Event-triggered distributed model predictive control of linear systems with additional disturbances

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Abstract

This paper presents an event-triggered model predictive control scheme for distributed linear systems with additional disturbances. The subsystem states are coupled with each other and affected by unknown bounded disturbances. The communication among subsystems is assumed to be prompt and free from any information loss. A novel distributed event-triggered strategy is developed to balance communication resources and system control performance during asynchronous communication. This mechanism is meticulously designed to ensure optimal system performance while utilizing communication resources. The nominal system is introduced to construct a local optimization problem and a triggering mechanism considering the coupling influence is developed. To counter the additional disturbances, the dual-mode control approach has been implemented along with developing a robust terminal set. The terminal set is purposefully designed to maintain system stability in the presence of additive disturbances, achieved through a meticulously designed triggering mechanism. Then it is imperative to discuss the stability of the resulting closed-loop system and provide a proof process of the feasibility of the iterative optimization. Finally, the effectiveness of the proposed algorithm is validated through simulation results, thereby confirming its efficacy.

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RESEARCH ARTICLE

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This paper presents an event-triggered model predictive control scheme for distributed linear systems with additional disturbances. The subsystem states are coupled with each other and affected by unknown bounded disturbances. The communication among subsystems is assumed to be prompt and free from any information loss. A novel distributed event-triggered strategy is developed to balance communication resources and system control performance during asynchronous communication. This mechanism is meticulously designed to ensure optimal system performance while utilizing communication resources. The nominal system is introduced to construct a local optimization problem and a triggering mechanism considering the coupling influence is developed. To counter the additional disturbances, the dual-mode control approach has been implemented along with developing a robust terminal set. The terminal set is purposefully designed to maintain system stability in the presence of additive disturbances, achieved through a meticulously designed triggering mechanism. Then it is imperative to discuss the stability of the resulting closed-loop system and provide a proof process of the feasibility of the iterative optimization. Finally, the effectiveness of the proposed algorithm is validated through simulation results, thereby confirming its efficacy.

KEYWORDS
Model predictive control (MPC), Event-triggered mechanism, Robust control, Distributed control system

1 | INTRODUCTION

Model predictive control (MPC) is a highly effective control method that widely utilized in various industrial production processes[1], such as chemical processes[2], multi-vehicle systems[3] as well as power network systems[4,5]. With the increment of system size and variable dimensions, the adoption of the central control strategy significantly impacts control performance, thereby posing challenges in meeting the requirements of industrial processes. With advancements in communication technology and computer science, distributed model predictive control (DMPC) is extensively explored as an alternative solution for centralized control concerns[6,7,8]. By utilizing measurement values and neighboring operation information, subsystems can efficiently solve optimization problems. Consequently, this leads to a reduction in the scale of the optimization problem, resulting in a substantial enhancement of global control performance[9,10,11].

In recent years, the applications of DMPC and the coordinated control of distributed systems have garnered considerable attention. For addressing collision and obstacle avoidence problem in multiple linear second-order agents, a DMPC strategy is proposed in[12]. By designing the terminal controller, it is ensured that the terminal set is positively invariant and satisfies collision avoidance constraints and obstacle avoidance constraints. In[13], a collaborative strategy for DMPC is proposed for decomposing the hierarchical structure. The subsystems are divided into different layers according to the strength of the couplings. In the collaborative DMPC algorithm, the necessity for global communication is alleviated, as only intra-layer communication is required. This effectively reduces the communication burden without significant performance degradation. A DMPC method considering neighbouring optimization is implemented to effectively control the tension system in[14]. The input and state evolution information is considered in the optimization process, and thus a better control performance can be achieved. Then

Abbreviations: MPC, model predictive control; DMPC, Distributed model predictive control; OCP, optimal control problem.

the DMPC algorithm is put through simulations to verify its efficacy. In [15], a fully DMPC method is adopted to guarantee the safety of voltage and information of integrated hybrid energy systems. The local controllers are designed by considering the different requirements of voltage quality for each microgrid. The Nash equilibrium is attained, and several simulations are provided to demonstrate the designed algorithm. In [16], a serial DMPC approach is developed for connected automated vehicles. The asymptotic local stability is analysed and some simulations provide clear evidence that the proposed strategy surpasses traditional MPC in terms of performance. In [17], a DMPC algorithm for nonlinear systems with coupled constraints is introduced, using the inexact distributed sequential quadratic programming (QP) framework for fully distributed execution and early QP termination handling. The algorithm ensures recursive feasibility and stability, with simulations on multi-agent formation control demonstrating its effectiveness compared to benchmarks. These methods provide an excellent research foundation for distributed dynamic systems.

Note that external disturbances in real-world industrial production processes are inevitable. Consequently, several studies have been conducted to address the robust performance of distributed systems, aiming to handle constraints, disturbances, and uncertainties inherent in interconnected systems. In [18], a DMPC strategy is formulated to control the systems with multiplicative uncertainties. The local controller is updated sequentially to guarantee that the algorithm is feasible in the stochastic sense. In [19], the robust invariant regions of discrete-time systems are studied. The general framework of robust invariant regions is established. Some constraints are also given to ensure the optimization process is feasible. The consensus problem based on the robust DMPC for multi-agent systems affected by external interference is presented [20]. A consensus algorithm based on the nominal system is designed and the expected state and the actual state are limited in an invariant set. Thus, the robustness and feasibility of controllers are guaranteed. In [21], a robust DMPC formula is developed to stabilize a disturbed linear system. The independent terminal cost and local robust terminal region are designed. The approach employs a set membership degree approach to ensure compliance with coupling constraints in the presence of disturbances.

It is essential to highlight that the majority of the aforementioned research on distributed MPC algorithms relies on the time-triggered mechanism. This mechanism entails that each subsystem synchronously addresses optimization problems and carries out operational actions within each sampling period. However, this update and control strategy fails to fully account for the operational information of subsystems, which may impose great communication pressure [22,23,24]. Therefore, there has been growing interest in the utilization of event-triggered strategies, which have been demonstrated to be highly effective in mitigating communication pressure. The event-triggered method proves to be more efficient in utilizing communication resources compared to the time-triggered mechanism while achieving good system performance. In [25], an event-triggered DMPC method is put forward, and conditions for triggering are determined through the system stability analaysis. Consequently, the local controllers collaborate autonomously to address optimization problems and achieve synchronization among themselves. This collaborative effort leads to a reduction in communication traffic and fewer controller updates. The scheme of event-triggered DMPC for a multi-agent system is presented in [26]. The asynchronous coordination is realized by exchanging information only at the specific triggering time. The dynamic variable, incorporating neighbor information, is utilized to design triggering conditions. The control performance can be ensured by implementing rigorous theoretical conditions. A threshold-based event-triggered DMPC method is developed for effective traffic signal control in [27]. Through the investigation of the literature, only a few research works consider the system couplings and additional disturbance in event-triggered control of distributed systems. Developing an event-triggered mechanism for distributed systems can be quite challenging, mainly due to the existence of disturbances and system dynamic couplings. Moreover, asynchronous communication presents difficulties in maintaining the global stability of control systems.

