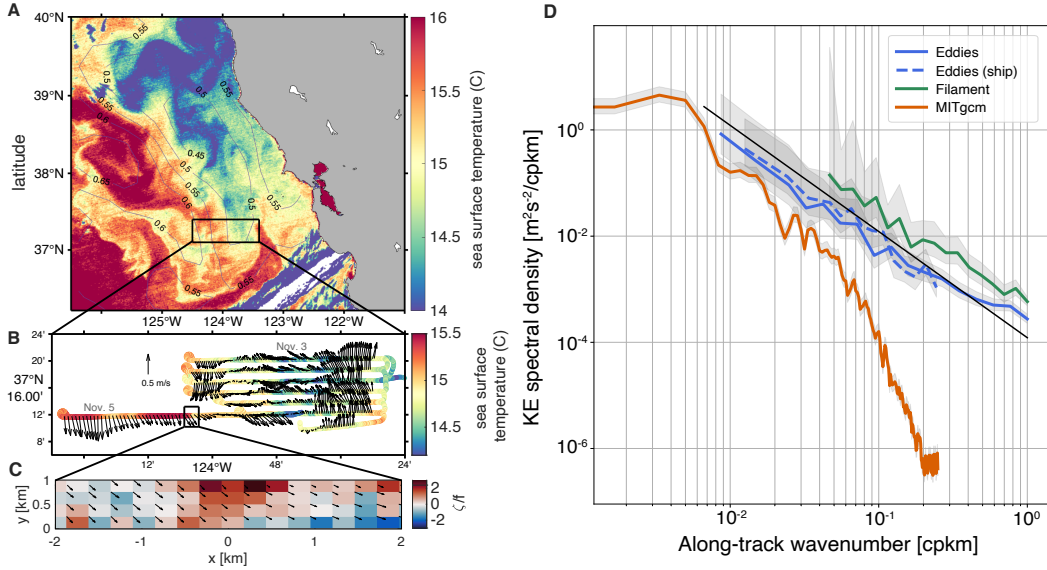


Figure 1. Velocity observations situated in the large-scale context using satellite observations. (A) Satellite (MODIS Aqua) sea surface temperature in the filament region on November 4, 2021 at 21:05Z. Contours show sea surface height from AVISO. (B) DoppVis velocity across the sampled filament (black rectangle in panel A) is shown as vectors with sea surface temperature from the infrared thermometer. Two days of observations are shown, November 3 and November 5. The filament had shifted on November 5. (C) Vorticity computed from DoppVis in the black rectangle in panel B with velocity vectors. This section is shown in Fig. 4A,B. Velocity gradients in Fig. 2 and 4C are computed from the whole transect collected on November 5. (D) Kinetic energy spectral density as a function of along-track wavenumber from a 2 km resolution regional MITgcm model and two observational regions – the eddy region (May 2021) and the filament region (November 2021; panel B) – and two measurement platforms during May 2021 – DoppVis and a ship. The black line shows a k^{-2} spectral slope.

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2.2 Spectra

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We analyze both the kinetic energy spectrum ($\hat{E}(f)$) and the cross spectrum ($\hat{S}(f)$) with 95% confidence intervals calculated following Bendat and Piersol (2011). Both the kinetic energy spectra and the cross-spectrum between along-track and across-track velocity are computed using Welch’s method with Hanning windows.

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3 Results

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3.1 Kinetic energy spectrum

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The multi-scale nature of the flow is quantified using energy spectra, which can also be used to make inferences about the dominant dynamics governing the flow (Callies & Ferrari, 2013). The filament region is more energetic than the eddy region (Figure 1D), with approximately twice the amount of energy at nearly all spatial scales sampled. The kinetic energy spectra of the DoppVis observations have slopes that are approximately k^{-2} (Figure 1D). The observed kinetic energy spectrum crossing the eddies has magnitude and spectral slope similar to that of the spectra from currents (15 m depth) taken with a vessel mounted acoustic Doppler current profiler (ADCP) on a nearby transect on the same day for 5–100 km scales. Differences between the spectra computed from

128 DoppVis and the ship are within uncertainty but could be due to vertical shear since the
 129 DoppVis observations are averages in the upper 2 m. Differences may also be attributable
 130 to aliasing in the observations from the ship, which took 30 hours to complete the tran-
 131 sect while DoppVis took 50 minutes.

132 This analysis extends the observations to smaller spatial scales than have been ob-
 133 served previously. Notably, these scales are smaller than those resolved by state-of-the-
 134 art global and regional models. As an example, we show the kinetic energy spectrum from
 135 a 2 km grid spacing MITgcm regional model of the California current system (Mazloff
 136 et al., 2020) (Figure 1D, red line). This model is forced with ERA5 atmospheric state,
 137 Hybrid Coordinate Ocean Model + Navy Coupled Ocean Data Assimilation boundary
 138 conditions, and both local and remote tides. The effective resolution of this model is 20 km
 139 with the velocity spectrum falling off steeply below that scale due to grid scale dissipa-
 140 tion. Even at larger scales, both regions are more energetic than the 2 km grid spacing
 141 ocean model of the same region (the eddy region is 5 times more energetic). The discrep-
 142 ancy between model and observations at lower wavenumbers is likely caused by an in-
 143 verse cascade of submesoscale energy energizing surface mesoscale features in ways that
 144 are not represented in the model (Lévy et al., 2001; Mahadevan & Tandon, 2006) and
 145 by biased observational sampling toward more energetic features. It is important to note
 146 that only the larger end of submesoscale dynamics are resolved by 2 km models (Su et
 147 al., 2018; Sinha et al., 2022). This is especially important to keep in mind when consid-
 148 ering cross-scale energy fluxes that may be modified by dynamics at small spatial scales.

149 The observed kinetic energy spectral slopes are consistent with previous observa-
 150 tions from this region: a comprehensive analysis of surface velocities measured from vessel-
 151 mounted ADCPs in the California Current region from 1993–2004 found that the kinetic
 152 energy spectral slope in this region is approximately k^{-2} to $k^{-5/3}$ at scales of 10 to 200 km
 153 (Chereskin et al., 2019). This is in contrast to the steeper spectral slope (k^{-3}) in more
 154 energetic regions such as the Antarctic Circumpolar Current, which implies geostrophic
 155 dynamics (Rocha et al., 2016). Modeling studies in the California Current System have
 156 found the kinetic energy spectrum to be continuous from the mesoscale to submesoscale,
 157 with a slope of approximately k^{-2} (Capet et al., 2008a).

