Different wave structures for a new extended shallow water wave equation in (3+1) dimension

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Abstract

In this work, a new extended shallow water wave equation in (3+1) dimensions is studied, which represents abundant physical meaning in nonlinear shallow water wave. We discuss the interaction between lump wave and single solitary wave, which is an inelastic collision. Further, the interaction between lump wave and two solitary waves, and the interaction between lump wave and periodic wave are also studied. Finally, the interaction among lump, periodic and one solitary waves is investigated. The dynamic properties of the obtained results are shown and analyzed by some three-dimensional images.

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Abstract In this work, a new extended shallow water wave equation in (3+1) dimensions is studied, which represents abundant physical meaning in nonlinear shallow water wave. We discuss the interaction between lump wave and single solitary wave, which is an inelastic collision. Further, the interaction between lump wave and two solitary waves, and the interaction between lump wave and periodic wave are also studied. Finally, the interaction among lump, periodic and one solitary waves is investigated. The dynamic properties of the obtained results are shown and analyzed by some three-dimensional images.

Keywords shallow water wave, lump wave, periodic wave, dynamic properties.

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1 Introduction

Lump wave is a rational soliton with a large amplitude, which is localized only in space and will not disappear due to time change. In 2015, an effective algebraic method to obtain lump solutions of integrable systems was proposed by Ma [1]. Subsequently, He also provided theoretical support for this method and proved it [2], which made great progress in the solution of lump wave and

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attracted the attention of a large number of researchers, such as Tian [3-5], Lü [6-8], He [9-11], Wen [12-14], Su [15-17], Lan [18-20], Chen [21-23] et al.

The shallow water wave equation is a hot topic in recent years. The wave equation for shallow water is a model in which the depth of water is less than the wavelength of the free surface disturbance. Examples of shallow water wave equations are widely used in the field of oceanography and atmosphere to simulate the dynamic behavior of water wave propagation. For example, shallow water wave equation in (2+1) dimension:

$$\Phi_{yt} - 3\Phi_x\Phi_{xy} - 3\Phi_y\Phi_{xx} + \Phi_{xxxy} = 0, \tag{1}$$

and

$$\Phi_{yt} - 4\Phi_x\Phi_{xy} - 2\Phi_y\Phi_{xx} + \Phi_{xxxy} = 0, \tag{2}$$

shallow water wave equation in (3+1) dimension:

$$\Phi_{yt} - \Phi_{xz} - 3\Phi_x\,\Phi_{xy} - 3\,\Phi_y\,\Phi_{xx} + \Phi_{xxxy} = 0. \tag{3}$$

Recently, a new extended shallow water wave equation in (3+1) dimension was proposed by Wazwaz as follow [24]

$$\alpha\Phi_{xx} + \beta\Phi_{yy} + \gamma\Phi_{xy} + \delta\Phi_{yz} + \Phi_{yt} - 3\Phi_x\Phi_{xy} - 3\Phi_y\Phi_{xx} + \Phi_{xxxy} = 0, \tag{4}$$

where $\Phi = \Phi(x, y, z, t)$, α, β, γ and δ are arbitrary constants. Wazwaz obtained the multiple soliton and lump solutions. In addition, this equation has not been studied in other literatures.

Under the transformation

$$\Phi = -2\,(\ln\xi)_x, \tag{5}$$

Eq. (4) has the following Hirota bilinear form

$$\begin{aligned}&[D_yD_t + D_x^3D_y + \beta D_y^2 + \alpha D_x^2 + \gamma D_yD_x + \delta D_yD_z]\xi\cdot\xi = -\alpha\xi_x^2 - \beta\xi_y^2\\&+3\xi_{xy}\xi_{xx} - \xi_y\xi_{xxx} + \xi\,(\alpha\xi_{xx} + \gamma\xi_{xy} + \delta\xi_{yz} + \beta\xi_{yy} + \xi_{xxxy} + \xi_{yt})\\&-\xi_t\xi_y - \gamma\xi_x\xi_y - \delta\xi_z\xi_y - 3\xi_x\xi_{xxy} = 0,\end{aligned} \tag{6}$$

where $\xi = \xi(x, y, z, t)$.

The organization of this paper is as follows: Section 2 investigates the interaction between lump wave and one soliton. Section 3 obtained the interaction solutions between lump and two solitary waves. Section 4 studies the interaction between lump and periodic wave; Section 5 discusses the interaction among lump, periodic and one solitary waves. Section 6 makes this conclusions.

2 Lump-1-soliton solution

The lump solutions of Eq. (1) have been obtained by Wazwaz. On this basis, we will further investigate the interaction between lump wave and one soliton. for this reason, we assume

$$\begin{aligned}\xi &= (\mathcal{G}_4 t + \mathcal{G}_1 x + \mathcal{G}_2 y + \mathcal{G}_3 z)^2 + (\mathcal{G}_8 t + \mathcal{G}_5 x + \mathcal{G}_6 y + \mathcal{G}_7 z)^2 \\ &+ \mathcal{G}_9 + k_1 e^{\mathcal{G}_{13} t + \mathcal{G}_{10} x + \mathcal{G}_{11} y + \mathcal{G}_{12} z},\end{aligned} \tag{7}$$

where $\mathcal{G}_i (i = 1, 2, \cdots, 12)$ and k_1 are undetermined constants. Substituting Eq. (7) into Eq. (6), we obtain