Inspired by the aforementioned observations, the event-triggered DMPC problem of dynamic coupling systems is studied in this paper. The primary challenge is to guarantee the practicality of the event-triggered DMPC method while maintaining the stability of the whole control system, even under bounded disturbances and couplings. The following are the primary contributions of this study: (i) A DMPC optimization problem is formulated under an event-triggered mechanism, utilizing the introduced nominal system. The inclusion of certain constraints is developed to guarantee the proposed algorithm is feasible. (ii) Taking into account state couplings and additional disturbances, the triggering conditions are determined by comparing the error between current and predicted states. Building upon the designed triggering mechanism, an event-triggered distributed strategy is proposed and a dual-mode MPC method strategy is implemented to effectively conserve communication resources while system performance can be guaranteed. (iii) The feasibility of the developed DMPC algorithm is ensured by establishing sufficient conditions. Additionally, a thorough analysis of the stability and robustness is also conducted. The analysis demonstrates that the system state can eventually converge to robust invariant regions.

The content of this article is structured as follows: The full text consists of six chapters, with section 2 describing the investigated problem and section 3 developing a DMPC framework and the event-triggering mechanism. In section 4, The focus is on ensuring the stability of the closed-loop system and determining the feasibility of the optimization algorithm. In section 5, some simulations are conducted to validate the effectiveness of the proposed control strategy. The conclusions are drawn in section 6.

Notation: The mathematical symbol $\mathbb{R}^n$ represents an Euclidean space with n dimensions. $\mathcal{N}$ denotes a set with N integers. $\|x\|$ denotes the 2-norm for a vector x, defined as $\|x\| = \sqrt{x^{\mathrm{T}}x}$, and $\|x\|^2_{\mathcal{H}}$ represents $x^{\mathrm{T}}\mathcal{H}x$. $\underline{\lambda}(P)$ represents minimum eigenvalue and $\bar{\lambda}(P)$ represents the maximum eigenvalue of matrix P.

2 | PROBLEM DESCRIPTION AND PRELIMINARIES

A discrete-time linear system consisting of N interdependent subsystems is the focus of this paper. Generally, the dynamics of the i-th subsystem are described as

$$x_i(k+1) = A_{ii}x_i(k) + B_iu_i(k) + \sum_{j\in\mathbb{N}_i} A_{ij}x_j(k) + d_i(k) \tag{1}$$

where $x_i(k) \in \mathbb{R}^{n_i}$ and $u_i(k) \in \mathbb{R}^{m_i}$ represent the states and control inputs of subsystem i, $i \in \mathcal{N}$, respectively; $x_j(k) \in \mathbb{R}^{n_j}$ is the state vector of neighbouring subsystem j, $j \in \mathbb{N}_i$, where $\mathbb{N}_i$ is the set of neighbours interacting with subsystem i; $A_{ii} \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times m_i}$ and $A_{ij} \in \mathbb{R}^{n_i\times n_j}$ are constant matrices. The additional disturbances $d_i(k)$ are bounded by $\|d_i(k)\| \leqslant \tilde{d}_i$.

Define $x(k) = [x_1^{\mathrm{T}}(k), \ldots, x_N^{\mathrm{T}}(k)]^{\mathrm{T}}$, $u(k) = [u_1^{\mathrm{T}}(k), \ldots, u_N^{\mathrm{T}}(k)]^{\mathrm{T}}$ and $d(k) = [d_1^{\mathrm{T}}(k), \ldots, d_N^{\mathrm{T}}(k)]^{\mathrm{T}}$. The overall system (1) can be given as

$$x(k+1) = Ax(k) + Bu(k) + d(k) \tag{2}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ are the states and the input of the whole system; the upper bound of disturbances $d(k)$ can be defined as $\|d(k)\| \leqslant \|\tilde{d}'\|$, $\tilde{d}' = (\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_N)$; the matrix $B = \mathrm{diag}\,(B_1, \ldots, B_N)$, and the system matrix A is a block matrix consisting of $N \times N$ blocks.

Throughout this paper, we adopt the following assumptions.

Assumption 1. For the system described in (1), if (A_{ii}, B_i) is stabilized, then a state feedback gain K_i can be found such that $A_{c_i} = A_{ii} + B_iK_i, \forall i \in \mathcal{N}$ is Schur and $A_c = A + BK$ is Schur, where $K = \mathrm{diag}(K_1, \ldots, K_N)$.

Assumption 2. Given the matrices $\mathcal{H}_i > 0$ and $\mathcal{C}_i > 0$, there exists a positive definite matrix P_i such that $A_{c_i}^{\mathrm{T}}P_iA_{c_i} - P_i \leqslant -\left(\mathcal{H}_i + K_i^{\mathrm{T}}\mathcal{C}_iK_i\right)$ for the i-th subsystem. Let $\mathcal{H} = \mathrm{diag}\,(\mathcal{H}_1, \ldots, \mathcal{H}_N)$, $\mathcal{C} = \mathrm{diag}\,(\mathcal{C}_1, \ldots, \mathcal{C}_N)$ and $P = \mathrm{diag}\,(P_1, \ldots, P_N)$, $A_c^{\mathrm{T}}PA_c - P \leqslant -\left(\mathcal{H} + K^{\mathrm{T}}\mathcal{C}K\right)$ can be satisfied for the whole system.

Assumption 3. It is assumed that every subsystem can communicate ideally without delay.

Lemma 1. *Given the fulfillment of Assumptions 1-3 and weight matrices $\mathcal{H}_i$ and $\mathcal{C}_i$ with proper dimensions. Then, there exists a constant $\sigma > 0$ such that $\Omega(\sigma) \triangleq \left\{x \in \mathbb{R}^n : \|x\|_P \leqslant \sigma\right\}$ remains invariant for the nominal system of* (2)*, namely, for system* (2) *under the control input $u(k) = Kx(k)$ when the additive disturbances $d(k) = 0$.*

Proof. For the i-th subsystem, $V_i(x_i(k)) = \|x_i(k)\|^2_{P_i}$ is a Lyapunov function candidate. The stage cost is defined as $J_{is}(k) = \|x_i(k)\|^2_{\mathcal{H}_i} + \|u_i(k)\|^2_{\mathcal{C}_i}$. Subsequently, the property of $V_i(x_i(k))$ is exhibited as follows:

$$\begin{aligned}
&\sum_{i=1}^{N}(V_i(x_i(k+\mathcal{P}+1)) - V_i(x_i(k+\mathcal{P})) + J_{il}(k+\mathcal{P})) \\
&= \sum_{i=1}^{N}\left\{x_i^{\mathrm{T}}(k+\mathcal{P}+1)P_ix_i(k+\mathcal{P}+1) - x_i^{\mathrm{T}}(k+\mathcal{P})P_ix_i(k+\mathcal{P}) + \|x_i(k+\mathcal{P})\|^2_{\mathcal{H}_i} + \|u_i(k+\mathcal{P})\|^2_{\mathcal{C}_i}\right\} \\
&= (A_cx(k+\mathcal{P}))^{\mathrm{T}}P(A_cx(k+\mathcal{P})) - (x(k+\mathcal{P}))^{\mathrm{T}}P(x(k+\mathcal{P})) + \|x(k+\mathcal{P})\|^2_{\mathcal{H}} + \|u(k+\mathcal{P})\|^2_{\mathcal{C}} \\
&= x^{\mathrm{T}}(k+\mathcal{P})(A_c^{\mathrm{T}}PA_c - P + \mathcal{H} + K^{\mathrm{T}}\mathcal{C}K)x(k+\mathcal{P})
\end{aligned} \tag{3}$$

Consequently, the dynamics of the whole system can be stabilized by distributed controllers separately when states are in the invariant set $\Omega(\sigma)$. This implies that all trajectories will converge to the origin and stay inside if the states of the closed-loop system start within $\Omega(\sigma)$. Additionally, for all $x_i \in \Omega_i(\sigma_i)$, there is a constant $\sigma_i > 0$ that the state can converge to zero under the input $u_i(k) = K_i x_i(k)$. The proof of Lemma 1 is now completed. □

3 | EVENT-TRIGGERED MECHANISM AND DMPC STRATEGY

In this section, the distributed optimal control problem (OCP) utilizing an event-triggered mechanism is presented. Moreover, triggering conditions involving state couplings and disturbances are provided, and the algorithm of event-triggered DMPC is also developed.