158 A range of dynamics could result in the observed spectral slope including internal
 159 gravity waves (k^{-2}), surface quasigeostrophy ($k^{-5/3}$), and fronts (k^{-2}) (Boyd, 1992; Lapeyre
 160 & Klein, 2006). The observations available in this study cannot sufficiently distinguish
 161 these spectral slopes, nor can we identify whether the observed slope has transitions in
 162 the submesoscale regime from current methods. We therefore rely on further analysis
 163 to infer the dynamics in this region.

164 3.2 Distributions of vorticity, divergence, and strain rate

165 One of the implications of a kinetic energy spectrum $E(k)$ with a k^{-2} slope is that
 166 the velocity derivative spectrum $V(k)$ is flat because the spectra are linked through the
 167 relationship $V(k) = k^2 E(k)$. The key velocity derivative quantities, divergence ($\delta =$
 168 $u_x + v_y$), vorticity ($\zeta = v_x - u_y$), and strain are related to each other through a sys-
 169 tem of coupled non-linear ordinary differential equations (c.f. Barkan et al., 2019). The
 170 strain is composed of shear strain ($\sigma_s = v_x + u_y$) and normal strain ($\sigma_n = u_x - v_y$).

171 In the observations that allow for computing gradient across the track, which are
 172 only available for one track in the filament case (Figure 1B,C), we find that the vortic-
 173 ity is skewed positive (skewness 0.54, 90% confidence interval [0.49,0.69]), and the di-
 174 vergence is skewed negative (skewness -0.081, 90% confidence interval [-0.17, 0.043]) (Fig-
 175 ure 2A,B), consistent with previous (shipboard) observations (Shcherbina et al., 2009;
 176 Rudnick, 2001). This skewness can arise from conservation of potential vorticity at fronts.
 177 Strain-driven frontogenesis at the sea surface, in the absence of dissipation, results in an
 178 infinitely sharp front in finite time with ageostrophic flow that has skewed distributions
 179 of divergence (negative) and vorticity (positive) (Hoskins & Bretherton, 1972; Barkan
 180 et al., 2019). In addition, the dynamical feedbacks are such that large negative relative

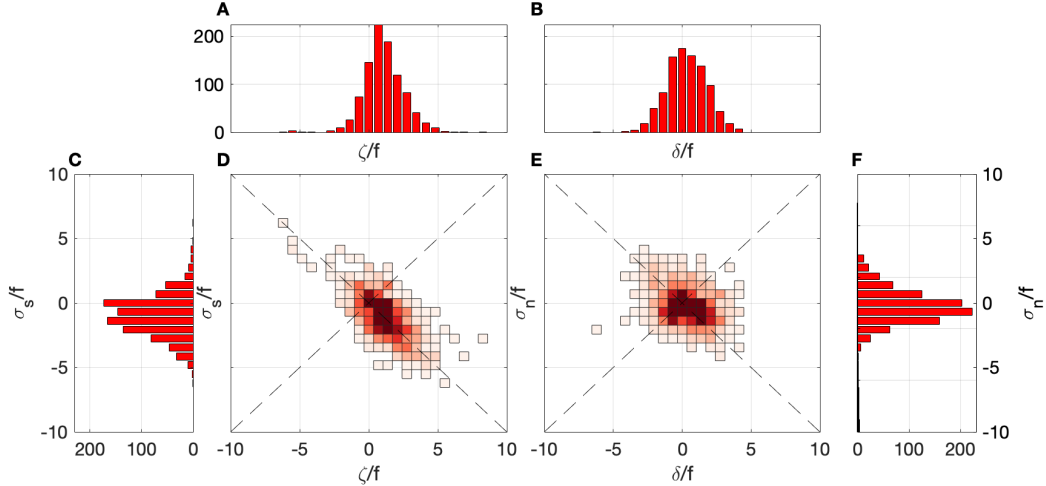


Figure 2. Velocity gradients in the filament observed on October 5 displayed as PDFs of (A) vorticity (ζ), (B) divergence (δ) (C) shear strain (σ_s), and (F) normal strain (σ_n), all normalized by f , along with joint PDFs of vorticity and shear strain (D) and divergence and normal strain (E).

181 vorticity is typically unstable to symmetric and centrifugal instabilities but positive relative
 182 vorticity stabilizes the flow to these instabilities, and therefore a skewed distribution
 183 develops (Rudnick, 2001; Buckingham et al., 2016). Compared with anticyclonic fronts,
 184 cyclonic fronts also progress more slowly to singularities during frontogenesis, which could
 185 result in longer lived cyclonic fronts (Shakespeare, 2016). However, it is notable that strain-
 186 driven frontogenesis can suppress the growth of symmetric instability (Thomas, 2012).
 187 In boundary layers, negative potential vorticity can arise from frictional and atmospheric
 188 forcing, which can trigger symmetric instability.

189 In the filament observations studied here, vorticity is strongly correlated with shear
 190 strain (Figure 2D). This arises not because of direct forcing of vorticity by shear strain
 191 but instead because $\sigma_s \approx u_y \approx -\zeta$ in geographic coordinates (x points eastward and
 192 y points northward) over much of the sampled domain (but $\zeta \approx v_x$ at the front shown
 193 in Figure 4). Shear strain and vorticity are correlated due to the relative stability of cy-
 194 clonic vorticity at straight fronts (Buckingham et al., 2021). This provides an explana-
 195 tion for the strain–vorticity relationship that has been observed in high-resolution simu-
 196 lations (Balwada et al., 2021). However, there is not a strong correlation between di-
 197 vergence and normal strain ($\sigma_n = u_x - v_y$); while non-zero vorticity can be maintained
 198 in an adiabatic system in the absence of divergence and vertical motion, a similar bal-
 199 ance does not exist for divergence (Figure 2E).