The dynamics of system (1) ignoring the disturbance is introduced as follows:

$$\breve{x}_i(k+1) = A_{ii}\breve{x}_i(k) + B_i\breve{u}_i(k) + \sum_{j\in\mathbb{N}_i} A_{ij}\breve{x}_j(k) \tag{4}$$

From equation (4), the global system is defined as

$$\breve{x}(k+1) = A\breve{x}(k) + B\breve{u}(k) \tag{5}$$

where $\breve{x}(k) = [\breve{x}_1^{\mathrm{T}}(k), \ldots, \breve{x}_N^{\mathrm{T}}(k)]^{\mathrm{T}}$, $\breve{u}(k) = [\breve{u}_1^{\mathrm{T}}(k), \ldots, \breve{u}_N^{\mathrm{T}}(k)]^{\mathrm{T}}$. Furthermore, the cost function of the i-th subsystem can be written as:

$$J_i(\breve{x}_i(k), \breve{u}_i(k)) = \sum_{q=0}^{\mathcal{P}-1}(\|\breve{x}_i(k+q|k)\|^2_{\mathcal{H}_i} + \|\breve{u}_i(k+q|k)\|^2_{\mathcal{C}_i}) + \|\breve{x}_i(k+\mathcal{P}|k)\|^2_{P_i} \tag{6}$$

where $\mathcal{H}_i$ and $\mathcal{C}_i$ are positive definite matrices; P_i is chosen to satisfy Lemma 1. In this paper, we define the stage cost $J_{iq}(k) = \sum_{q=0}^{\mathcal{P}-1}(\|\breve{x}_i(k+q|k)\|^2_{\mathcal{H}_i} + \|\breve{u}_i(k+q|k)\|^2_{\mathcal{C}_i})$ and the terminal cost $J_{if} = \|\breve{x}_i(k+\mathcal{P}|k)\|^2_{P_i}$, respectively.

The introduction of nominal systems aims to address the challenge of obtaining accurate states due to unknown disturbances. Thus, the use of nominal states is necessary for designing the event-triggered DMPC condition, as unknown disturbances can make it difficult to obtain accurate states. Each local controller is considered to solve a highly demanding optimization problem in a finite time, and triggering conditions are designed with consideration for the uncertainties and performance of systems within the DMPC framework. When these conditions are fulfilled, the optimization problem is solved by local controllers, with the state information subsequently transmitted to neighbouring subsystems to get coordination between controllers. The optimization control problem is defined in (7):

$$\min_{\breve{u}_i(k|t_k^i),\cdots,\breve{u}_i(k+\mathcal{P}-1|t_k^i)} J_i(\breve{x}_i(t_k^i), \breve{u}_i(t_k^i)) \tag{7a}$$

subject to:

$$\breve{x}_i(k+q+1|t_k^i) = A_{ii}\breve{x}_i(k+q|t_k^i) + B_i\breve{u}_i(k+q|t_k^i) + \sum_{j\in\mathbb{N}_i} A_{ij}\breve{x}_j^*(k+q|\varphi^j(t_k^i)), q = 0, \ldots, \mathcal{P}-1 \tag{7b}$$

$$\sum_{r=0}^{q-1} \bar{\lambda}^{\frac{1}{2}}\left(\eta_i{}^{\mathrm{T}} P_i \eta_i\right) \|\breve{x}_i(k+r|t_k^i) - \breve{x}_i(k+r|t_{k-1}^i)\| \leqslant \frac{\gamma\kappa\sigma_i}{m_1}, q = 1, \ldots, \mathcal{P}-1 \tag{7c}$$

$$\|\breve{x}_i(k+\mathcal{P}-1|t_k^i)\|_{P_i} \leq (1-\kappa)\sigma_i \tag{7d}$$

$$\breve{x}_i(k|t_k^i) = x_i(t_k^i), k = t_k^i \tag{7e}$$

$$\breve{x}_i(k+\mathcal{P}|t_k^i) \in \Omega_i(\sigma_i) \tag{7f}$$

$$\breve{u}_i(k+q|t_k^i) \in \mathbb{U}_i \tag{7g}$$

where t_k^i denotes the triggering instant of subsystem i and t_{k-1}^i is the last triggering instant. $\varphi^j(t_k^i)$ is the triggering time of system j satisfying $\varphi^j(t_k^i) = \max_{k\in Z}\{\varphi^j(t_k^i)|\varphi^j(t_k^i) < t_k^i, j \in \mathbb{N}_i\}$, which is adopted to describe the triggering instant latest to t_k^i of subsystem j and the sampling time $k \in Z$ with Z being the integer set. $\breve{x}_j^*(\varphi^j(t_k^i))$ represents an optimal state of neighbouring subsystem at the triggering instant $\varphi^j(t_k^i)$. And $\eta_j = A_{ii}^{q-r-1}A_{ij}$, the parameter γ and κ are two constants satisfying $0 < \gamma < 1$ and $0 < \kappa < 1$, and $n_1 = \max\limits_{i\in\mathcal{N}}|\mathbb{N}_i|$ is defined to describe the maximum number of neighbouring subsystems interacting with subsystem i.

Remark 1. The dynamics of subsystem i need its neighbouring information due to state coupling. However, The triggering instants of subsystem i and the neighbouring subsystem j are not synchronous in the event-triggered framework. It implies that the state of neighbouring subsystem $\breve{x}_j(t_k^i)$ cannot be obtained directly. Then the state $\breve{x}_j^*(\varphi^j(t_k^i))$, which is an optimal state of neighbouring subsystem at the triggering instant closest to t_k^i, is introduced.

For each subsystem i, the optimal control input is determined by solving the OCP (7) at the triggering time t_k^i. To maintain the stability of the distributed system and alleviate the calculation and communication burden, the control strategy operates in dual modes. Lemma 1 indicates that the control input $u_i(k) = K_ix_i(k)$ can be used to stabilize subsystem i when $x_i \in \Omega_i(\sigma_i)$. Furthermore, outside of the terminal region, the optimal control input $u_i^*(t_k^i)$ acts on the local subsystem i. If states of subsystems enter the terminal region, i.e., $x_i \in \Omega_i(\sigma_i)$, $u_i(k) = K_ix_i(k)$ will be used.

The optimal control input is defined as $\breve{u}_i^*\left(k + q|t_k^i\right)$ and the corresponding optimal state is $\breve{x}_i^*\left(k + q|t_k^i\right)$.

According to the dual-mode control method mentioned above, a feasible control input of the i-th subsystem at k is constructed as:

$$\bar{u}_i(k+q|k) = \begin{cases} \breve{u}_i^*(k+q|\varphi^i(k)), & q \in \{0,\ldots,\mathcal{P}-2\} \\ K_i\bar{x}_i(k+q|k), & q = \mathcal{P}-1 \end{cases} \tag{8}$$

where $K_i\bar{x}_i(k+q|k)$ serves as the terminal control law and $\varphi^i(k)$ represents the triggering instant of system i, satisfying $\varphi^i(k) = \max_{k\in Z}\{\varphi^i(k)|\varphi^i(k) < k\}$. As shown in the formula (8), the optimal control inputs from the last time will be adopted at the non-triggered instant as the feasible inputs to achieve iterative optimization of local MPC. Then the feasible state can be approximately defined by the optimal state at the last triggering instant. The feasible initial state $\bar{x}_i(k|k)$ and the feasible state $\bar{x}_i(k+q|k)$ can be gained using equation (8).

For any $q = 1, ..., \mathcal{P}, \bar{x}_i(k+q|k)$ can be obtained as:

$$\bar{x}_i(k+q+1|k) = A_{ii}\bar{x}_i(k+q|k) + B_{ii}\bar{u}_i(k+q|k) + \sum_{j\in\mathbb{N}_i} A_{ij}\breve{x}_j^*(k+q|\varphi^j(k)) \tag{9}$$

The estimated value of the terminal state $\hat{x}_i(k+\mathcal{P}|k)$ is calculated through a iterative process, with its definition established for each sampling period.

$$\hat{x}_i(k+\mathcal{P}|k) = A_{c_i}\bar{x}_i(k+\mathcal{P}-1|k-1) + \sum_{j\in\mathbb{N}_i} A_{ij}\breve{x}_j^*(k+\mathcal{P}-1|\varphi^j(k)) \tag{10}$$

By solving the optimal problem (7), the optimized state trajectory $\breve{x}_i^*(t_k^i)$ can be described as:

$$\breve{x}_i^*(t_k^i+q+1|t_k^i) = A_{ii}\breve{x}_i^*(t_k^i+q|t_k^i) + B_i\breve{u}_i^*(t_k^i+q|t_k^i) + \sum_{j\in\mathbb{N}_i} A_{ij}\breve{x}_j^*(t_k^i+q|\varphi^j(t_k^i)) \tag{11}$$

Moreover, actual state trajectories $x_i^*(t_k^i)$ are defined as:

$$x_i^*(t_k^i+q+1|t_k^i) = A_{ii}x_i^*(t_k^i+q|t_k^i) + B_i\breve{u}_i^*(t_k^i+q|t_k^i) + \sum_{j\in\mathbb{N}_i} A_{ij}x_j^*(t_k^i+q|\varphi^j(t_k^i)) + d_i(t_k^i) \tag{12}$$

For formulas (11) and (12), the optimal inputs are obtained by solving the OCP. It implies that the actual input $u_i^*(k)$ and neighbouring states $x_j(t_k^i)$ are difficult to obtain for subsystem i at triggering instant t_k^i. In addition, the unknown disturbances also bring a great challenge to the accuracy of control. On this basis, the event-triggered strategy is proposed to ensure system performance while reducing communication resource consumption.

Define the triggering condition of subsystem $i, i \in \mathcal{N}$ as:

$$\left\| x_i(k) - \breve{x}_i^*(k|t_k^i) \right\|_{P_i} \geq \delta_i, \tag{13}$$

where $\breve{x}_i^*(k|\varphi^i(t_k^i))$ denotes the feasible state that describes a state trajectory at a non-triggered instant. δ_i is a constant satisfying $\delta_i \geq 0$. This condition imposes limits on the error of the subsystem at non-triggered instants, ensuring that it remains within an allowable range and thereby guaranteeing global performance.

Considering the prediction horizon of local controllers, the state information received from neighbours has at most $\mathcal{P}$ values. Therefore, the triggering condition (14)will be set at $t_k^i + \mathcal{P}$ to guarantee that the optimization control process will be addressed even if the condition (13) is not met throughout the entire prediction horizon.

$$k \geq t_k^i + \mathcal{P}. \tag{14}$$

Then the optimal input $\breve{u}_i^*(t_{k+1}^i + q|t_{k+1}^i)$ can be gained by solving the OCP, and the state $\breve{x}_i^*(t_{k+1}^i + q|t_{k+1}^i)$ will be transmitted to the neighbouring subsystems. The investigated algorithm is summarised as Algorithm 1.

Algorithm 1 Event-triggered Model Predictive Control algorithm

Initialization: For the i-th local controller, select matrices $\mathcal{H}_i$ and $\mathcal{C}_i$ with proper dimensions and calculate K_i and P_i; select parameters κ and δ_i; set initial values $x_i(0)$ and $u_i(0)$.
for $x_i(t_k^i) \notin \Omega_i(\sigma_i)$ **do**
  if k=0 **then**
    Solve the OCP (7) and gain the optimal input $\breve{u}_i^*(t_0^i + q|t_0^i)$, $q = 0, \ldots, \mathcal{P} - 1$.
    The first term of optimal input $\breve{u}_i^*(t_0^i|t_0^i)$ will be applied to subsystem i.
    Transmit $\breve{x}_i^*(t_0^i + q|t_0^i)$, $q = 0, \ldots, \mathcal{P}$ to all the neighbouring subsystems j, $j \in \mathbb{N}_i$.
  end if
  Check the event-triggered conditions (13) and (14).
  while Conditions (13) and (14) are not satisfied **do**
    Implement $\breve{u}_i^*(k + q|\varphi^i(k))$ on the subsystem i.
  end while
  Solve the optimal problem (7) and gain the optimal input $\breve{u}_i^*(t_k^i + q|t_k^i)$, $q = 0, \ldots, \mathcal{P} - 1$.
  The first term of optimal input $\breve{u}_i^*(t_k^i|t_k^i)$ will be applied to subsystem i.
  Transmit $\breve{x}_i^*(k + q|t_k^i)$, $q = 0, \ldots, \mathcal{P}$ to all neighbouring subsystems j, $j \in \mathbb{N}_i$.
  $k \longleftarrow k + 1$
end for
Implement the control law $\bar{u}_i(k) = K_i \bar{x}_i(k)$ on the subsystem i.

4 | PROPERTIES OF THE CLOSED-LOOP SYSTEM

The feasibility of the distributed optimization process and the investigation of adequate conditions to ensure the stability of the closed-loop system are examined in this section. Additionally, a robustness analysis of the system is presented.

The discussion below assesses the feasibility of the OCP to guarantee a smooth local MPC iterative process. The optimization control problem (7) is assumed to be solved at t_k^i. Then it's necessary to ensure the optimization process can proceed at t_{k+1}^i. The feasible control inputs are constructed in (8), which implies the feasible input of non-triggered instant can be selected as the optimal control input at the most recent triggering instant. Besides, the terminal control law can stabilize the subsystem based on

Lemma 1. Therefore, the optimization problem is solvable at t_{k+1}^i. The feasible state is described as:

$$\bar{x}_i(k+q|k) = A_{ii}^q x_i(k) + \sum_{r=0}^{q-1} A_{ii}^{q-r-1} B_i \bar{u}_i(k+r|k) + \sum_{j\in\mathbb{N}_i}\sum_{r=0}^{q-1} A_{ii}^{q-r-1} A_{ij} \breve{x}_j^* \left(k+r|\varphi^i(k)\right) \tag{15}$$

Based on equation (11), it can be inferred that:

$$\breve{x}_i^*(k+q|t_k^i) = A_{ii}^q \breve{x}_i^*(t_k^i) + \sum_{r=0}^{q-1} A_{ii}^{q-r-1} B_i \breve{u}_i^*(k+r|t_k^i) + \sum_{j\in\mathbb{N}_i}\sum_{r=0}^{q-1} A_{ii}^{q-r-1} A_{ij} \breve{x}_j^* \left(k+r|\varphi^i(t_k^i)\right) \tag{16}$$

The distinction between the feasible state and the optimal state can be described as follows:

$$\begin{aligned}
&\bar{x}_i(k+q|k) - \breve{x}_i^*\left(k+q|t_k^i\right) \\
=&A_{ii}^q x_i(k) - \breve{x}_i^*(k|t_k^i) + \sum_{j\in\mathbb{N}_i}\sum_{r=0}^{q-1} A_{ii}^{q-r-1} A_{ij}\left[\breve{x}_j^*(k+r|\varphi^i(k)) - \breve{x}_j^*(k+r|\varphi^i(t_k^i))\right] \\
=&A_{ii}^q e_i(k) + \sum_{j\in\mathbb{N}_i}\sum_{r=0}^{q-1} \eta_j(e_j(k+r|\varphi^i(k)))
\end{aligned} \tag{17}$$

where $e_j(k+r|\varphi^i(k)) = \breve{x}_j^*(k+r|\varphi^i(k)) - \breve{x}_j^*(k+r|\varphi^i(t_k^i))$, $e_i(k) = x_i(k) - \breve{x}_i^*(k|t_k^i)$.