200 3.3 Non-linear interactions

At the submesoscale ($Ro \sim \mathcal{O}(1)$), the flow becomes more fully three dimensional
 (velocity divergence $\delta \sim U/L$). A key dynamical transitions at the submesoscale is that
 temporal changes in velocity gradient quantities (vorticity, divergence, and strain) are
 coupled such that non-linear feedbacks become important (Barkan et al., 2019). For ex-
 ample, the rate of change of vorticity in an adiabatic system is

$$\frac{D\zeta}{Dt} = -f\delta - \delta\zeta - w_x v_z + w_y u_z. \quad (1)$$

201 Only the first term, which does not involve a feedback, is present in a quasigeostrophic
 202 system. In addition, at the submesoscale the inertial term in the equations of motion ($u \cdot$

203 $\nabla u \sim U^2/L$) is of the same order as the Coriolis term ($uf \sim Uf$), facilitating cross-
 204 scale kinetic energy transfers.

205 **3.3.1 Interactions between rotational and divergent flow**

206 The approximately k^{-2} spectral slope in both the filament and eddy regions is in-
 207 formative but inconclusive about the about the dominant dynamics operating in these
 208 regions. The nearly uniform slope across the observed spatial scales leaves open ques-
 209 tions about the scales at which a transition to submesoscale dynamics may occur.

210 The submesoscale feedback between vorticity and divergence (equation 1) results
 211 in a correlation between the geostrophically balanced rotational (streamfunction) flow
 212 and the divergent (potential) component of the velocity. We diagnose when the stream-
 213 function and potential become correlated using the cross spectrum between the along-
 214 track (u) and cross track (v) velocity components.

215 When the streamfunction and potential are uncorrelated, as is typically true at the
 216 mesoscale and larger, the cross spectrum of the u and v velocity components is a super-
 217 position of the spectra of the streamfunction and velocity potential. In this case, since
 218 spectra are real, the cross spectrum is real (Bühler et al., 2017). This is a key assump-
 219 tion of the ‘wave–vortex’ decomposition introduced by Bühler et al. (2014). However,
 220 when the rotational and divergent flow components interact, the cross spectrum between
 221 the along-track and across-track velocity (\hat{S}_{uv}) is complex. We are therefore able to di-
 222 agnose the spatial scale where a shift to submesoscale dynamics occurs as the scale at
 223 which the cross spectrum becomes complex. The cross spectral phase should only be in-
 224 terpreted in this manner if the coherence, which is the normalized cross spectrum be-
 225 tween the along-track and across track velocity, is significant. Coherence is related to flow
 226 anisotropy because $\mathbb{E}(uv) = 0$ for isotropic flows, but it is not a quantitative measure
 227 of anisotropy. For isotropic flows, the Bühler et al. (2014) decomposition may be applied
 228 even if the streamfunction and potential are correlated (Callies et al., 2016).

229 The squared coherence has contrasting dependence on spatial scale in the two re-
 230 gions studied here (Figure 3A). In the eddy region (blue lines in Figure 3), the squared
 231 coherence is large at the largest spatial scales sampled (~ 100 km) and decreases steadily
 232 to 10 km scales after which it flattens out, but remains significantly different from zero.
 233 In the filament region (green lines), the squared coherence is also large at the largest spa-
 234 tial scales sampled (~ 10 km), decreases at scales larger than 6 km, and then increases
 235 again toward the smallest spatial scales sampled (~ 1 km). Fronts and filaments are ex-
 236 pected to be anisotropic at the scale of the feature, as is observed. In all observations
 237 considered here, the coherence is large enough to be statistically significant, allowing for
 238 analysis of the cross-spectral phase. The only exception is in the eddy region between
 239 0.1 and 0.6 cpkm where the coherence falls below the significance threshold, suggesting
 240 that the eddy region is relatively isotropic at mesoscale.

241 The cross-spectral phase summarizes the relationship between the real and imag-
 242 inary parts of the cross spectrum. When the cross spectrum is purely real, the phase is
 243 0° or 180° ; when it is purely imaginary, the phase is $\pm 90^\circ$. We find abrupt transitions
 244 at a scale slightly smaller than 10 km in the eddy region and 6 km in the filament re-
 245 gion, where the imaginary part of the cross spectrum becomes larger than the real part
 246 (Figure 3B). This 6 km spatial scale is the same scale where the coherence increases in
 247 the filament region (suggesting increased anisotropy), providing consistent evidence of
 248 a change to increasingly non-linear frontal dynamics at these scales. By contrast, in a
 249 surface quasigeostrophic model, which neglects ageostrophic advection, the real part of
 250 the cross spectrum dominates at all spatial scales (Figure S3). During the eddy obser-
 251 vations, the mixed-layer depth was 40–55 m with some regions as shallow as 15 m. In
 252 contrast, for the filament observations, the mixed-layer depth was approximately 35 m
 253 or shallower and the stratification was approximately $3 \times 10^{-5} \text{ s}^{-2}$. Therefore, the mixed-
 254 layer deformation radius was 2–4 km for these locations, implying that the fastest grow-
 255 ing baroclinic mode is around 8–24 km (Dong et al., 2020). Thus, the transition to non-

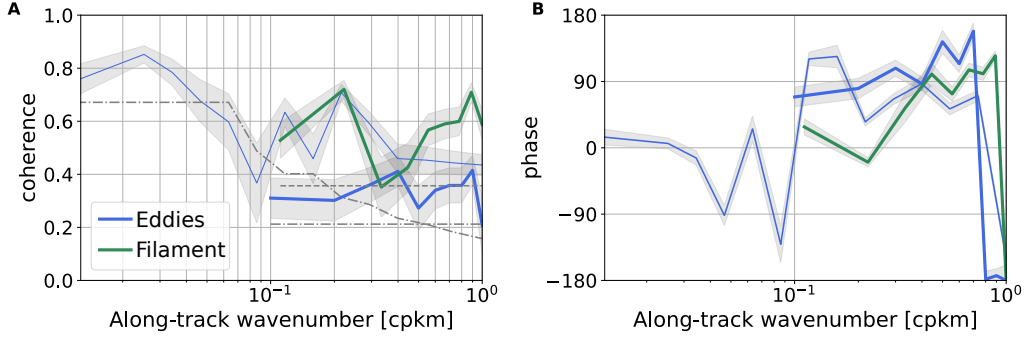


Figure 3. Flow anisotropy and non-linearity revealed by analysis of velocity cross spectrum. (A) Squared coherence as a function of wavenumber. The gray lines show the significance threshold (dashed and dot dash show filament and eddies, respectively) and the shading shows standard deviation. (B) Cross-spectrum phase. The thin lines in show the squared coherence computed from two long transects in the eddy region with 80 km windows while the thick lines show the squared coherence computed from the two long sections and four shorter sections (which crossed the eddy nearly perpendicularly, SI Fig. S1) using 10 km windows.

linear ageostrophic dynamics observed here occurs in the approximate range of the scale of mixed-layer baroclinic instability.

There are a number of mechanisms that could be responsible for the interaction between rotational and divergent velocity. In the filament case, the interaction of the ageostrophic frontal divergence and larger scale geostrophic flow is likely the dominant mechanism. Here we find that the shift to a mostly imaginary cross spectrum is localized in the regions of largest velocity gradient (Figure S4). The eddy case likely encompasses a larger range of dynamics, including near-inertial oscillations modified by the vorticity of the observed features, frontal dynamics, and submesoscale vortices.

3.3.2 Spectral energy transfers

The distribution of kinetic energy across spatial scales reflects dynamics that are local in wavenumber, but importantly also reflects energy transfers across scales. At the submesoscale, major open questions remain regarding the direction of the energy cascade, the mechanisms that lead to a forward energy, and the rate of the forward energy cascade (Müller et al., 2005; McWilliams, 2016). Forward energy flux precedes dissipation at small spatial scales by turbulent processes.

The energy transfer across scales can be quantified using coarse graining (Germano, 1992; Eyink, 2005; Aluie et al., 2018). The kinetic energy flux is defined here as

$$\Pi = -(\tau_{uv}(u_y + v_x) + \tau_{uu}u_x + \tau_{vv}v_y) \quad (2)$$

where $\tau_{ab} = \overline{ab} - \bar{a}\bar{b}$ and $\bar{\cdot}$ is a top hat filter. Positive (negative) values indicate a flux of energy toward smaller (larger) spatial scales. We use velocity observed on a 256 m grid and a top hat filter with a scale of 1 km to compute an instantaneous energy flux across the observed transect. Error is estimated using a bootstrapped confidence intervals and a velocity error of 0.05 m s⁻¹ (Lenain et al., 2023).

In the frontal regions in this flow, there is a strong forward energy flux localized in a 1 km region at the frontal outcrop where there is also a peak in frontogenesis (Figure 4A,B). The energy flux to smaller spatial scales is driven by the first term in equation 2 (Figure S5). This term involves the shear strain multiplied by the scale-dependent

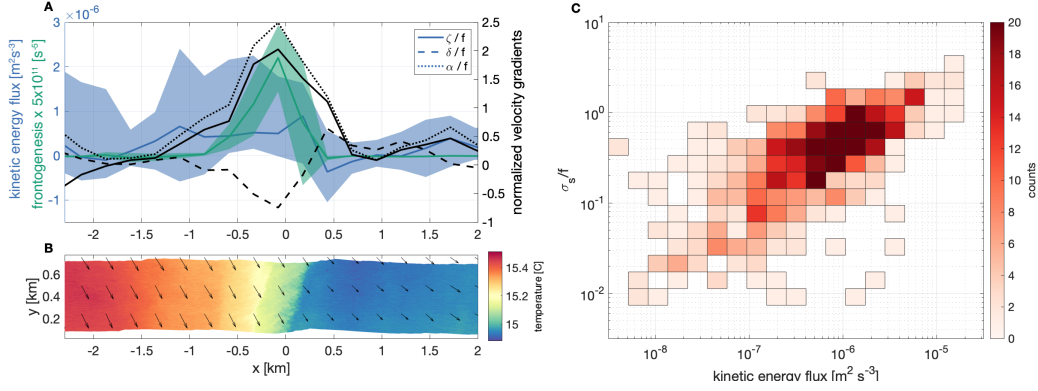


Figure 4. Spatial distribution of kinetic energy flux and frontogenesis. (A) Kinetic energy flux across 1 km and buoyancy frontogenesis (shading shows the bootstrapped 95% confidence interval) and vorticity, divergence, and strain ($\alpha = \sqrt{\sigma_n^2 + \sigma_s^2}$) at one of the fronts on the sampled transect. (B) Sea surface temperature measured from long wave infrared. Velocity is shown with vectors. (C) Joint probability density function of kinetic energy flux from the whole transect and shear strain.

281 covariance between the along track and cross track velocity. In fact, over the entire 60 km
 282 section, there is a strong correlation between the shear strain and the kinetic energy flux
 283 (Figure 4C).

284 The influence of the shear strain on the kinetic energy flux is modulated by the co-
 285 variance between the u and v velocity components (or, equivalently, the anisotropy of the
 286 flow), which becomes large below scales of 6 km in this filament region (Figure 3A). Barotropic
 287 shear instabilities extract kinetic energy from sheared mean flows when smaller scale fea-
 288 tures lean into the shear, resulting in a forward energy cascade.

289 The observed kinetic energy flux is patchy (Figure 4C), with the largest flux con-
 290 centrated in small spatial scales even within the 60 km filament region observed here.
 291 Within the larger filament region (the 60 km sampling region), the kinetic energy flux
 292 varies over three orders of magnitude (Figure 4C). The typical kinetic energy flux across
 293 1 km in the filament region is $\mathcal{O}(10^{-6} \text{ m}^2\text{s}^{-3})$. This rate is about an order of magnitude
 294 larger than the kinetic energy flux obtained from mooring based observations using a fil-
 295 ter scale of 5 days (Naveira Garabato et al., 2022) and in a modeling study at 500 m spa-
 296 tial resolution (Srinivasan et al., 2023). Given that we present direct observations of the
 297 kinetic energy flux terms, this suggests that the magnitude of instantaneous kinetic en-
 298 ergy flux has been underestimated by previous modeling and observational work.