The following lemma demonstrates that possible states will eventually reach the terminal region $\Omega_i(\sigma_i)$ under the designed local model predictive controller.

Lemma 2. *Consider the i-th subsystem described in* (1) *and suppose that Assumptions 1–3 hold. If the following conditions are satisfied, the OCP can be solved at each* t_{k+1}^i.

$$\delta_i \leqslant \frac{\kappa\sigma_i(1-\gamma)}{\max\limits_{q\in\{1,\ldots,\mathcal{P}-1\}} \bar{\lambda}^{-\frac{1}{2}}\left(P_i^{-\frac{1}{2}}\left(A_{ii}^q\right)^{\mathrm{T}} P_i A_{ii}^q P_i^{-\frac{1}{2}}\right)} \tag{18}$$

where γ is a constant satisfying $\gamma \in (0,1)$. Then, we can get $\bar{x}_i(k+\mathcal{P}-1|k) \in \Omega_i(\sigma_i)$.

Proof. Applying the triangle inequality and combining formula (17), we have

$$\begin{aligned}
&\|\bar{x}_i(k+q|k)\|_{P_i} - \left\|\breve{x}_i^*\left(k+q|t_k^i\right)\right\|_{P_i} \\
\leqslant&\left\|\bar{x}_i(k+q|k) - \breve{x}_i^*\left(k+q|t_k^i\right)\right\|_{P_i} \\
\leqslant&\left\|P_i^{\frac{1}{2}} e_i(k)\right\|_{P_i^{-\frac{1}{2}}(A_{ii}^q)^{\mathrm{T}} P_i A_{ii}^q P_i^{-\frac{1}{2}}} + \sum_{j\in\mathbb{N}_i}\sum_{r=0}^{q-1}\left\|\eta_j(e_j(k+r|\varphi^i(k)))\right\|_{P_i} \\
\leqslant&\bar{\lambda}^{\frac{1}{2}}\left(P_i^{-\frac{1}{2}}\left(A_{ii}^q\right)^{\mathrm{T}} P_i A_{ii}^q P_i^{-\frac{1}{2}}\right)\|e_i(k)\|_{P_i} + \sum_{j\in\mathbb{N}_i}\sum_{r=0}^{q-1}\bar{\lambda}^{\frac{1}{2}}\left(\eta_i{}^{\mathrm{T}} P_i \eta_i\right)\left\|e_j(k+r|\varphi^i(k))\right\|
\end{aligned} \tag{19}$$

It can be known that the constrain (7c) is satisfied at the triggering instant. Hence, if the condition (18) holds, we can have:

$$\begin{aligned}
&\|\bar{x}_i(k+q|k)\|_{P_i} - \left\|\breve{x}_i^*\left(k+q|t_k^i\right)\right\|_{P_i} \\
\leqslant&\bar{\lambda}^{\frac{1}{2}}\left(P_i^{-\frac{1}{2}}\left(A_{ii}^q\right)^{\mathrm{T}} P_i A_{ii}^q P_i^{-\frac{1}{2}}\right)\delta_i + n_1 \times \frac{\gamma\kappa\sigma_i}{n_1} \\
\leqslant&(1-\gamma)\kappa\sigma_i + \gamma\kappa\sigma_i < \kappa\sigma_i
\end{aligned} \tag{20}$$

When $q = \mathcal{P}-1$, using the constrain (7d) can obtain that

$$\|\bar{x}_i(k+\mathcal{P}-1|k)\|_{P_i} \leqslant \|\breve{x}_i^*(k+\mathcal{P}-1|t_k^i)\|_{P_i} + \kappa\sigma_i \leqslant \sigma_i \tag{21}$$

It implies that $\bar{x}_i\,(k+\mathcal{P}-1|k) \in \Omega_i\,(\sigma_i)$. □

Then, the feasibility of terminal states constraint is discussed as follows.

Lemma 3. *For i−th subsystem, if Assumptions 1–3 hold, terminal states will be in the invariant set under the following condition:*

$$1-\underline{\lambda}\left(P_i^{-\frac{1}{2}}\mathcal{H}_i^* P_i^{-\frac{1}{2}}\right) \leqslant (1-\kappa)^2 \tag{22}$$

where $\mathcal{H}_i^* = \mathcal{H}_i + K_i^{\mathrm{T}}\mathcal{C}_i K_i$

Proof. It is shown in Lemma 1 that the feedback control input can stabilize the control system when states are within the designed invariant region $\Omega_i(\sigma_i)$. The estimated terminal state is described as $\hat{x}_i(k+\mathcal{P}|k)$ shown in (10). The difference between feasible terminal states and estimated terminal states is expressed as:

$$\left\|\bar{x}_i(k+\mathcal{P}|k)\right\|_{P_i} - \left\|\hat{x}_i\,(k+\mathcal{P}|k)\right\|_{P_i} \leqslant \left\|A_{c_i}[\bar{x}_i(k+\mathcal{P}-1|k) - \hat{x}_i\,(k+\mathcal{P}-1|k-1)]\right\|_{P_i} \tag{23}$$

By utilizing Lemma 1 alongside Assumptions 1 to 3,

$$\begin{aligned}
&\left\|\hat{x}_i(k+\mathcal{P}|k)\right\|_{P_i}^2 - \left\|\bar{x}_i(k+\mathcal{P}-1|k-1)\right\|_{P_i}^2 \\
\leqslant & \bar{x}_i^{\mathrm{T}}(k+\mathcal{P}|k-1)(A_{c_i}^{\mathrm{T}} P A_{c_i})\bar{x}_i(k+\mathcal{P}|k-1) \\
\leqslant & -\left\|\bar{x}_i(k+\mathcal{P}-1|k-1)\right\|^2_{P_i^{-\frac{1}{2}}\mathcal{H}_i^* P_i^{-\frac{1}{2}}} \\
\leqslant & -\underline{\lambda}\left(P_i^{-\frac{1}{2}}\mathcal{H}_i^* P_i^{-\frac{1}{2}}\right)\left\|\bar{x}_i(k+\mathcal{P}-1|k-1)\right\|_{P_i}^2
\end{aligned} \tag{24}$$

Combining formulas (22) and (24) yields that

$$\begin{aligned}
&\left\|\hat{x}_i(k+\mathcal{P}|k)\right\|_{P_i} \\
\leqslant & \sqrt{1-\underline{\lambda}\left(P_i^{-\frac{1}{2}}\mathcal{H}_i^* P_i^{-\frac{1}{2}}\right)}\left\|\bar{x}_i(k+\mathcal{P}-1|k-1)\right\|_{P_i} \\
\leqslant & \sqrt{1-\underline{\lambda}\left(P_i^{-\frac{1}{2}}\mathcal{H}_i^* P_i^{-\frac{1}{2}}\right)} \times \sigma_i \leqslant (1-\kappa)\sigma_i
\end{aligned} \tag{25}$$

Therefore, $\left\|\bar{x}_i(k+\mathcal{P}|k)\right\|_{P_i} \leqslant \sigma_i$, i.e., the feasible terminal state is in the terminal set. It is also shown that $\tilde{x}_i(k+\mathcal{P}) \in \Omega_i(\sigma_i)$ according to the designed feasible control input. □

Remark 2. According to the above analysis, the selection of parameters γ and κ is an important factor affecting the feasibility. It can be seen in formula (18), the upper bound of triggering threshold δ_i is determined by γ and κ. If the parameter κ increases or γ decreases, the upper bound of triggering level δ_i will be larger inevitably. Then the larger the selected κ, the smaller $(1-\kappa)$ will be, which implies that the constraints on the optimal prediction state are more strict based on (7d). Moreover, the value range of κ can be obtained by (22).