299 4 Discussion and conclusion

300 The airborne observations presented here reveal a transition to non-linear submeso-
 301 scale dynamics at scales of 6–10 km with implications for kinetic energy flux. The syn-
 302 optical sampling from submesoscale to mesoscale allows us to extend an observational ki-
 303 netic energy spectrum to scales below 1 km. Dense filaments such as the one observed
 304 here have an important role in the energetics of upwelling systems with submesoscale
 305 dynamics influencing the fate of upwelled waters.

306 We demonstrate that although there is not a clear change in the kinetic energy spec-
 307 tral slope, there is a transition in the dynamics to non-linear interactions that charac-
 308 terize submesoscales at scales of $\mathcal{O}(1 \text{ km})$. In particular, this transition is character-
 309 ized by the interaction between divergent and rotational velocity components. This transi-

tion would not occur with surface quasigeostrophic dynamics and we attribute it instead to a dominance of ageostrophic dynamics in the observations.

The observed transition to non-linearity has important implications for observations of ocean velocity from remote sensing. For example, the SWOT mission aims to infer mesoscale to submesoscale velocities through observation of sea surface height. These velocities are computed through geostrophic balance, which only accounts for the rotational component of the flow. Not only do we find that a significant amount of the kinetic energy is likely in the divergent component of the flow at scales below 10 km in this region — and potentially at larger scales in more energetic regions (Callies et al., 2015) — but also that the rotational and divergent flows interact such that filtering of the divergent processes (e.g. waves) will not result in recovering the rotational component of the flow.

These observations are also the first direct observations of *snapshots* of kinetic energy flux and frontogenesis in the ocean. This allows us to investigate the relationships between the kinetic energy flux and hydrographic features. We find that kinetic energy flux is patchy but can be large ($10^{-6} \text{ m}^2\text{s}^{-3}$) at submesoscale fronts. The patchiness of kinetic energy flux has important implications for resolving the dynamics that contribute to an energy cascade. Due to the difficulty resolving scales ranging from mesoscale straining to turbulent dissipation in models, these observations — where that challenge is observationally addressed using a novel remote sensing platform — are particularly valuable. These aircraft measurements provide a precursor to what might be possible from future satellite-based radar snapshots from platforms such as Harmony and Seastar (Gommenginger et al., 2019; López-Dekker et al., 2019). In these observations, kinetic energy is transferred both downscale and upscale from 1 km.

Recent modeling work has suggested that resolving frontogenesis is essential to accurate representation of submesoscale kinetic energy transfers (Naveira Garabato et al., 2022; Srinivasan et al., 2023). The observations analyzed here demonstrate a large forward energy transfer localized at fronts, although not exclusively during active large-scale frontogenesis. Recent work in the Gulf of Mexico, another region with an active submesoscale, has hinted that a forward cascade of kinetic energy occurs at scales of 500 m–5 km (Balwada et al., 2022) in observations (with smaller scales during the summer) and at scales of 5 km in models (Srinivasan et al., 2023).

The modeling study of Sullivan and McWilliams (2018), which simulated a dense filament, also found an important role for the horizontal Reynolds stress term ($u'v'v_x$) during the frontal arrest phase of a dense filament, which is consistent with our observation that the shear strain term dominated kinetic energy flux. This relationship may arise from certain aspects of the feature studied here and may not generalize to all fronts. For example, Srinivasan et al. (2023) analyzed kinetic energy fluxes in 500 m and 2 km resolution ocean models, which resolve dynamics at larger scales than those that are the focus of our study. They find an equipartition between strain-driven and convergence-driven forward energy cascade at submesoscale scales (Srinivasan et al., 2023). While we observe that the forward energy transfer is strain-driven in our observations, it is important to note that we have only one snapshot of a filament that appears to be partially restratifying, so this does not invalidate the role of convergence in forward energy flux.

These results suggest an out-sized role for fronts and filaments as hotspots of surface kinetic energy flux. Barotropic energy transfer is enabled by interactions between the rotational and divergent components of the flow field at submesoscale fronts. Fronts are spatially inhomogeneously distributed in the ocean and vary seasonally (Drushka et al., 2019; Mauzole et al., 2020), but the distributions of fronts are distinct from the distributions of mesoscale kinetic energy (Busecke & Abernathey, 2019). Surface kinetic energy dissipation may similarly vary substantially in space and time, but understanding how it varies relies on increased mechanistic understanding of the energetics of submesoscale features. Disentangling these would require more observations to establish the effect of particular submesoscale features on the regional statistics.

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Open research

All presented data are available at UCSD Library Digital Collection, <https://doi.org/10.6075/J0F76CRK>.

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Supporting Information for “Non-linear processes characterize the transition to submesoscale dynamics in observations of a dense filament”

Mara Freilich¹, Luc Lenain¹, Sarah T. Gille¹

¹Scripps Institution of Oceanography

Contents of this file

1. Definitions
2. Cross spectrum, coherence, and phase
3. Surface quasigeostrophic model
4. Vertical kinetic energy flux terms
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1. Definitions

The velocity components can be written in terms of the sum of the velocity streamfunction (ψ) and velocity potential (ϕ)

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y} \tag{1}$$

$$v = \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}. \tag{2}$$

The velocity potential is divergent while the streamfunction is rotational.

2. Cross spectrum, coherence, and phase

The cross spectrum ($\hat{S}(f)$) is defined as

$$\hat{S}_{uv}(f) = \frac{\langle \hat{u}^* \hat{v} \rangle}{T}, \quad (3)$$

where \hat{u} is the Fourier transform of the u velocity, and $*$ indicates the complex conjugate (Bendat & Piersol, 2011). The angle brackets indicate averages over ν realizations. u is the along-track velocity and v is the cross-track velocity. The squared coherence is given by

$$\gamma_{uv}^2(f) = \frac{|\hat{S}_{uv}(f)|^2}{\hat{S}_u(f)\hat{S}_v(f)}. \quad (4)$$

$\hat{S}(f) = \hat{C}(f) + i\hat{Q}(f)$ is complex. The phase is given by

$$\tan(\phi_{uv}(f)) = \left(\frac{-\hat{Q}(f)}{\hat{C}(f)} \right). \quad (5)$$

The 95% percent confidence interval of the kinetic energy spectrum is estimated using a standard method by assuming that the ratio of estimated to true spectrum has a χ^2 distribution with expectation ν where ν is the number of segments. The significance of the estimated coherence is assessed using two methods. The 95% significance level is computed as $\sqrt{1 - \alpha^{1/(\nu-1)}}$ where $\alpha = 0.05$. The standard deviation of the coherence is calculated as

$$std_{\gamma_{uv}^2} = \frac{\sqrt{2}(1 - \gamma_{uv}^2)}{|\gamma_{uv}|\sqrt{2\nu}} \quad (6)$$

The standard error of the phase spectrum is calculated as

$$std_{\phi_{uv}} = \frac{\sqrt{1 - \gamma_{uv}^2}}{|\gamma_{uv}|\sqrt{2\nu}} \quad (7)$$

3. Surface quasigeostrophic model

A surface quasigeostrophic model (SQG) is used to validate the dynamical interpretation of the cross spectrum as representing contributions from ageostrophic advection. The surface quasigeostrophic model describes a flow field where the interior potential vorticity is zero and the full 3D dynamics are described by the 2D surface flow field. This is a suitable null model for this analysis because it is the simplest model that includes surface fronts but not ageostrophic advection. In this model, the interior potential vorticity is zero

$$\nabla^2\psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N_0^2} \frac{\partial\psi}{\partial z} \right) = 0, \quad (8)$$

where ψ is a streamfunction, f_0 is the Coriolis frequency, and N_0 is the buoyancy frequency. f_0 and N_0 are both constant. At depth (as $z \rightarrow -\infty$), $\psi = 0$. Surface density gradients are advected by and feedback on the streamfunction ψ

$$\frac{\partial b}{\partial t} + J(\psi, b) = 0. \quad (9)$$

The surface buoyancy gradients are related to the streamfunction through the hydrostatic relationship $b = \psi_z$. We can solve equation 8 for the streamfunction in Fourier space and obtain

$$\hat{\psi} = \frac{f_0}{N_0} \frac{1}{\kappa} \hat{b} \quad (10)$$

We initialize PyQG (Abernathey et al., 2022), a Python implementation of a surface quasigeostrophic model with two counter-rotating eddies. These eddies evolve to form filaments, but the net kinetic energy flux of the resolved dynamics is always upscale (Capet et al., 2008). A cross spectrum of the u and v velocity components in this model

shows that the cross spectrum is mostly real, particularly at the smallest wavenumbers, in contrast to the observed patterns (Figure S3).

4. Vertical kinetic energy flux terms

While typically only the horizontal terms are considered in spectral kinetic energy fluxes (Aluie et al., 2018; Balwada et al., 2022; Srinivasan et al., 2023), these observations allow us to diagnose kinetic energy fluxes that are associated with the vertical shear. The full expression for the kinetic energy flux is

$$\Pi_a = - \underbrace{(\tau_{uv}(u_y + v_x) + \tau_{uu}u_x + \tau_{vv}v_y)}_{\Pi_h} - \underbrace{(\tau_{uw}u_z + \tau_{vw}v_z)}_{\Pi_v} - \underbrace{(\tau_{ww}w_z)}_{\Pi_w} \quad (11)$$

The first term Π_h is shown in the main text. The second term, Π_v , is associated with baroclinicity of the flow. Here we diagnose this term using the velocity and sea surface temperature observations. The third term, Π_w , cannot be diagnosed with the available observations, but it is expected to be small. We compute Π_v by calculating the vertical velocity as $w = \delta \times h$ where δ is the surface divergence and h is the integration depth. Here, the integration depth is 1 meter, which is the approximate depth over which the surface velocity observations have been averaged (Lenain et al., 2023). The shear terms are computed from thermal wind balance. However, given that only sea surface temperature observations are available, we convert sea surface temperature to density using thermosalinograph observations from a ship that was nearby. The observed fronts are partially salinity compensated so the density is computed from temperature using a relationship within the observed temperature range. We find that Π_v peaks at the front, where δ is large, and is negative while Π_h is positive (Figure S6). However, Π_v is still at least an order of magnitude smaller than Π_h .

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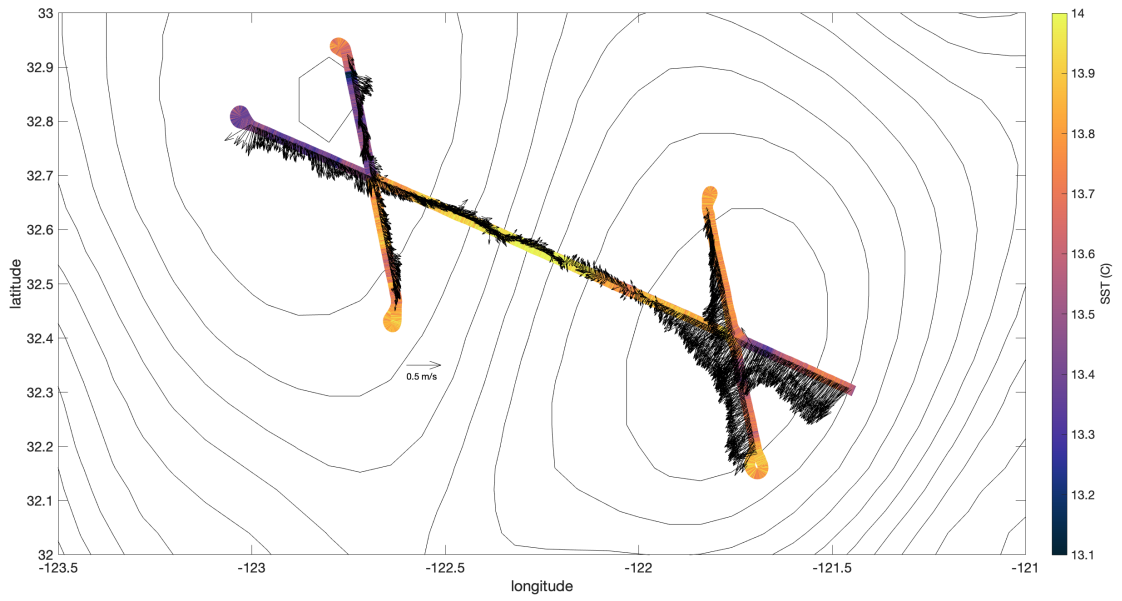


Figure S1. DoppVis velocity across the eddy region as vectors with sea surface temperature from the infrared thermometer. The contours are HYCOM sea surface height from May 19, 2021.