With the feasibility of iteration and constraints now guaranteed, the following section will address the closed-loop stability. The analysis demonstrates that even under the unknown disturbances, and the state trajectories will converge to a bounded region. Additionally, this also illustrates that the closed-loop system will retain stability when there are no external disturbances as per equation (2).

Theorem 1. *Given that Assumptions 1-3 and the feasibility conditions (18) and (22) are hold for the subsystem i. The parameters of parameters κ and $\mathcal{P}$ are chosen to satisfy the condition given below.*

$$\frac{(\mathcal{P}-2)\bar{\lambda}\,(\mathcal{H}_i)}{\underline{\lambda}\,(P_i)}\kappa\sigma_i(2-\kappa) + 2\kappa\sigma_i \leqslant \frac{\bar{\lambda}\,(\mathcal{H}_i)}{\underline{\lambda}\,(P_i)}(\sigma_i - 2(1-\kappa)\delta_i), \tag{26}$$

then the stability of the closed-loop system without disturbances can be guaranteed.

Proof. According to the definition of the cost function in (6), the local optimal cost function can be described as:

$$J_i^*(\breve{x}_i(t_k^i), \breve{u}_i(t_k^i)) = \sum_{q=0}^{\mathcal{P}-1}(\|\breve{x}_i^*(k+q|t_k^i)\|_{\mathcal{H}_i}^2 + \|\breve{u}_i^*(k+q|t_k^i)\|_{\mathcal{C}_i}^2) + \|\breve{x}_i^*(k+\mathcal{P}|t_k^i)\|_{P_i}^2 \tag{27}$$

The Chosen Lyapunov function candidate $V(\breve{x}, \breve{u})$ is defined as $V(\breve{x}, \breve{u}) = \sum_{i=1}^{N} J_i^*(\breve{x}_i(t_k^i), \breve{u}_i(t_k^i))$. For such a Lyapunov function, its difference can be described as

$$\begin{aligned} &\Delta V(\breve{x}, \breve{u}) \\ &= \sum_{i=1}^{N} J_i^*(\breve{x}_i(t_{k+1}^i), \breve{u}_i(t_{k+1}^i)) - \sum_{i=1}^{N} J_i^*(\breve{x}_i(t_k^i), \breve{u}_i(t_k^i)) \\ &\leqslant \sum_{i=1}^{N} \bar{J}_i(\bar{x}_i(t_k^i+1), \bar{u}_i(t_k^i+1)) - \sum_{i=1}^{N} J_i^*(\breve{x}_i(t_k^i), \breve{u}_i(t_k^i)) \end{aligned} \tag{28}$$

where the feasible cost function at $t_k^i + 1$ can be defined as:

$$\bar{J}_i(\bar{x}_i(t_k^i+1), \bar{u}_i(t_k^i+1)) = \sum_{q=0}^{\mathcal{P}-1}(\|\bar{x}_i(k+q+1|t_k^i+1)\|_{\mathcal{H}_i}^2 + \|\bar{u}_i(k+q+1|t_k^i+1)\|_{\mathcal{C}_i}^2) + \|\bar{x}_i(k+\mathcal{P}+1|t_k^i+1)\|_{P_i}^2 \tag{29}$$

According to formulas (27) and (29) and considering the definition of feasible control input $\bar{u}_i(k+q|k)$, the difference of $V(\breve{x}, \breve{u})$ is as follows:

$$\Delta V(\breve{x}, \breve{u}) \leqslant \Delta J_1 + \Delta J_2 + \Delta J_3 + \Delta J_4 \tag{30}$$

where

$$\Delta J_1 \triangleq \sum_{i=1}^{N}\sum_{q=1}^{\mathcal{P}-2}[\|\bar{x}_i(k+q+1|t_k^i+1)\|_{\mathcal{H}_i}^2 - \|\breve{x}_i^*(k+q+1|t_k^i)\|_{\mathcal{H}_i}^2] \tag{31}$$

$$\Delta J_2 \triangleq \sum_{i=1}^{N}[\|\bar{x}_i(k+\mathcal{P}|t_k^i+1)\|_{P_i}^2 - \|\breve{x}_i^*(k+\mathcal{P}|t_k^i)\|_{P_i}^2] \tag{32}$$

$$\Delta J_3 \triangleq \sum_{i=1}^{N}[\|\bar{x}_i(k+1|t_k^i+1)\|_{\mathcal{H}_i}^2 - \|\breve{x}_i^*(k+1|t_k^i)\|_{\mathcal{H}_i}^2] \tag{33}$$

$$\Delta J_4 \triangleq \sum_{i=1}^{N} -[\|\breve{x}_i^*(k|t_k^i)\|_{\mathcal{H}_i}^2 + \|\breve{u}_i^*(k|t_k^i)\|_{\mathcal{C}_i}^2] \tag{34}$$

In what follows, we investigate the detailed properties ΔJ_1, ΔJ_2, ΔJ_3 and ΔJ_4 separately. We obtain

$$\Delta J_1 \leqslant \sum_{i=1}^{N}\sum_{q=1}^{\mathcal{P}-2}\frac{\bar{\lambda}(\mathcal{H}_i)}{\underline{\lambda}(P_i)}[\|\bar{x}_i(k+q+1|t_k^i+1) - \breve{x}_i^*(k+q+1|t_k^i)\|_{P_i}(\|\bar{x}_i(k+q+1|t_k^i+1)\|_{P_i} + \|\breve{x}_i^*(k+q+1|t_k^i)\|_{P_i})] \tag{35}$$

Using the condition $\|\bar{x}_i(k+q+1|t_k^i+1)\|_{P_i} \leqslant \|\breve{x}_i^*(k+q+1|t_k^i)\|_{P_i} + \kappa\sigma_i$. Then, we have

$$\begin{aligned} \Delta J_1 &\leqslant \sum_{i=1}^{N}\sum_{q=1}^{\mathcal{P}-2}\frac{\bar{\lambda}(\mathcal{H}_i)}{\underline{\lambda}(P_i)}\{\kappa\sigma_i[2(1-\kappa)\sigma_i + \kappa\sigma_i]\} \\ &\leqslant \sum_{i=1}^{N}\frac{(\mathcal{P}-2)\bar{\lambda}(\mathcal{H}_i)}{\underline{\lambda}(P_i)}[\kappa\sigma_i^2(2-\kappa)] \end{aligned} \tag{36}$$

Analogously, we can obtain that $\|\bar{x}_i(k+\mathcal{P}|t_k^i+1)-\check{x}_i^*(k+\mathcal{P}|t_k^i)\|_{P_i}\leqslant\kappa\sigma_i$ and $\|\bar{x}_i(k+\mathcal{P}|t_k^i+1)\|_{P_i}+\|\check{x}_i^*(k+\mathcal{P}|t_k^i)\|_{P_i}\leqslant 2\sigma_i$, then

$$\Delta J_2 \leqslant \sum_{i=1}^{N} 2\kappa\sigma_i^2 \tag{37}$$

Based on condition (13), we can obtain

$$\Delta J_3 \leqslant \sum_{i=1}^{N} \frac{\bar{\lambda}(Q_i)}{\underline{\lambda}(P_i)} 2(1-\kappa)\sigma_i\delta_i \tag{38}$$

Since $x_i(k)\notin\Omega_i(\varepsilon_i)$, then the upper bound of ΔJ_4 can be given as

$$\Delta J_4 \leqslant \sum_{i=1}^{N} -\frac{\bar{\lambda}(\mathcal{H}_i)}{\underline{\lambda}(P_i)}\|\check{x}_i^*(k|t_k^i)\|_{P_i}^2 \leqslant \sum_{i=1}^{N} -\frac{\bar{\lambda}(\mathcal{H}_i)}{\underline{\lambda}(P_i)}\sigma_i^2 \tag{39}$$

Using the condition in Theorem 1, we have $\Delta V(\check{x},\check{u})=\sum_{i=1}^{N}[J_i^*(\check{x}_i(t_{k+1}^i),\check{u}_i(t_{k+1}^i))-J_i^*(\check{x}_i(t_k^i),\check{u}_i(t_k^i))]\leqslant 0$. This implies that states can converge to the region $\Omega(\sigma)$ within finite time. If the additive disturbance $d_i(k)=0$, it follows that the resulting closed-loop system exhibits asymptotic stability with respect to the origin. □

Theorem 2. *Considering the subsystem i in (1), assuming that Assumptions 1-3 hold, the feasibility conditions (18), (22),as well as the stability condition (26), are satisfied. Moreover, if the disturbance d(k) is bounded such that* $\|d(k)\|\leqslant\bar{d}$, *then the state will reach a bounded region* $\Omega(\bar{\sigma})$, *where* $\bar{\sigma}=\sqrt{\frac{\bar{\lambda}(P)}{\underline{\lambda}(\mathcal{H}^*)}\beta(2\sigma+\beta)}$ *and* $\beta\triangleq\bar{\lambda}(\sqrt{P})\bar{d}$.

Proof. Using the dual-model strategy, the terminal control input $u_i(k)=K_ix_i(k)$ is implemented for the subsystem i as shown in (1). The Lyapunov function $V_i(x_i(k))=\|x_i(k)\|_{P_i}^2$ is chosen. For the global system, one can obtain that

$$\begin{aligned}\Delta V(x(k)) =& \sum_{i=1}^{N}(V_i(x_i(k+1))-V_i(x_i(k)))\\ =&\|A_cx(k)+d(k)\|_P^2-\|x(k)\|_P^2\\ =&x^{\mathrm{T}}(k)(A_c^{\mathrm{T}}PA_c-P)x(k)+2x^{\mathrm{T}}(k)A_c^{\mathrm{T}}Pd(k)+d^{\mathrm{T}}(k)Pd(k)\end{aligned} \tag{40}$$

According to Lemma 1, one can infer that

$$\begin{aligned}\Delta V(x(k)) \leqslant& -\frac{\underline{\lambda}(\mathcal{H}^*)}{\bar{\lambda}(P)}\|x(k)\|_P^2+\|d(k)\|_P^2+2\|(A_cx(k))^{\mathrm{T}}\sqrt{P}\|\|\sqrt{P}\|\|d(k)\|\\ \leqslant&\frac{\underline{\lambda}(\mathcal{H}^*)}{\bar{\lambda}(P)}(-V(x(k))+\bar{\sigma}^2)\end{aligned} \tag{41}$$

Then we conclude that the state can reach $\Omega(\bar{\sigma})$ in a finite time, which implies state trajectories can eventually converge to the bounded region under unknown bounded disturbances. $\Omega(\bar{\sigma})$ is the robust invariant for the subsystem in (1). Finally, the proof of Theorem 2 is now concluded. □

5 | SIMULATION EXAMPLE

This section demonstrates that the developed method can achieve distributed control of a collection of dynamic coupling subsystems under disturbances. To assess the feasibility of the suggested distributed strategy, a benchmark scenario involving a four-tank system is employed. The illustration of the four-tank process is shown in Fig. 1, and the simplified model can be described as:

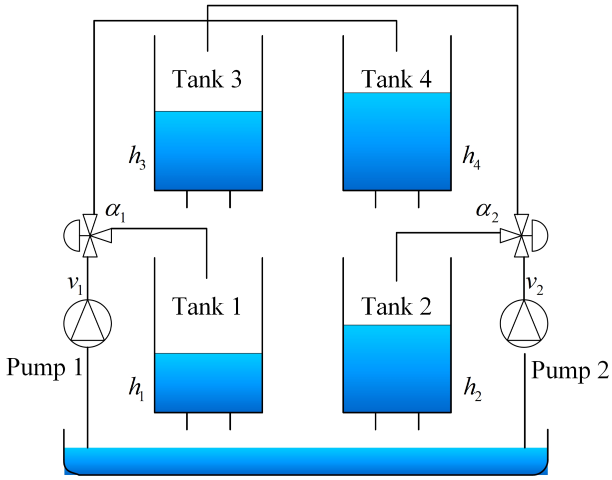

FIGURE 1 Illustration of the four-tank process

$$
\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_4}{A_4}\sqrt{2gh_4} + \frac{\alpha_1 k_1}{A_1}v_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{(1-\alpha_1)\,k_1}{A_2}v_1 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{a_2}{A_2}\sqrt{2gh_2} + \frac{\alpha_2 k_2}{A_3}v_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\alpha_2)\,k_2}{A_4}v_2
\end{aligned}
\tag{42}
$$

where the water tank level is represented by $h_i, i = 1, 2, 3, 4$. The corresponding cross-sectional area denoted by A_i, and the cross-sectional area of water tank outlet is represented by a_i. v_1 and v_2 are the voltages of two pumps, the parameter k_i is a conversion parameter. Moreover, α_1 and α_2 indicate the ration of two three-way valves, and g is the gravitational acceleration. The relevant parameters can be found in[28]: $A_1 = A_4 = 28$ cm^2, $A_2 = A_3 = 32$ cm^2, $a_1 = a_4 = 0.071$ cm^2, $a_2 = a_3 = 0.057$ cm^2, $k_1 = 3.35$ cm^3/Vs, $k_2 = 3.33$ cm^3/Vs, $\alpha_1 = 0.7$, and $\alpha_2 = 0.6$. Then the discrete-time linear model can be obtained:

$$
\begin{bmatrix} \delta h_1(k+1) \\ \delta h_2(k+1) \\ \delta h_3(k+1) \\ \delta h_4(k+1) \end{bmatrix} = \begin{bmatrix} 0.98 & 0 & 0 & 0.04 \\ 0 & 0.97 & 0 & 0 \\ 0 & 0.03 & 0.99 & 0 \\ 0 & 0 & 0 & 0.96 \end{bmatrix} \begin{bmatrix} \delta h_1(k) \\ \delta h_2(k) \\ \delta h_3(k) \\ \delta h_4(k) \end{bmatrix} + \begin{bmatrix} 0.08 & 0 \\ 0.03 & 0 \\ 0 & 0.06 \\ 0 & 0.05 \end{bmatrix} \begin{bmatrix} \delta v_1(k) \\ \delta v_2(k) \end{bmatrix}
\tag{43}
$$