May 30, 2023, 3:46am

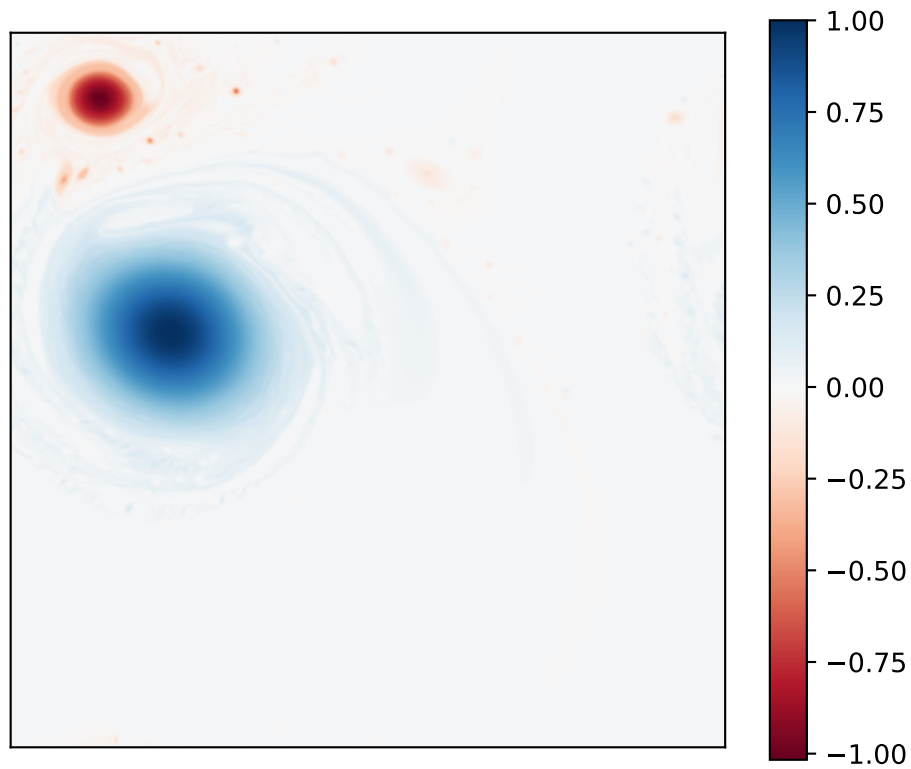


Figure S2. Snapshot of an SQG model initialized with an eddy and a filament. The SQG model used is PyQG. The model is non-dimensionalized with buoyancy frequency and Coriolis frequency equal, as is approximately the case in the surface layer here.

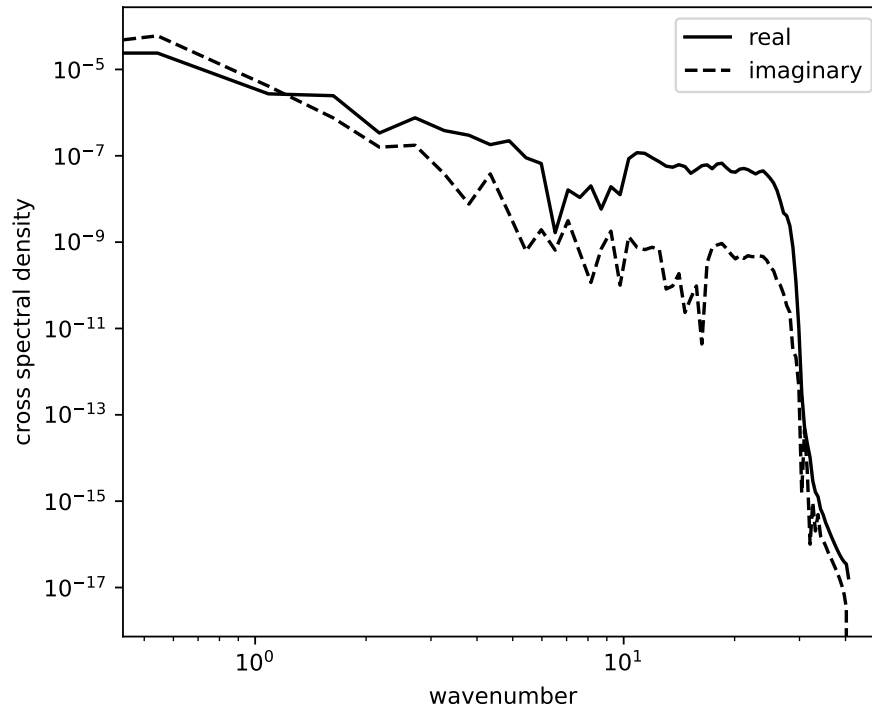


Figure S3. Cross spectrum as a function of wavenumber from a surface quasigeostrophic model initialized with two counter rotating eddies. The solid line is the real part of the cross spectrum while the dashed line is the imaginary part. In contrast to the observations where the imaginary part becomes relatively more important at small scales, in the SQG model the imaginary part becomes less important at small scales.

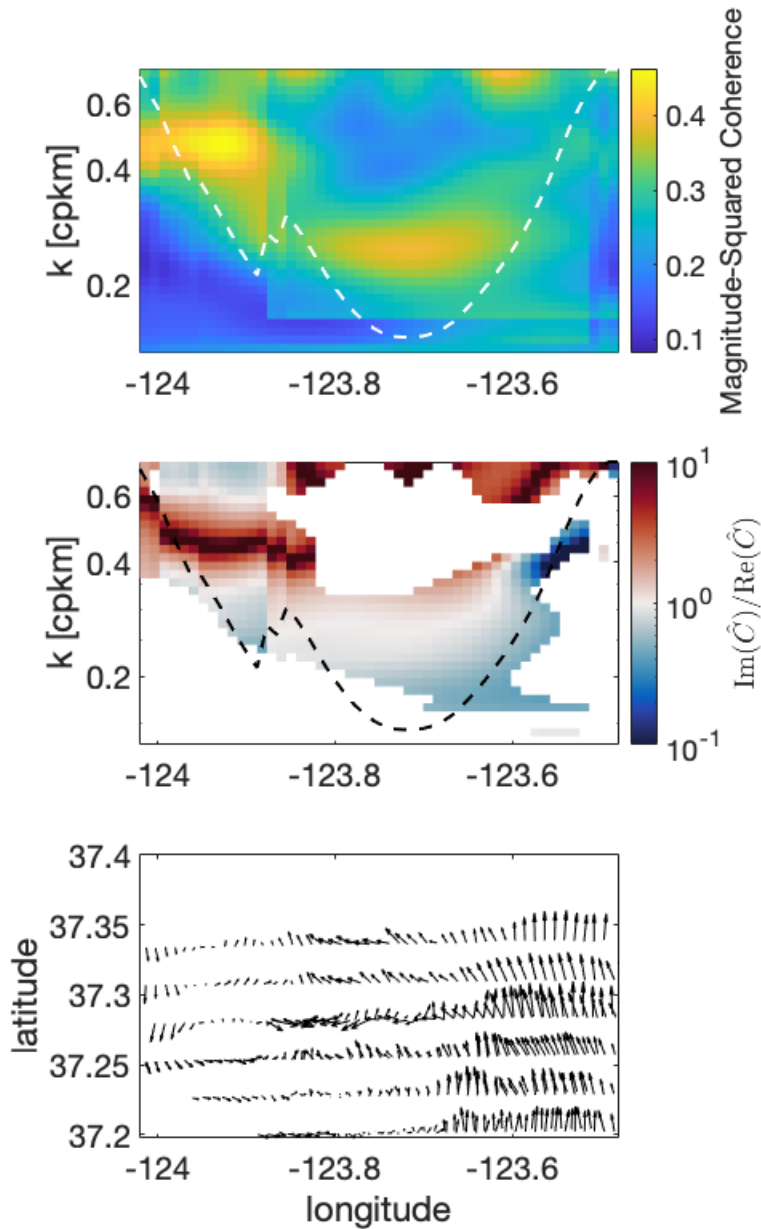


Figure S4. (top) Average wavelet coherence spectrum from S-MODE (middle) Ratio of the imaginary to real part of the cross spectrum. This ratio is only shown where the coherence value is above the significance threshold. (bottom) Velocity vectors. This analysis reveals that the shift from a predominantly real to a complex cross spectrum occurs at the strongest fronts in the region sampled, occurring at slightly larger scales at the western front and slightly smaller scales at the eastern fronts.

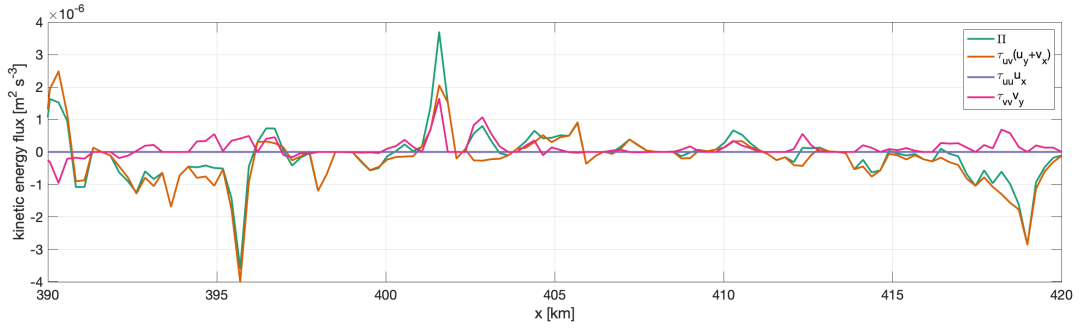


Figure S5. Kinetic energy flux across 1 km (Π) and its component parts $\Pi = -(\tau_{uv}(u_y + v_x) + \tau_{uu}u_x + \tau_{vv}v_y)$.

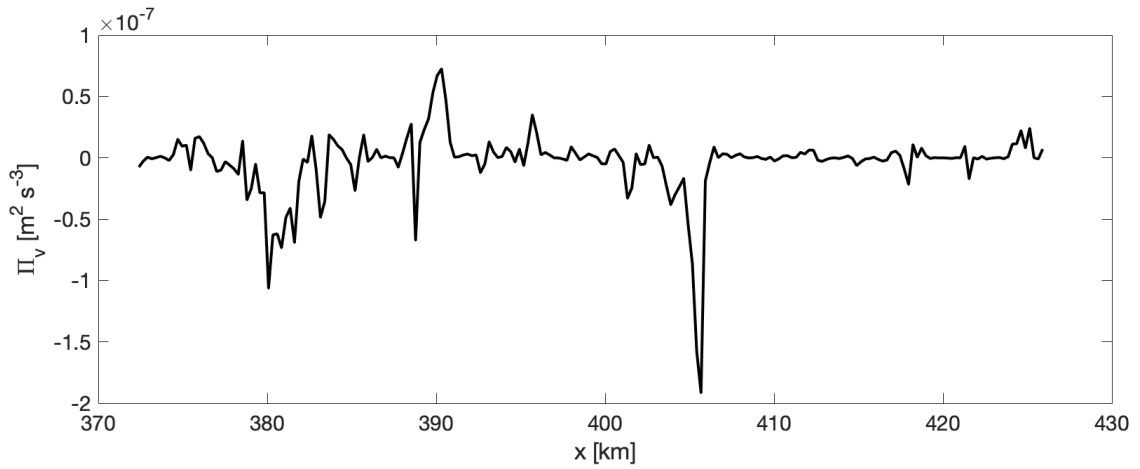


Figure S6. Kinetic energy flux associated with vertical shear Π_v across 1 km.