The system shown in Equation (43) can be divided into two coupled subsystems. The parameter matrices are given as $\mathcal{H}_i = I$ and $\mathcal{C}_i = 1$. The feedback control gain K_i and the symmetric matrix P_i are gained referring to the work of[25].

$$
\begin{aligned}
K_1 &= [-1.0439 - 0.2726], \\
K_2 &= [-1.2131 - 0.3562], \\
P_1 &= \begin{bmatrix} 19.0046 & -12.2731 \\ -12.2731 & 44.1559 \end{bmatrix}, \\
P_2 &= \begin{bmatrix} 34.8631 & -15.2952 \\ -15.2952 & 26.9819 \end{bmatrix}.
\end{aligned}
$$

The parameters of DMPC strategy are given as follows: $\delta_i = 0.6$, $\mathcal{P} = 5$, $\kappa = 0.0035$, $\gamma = 0.8$. The disturbance vector d_i is designed as a variable where each element varies the range of $[-0.01, 0.01]$. The initial state of two subsystems are

$x_1(0) = [0.825, 0.203]^{\mathrm{T}}$ and $x_2(0) = [0.647, 0.250]^{\mathrm{T}}$. Moreover, it can be verified that the designed parameters satisfy the sufficient conditions (26).

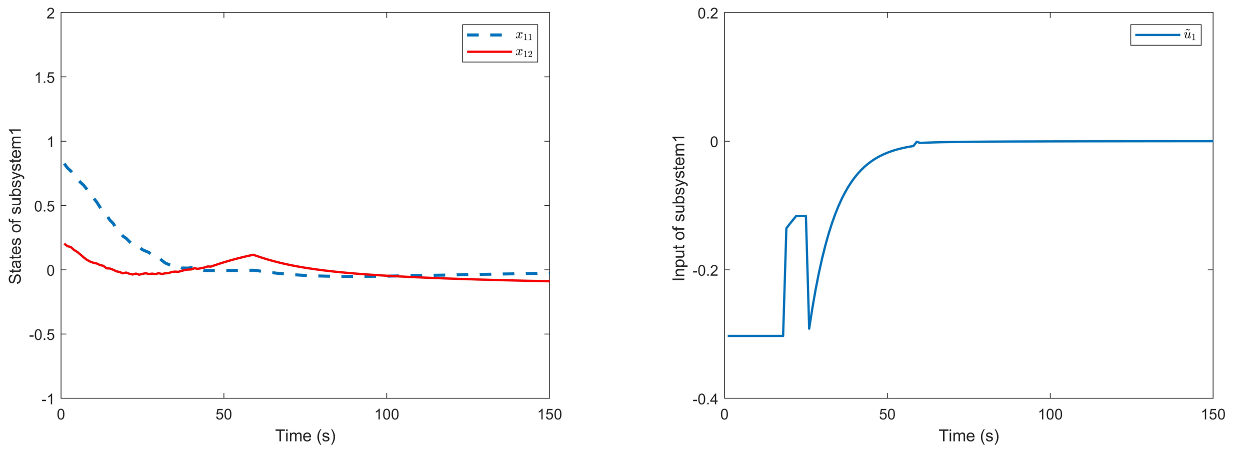

FIGURE 2 States and input of subsystem 1

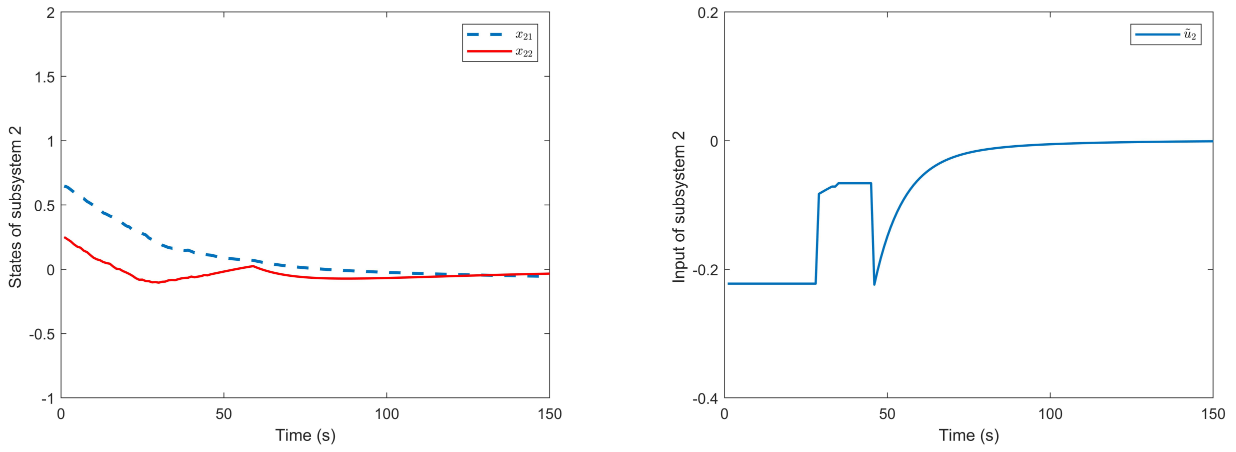

FIGURE 3 States and input of subsystem 2

The simulations are shown in the following Fig. 2 to Fig. 4. The states and inputs of two subsystems are displayed in Fig. 2 and Fig. 3. The state trajectories and control inputs of the four-tank system are convergent, and terminal states can enter a robust admissible invariant set under bounded random disturbances. This implies that the proposed distributed event-triggered predictive control strategy is feasible and robust. In addition, Fig. 4 denotes the triggering signals of the local model predictive controllers. Once the triggering conditions are met, a signal will be generated to initiate the optimization problem solving process. The local controller 1 triggers six times within the range of $k = 150$, where the triggering occurs in steps of $k = 1$, $k = 19$, $k = 20$, $k = 21$, $k = 22$ and $k = 59$, respectively. Then it can be also drawn that the local controller 2 triggers seven times during the iteration process. It shows that the designed event-triggered strategy greatly reduces the control time of the local model predictive controller, thereby reduces the consumption of communication resources. It is evident that the designed strategy can effectively ensure control performance while minimising communication and computational load.

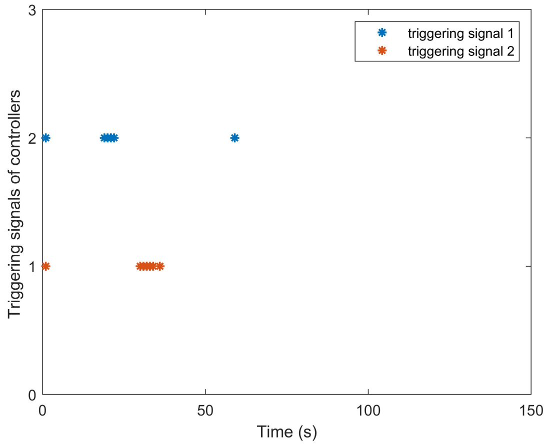

FIGURE 4 Triggering signals of controllers

6 | CONCLUSIONS

A novel event-triggered DMPC formulation for a collection of discrete-time linear systems subject to additive bounded disturbances is investigated in this paper. In the distributed control process, considerations include he neighbouring states and the asynchronous information transmission, employing the event-triggered mechanism and dual-mode control strategy to lessen the communication load and maintain the stability in the distributed system. Furthermore, special conditions are tailored specifically for ensuring the feasibility of optimization, stability of the entire system, and robustness of the proposed algorithm. Ultimately, the efficacy of the devised control strategy are validated through simulation results.

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CONFLICT OF INTEREST

The authors declare no conflict of interests.

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