$$\begin{aligned}(I)\quad &\mathcal{G}_3 = \frac{\alpha\left(-\mathcal{G}_2\mathcal{G}_1^2 - 2\mathcal{G}_5\mathcal{G}_6\mathcal{G}_1 + \mathcal{G}_2\mathcal{G}_5^2\right)}{\delta\left(\mathcal{G}_2^2 + \mathcal{G}_6^2\right)} - \frac{\beta\mathcal{G}_2 + \gamma\mathcal{G}_1 + \mathcal{G}_4}{\delta}, \\ &\mathcal{G}_8 = -[\mathcal{G}_6\left(\alpha\mathcal{G}_5^2 + \beta\mathcal{G}_6^2 + \gamma\mathcal{G}_6\mathcal{G}_5 + \delta\mathcal{G}_6\mathcal{G}_7\right) - \alpha\mathcal{G}_6\mathcal{G}_1^2 + 2\alpha\mathcal{G}_2\mathcal{G}_5\mathcal{G}_1 \\ &+\mathcal{G}_2^2\left(\beta\mathcal{G}_6 + \gamma\mathcal{G}_5 + \delta\mathcal{G}_7\right)]/(\mathcal{G}_2^2 + \mathcal{G}_6^2), \\ &\mathcal{G}_9 = -\frac{3\left(\mathcal{G}_1^2 + \mathcal{G}_5^2\right)\left(\mathcal{G}_1\mathcal{G}_2 + \mathcal{G}_5\mathcal{G}_6\right)\left(\mathcal{G}_2^2 + \mathcal{G}_6^2\right)}{\alpha\left(\mathcal{G}_2\mathcal{G}_5 - \mathcal{G}_1\mathcal{G}_6\right)^2}, \\ &\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}, \mathcal{G}_{11} = \epsilon_1\frac{\sqrt{\mathcal{G}_2^2 + \mathcal{G}_6^2}\mathcal{G}_{10}}{\sqrt{\mathcal{G}_1^2 + \mathcal{G}_5^2}}, \\ &\mathcal{G}_{10} = \epsilon_2\frac{\sqrt{\frac{2}{3}}\sqrt{\alpha\mathcal{G}_1\mathcal{G}_2 + \alpha\mathcal{G}_5\mathcal{G}_6 + \epsilon_1\alpha\sqrt{\mathcal{G}_1^2 + \mathcal{G}_5^2}\sqrt{\mathcal{G}_2^2 + \mathcal{G}_6^2}}}{\sqrt{\mathcal{G}_2^2 + \mathcal{G}_6^2}}.\end{aligned} \tag{8}$$

$$\begin{aligned}(II)\quad &\mathcal{G}_3 = \frac{\mathcal{G}_1\mathcal{G}_7}{\mathcal{G}_5}, \mathcal{G}_6 = -\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5}, \mathcal{G}_7 = \frac{\mathcal{G}_5\left(\alpha\mathcal{G}_5^2 - \beta\mathcal{G}_2^2 - \gamma\mathcal{G}_1\mathcal{G}_2 - \mathcal{G}_4\mathcal{G}_2\right)}{\delta\mathcal{G}_1\mathcal{G}_2}, \\ &\mathcal{G}_8 = \frac{\mathcal{G}_1^2\left(\beta\mathcal{G}_2^2 - \alpha\mathcal{G}_5^2\right) - \alpha\mathcal{G}_5^4 + \mathcal{G}_2\mathcal{G}_5^2\left(\beta\mathcal{G}_2 + \mathcal{G}_4\right)}{\mathcal{G}_1\mathcal{G}_2\mathcal{G}_5}, \mathcal{G}_9 = 0, \mathcal{G}_5 = \mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha}, \\ &\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}, \mathcal{G}_{11} = \pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}.\end{aligned} \tag{9}$$

$$\begin{aligned}(III)\quad &\mathcal{G}_3 = -\frac{\mathcal{G}_5\mathcal{G}_7}{\mathcal{G}_1}, \mathcal{G}_6 = -\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5}, \mathcal{G}_7 = \frac{\mathcal{G}_1\left(\mathcal{G}_2\left(\beta\mathcal{G}_2 + \gamma\mathcal{G}_1 + \mathcal{G}_4\right) - \alpha\mathcal{G}_5^2\right)}{\delta\mathcal{G}_2\mathcal{G}_5}, \\ &\mathcal{G}_8 = -\frac{\mathcal{G}_1\left(\gamma\mathcal{G}_1 + \mathcal{G}_4\right)}{\mathcal{G}_5} - \gamma\mathcal{G}_5, \mathcal{G}_9 = 0, \mathcal{G}_5 = \mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha}, \\ &\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}, \mathcal{G}_{11} = \pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}.\end{aligned} \tag{10}$$

By substituting the results of formula (8)-(10) into the transformation (6), we can get the following corresponding interaction solution

$$\Phi = -2\left(\ln\xi\right)_x. \tag{11}$$

In order to understand the dynamic properties of the interaction solutions between lump wave and one soliton, we take Eq. (8) as an example and select special values of parameters (see Fig. 1) to obtain a special solution of the equation as follow

$$\Phi = -576[73e^{\frac{19t}{6}+x}(7t-16x+9z)+192e^{\frac{2y}{3}+3z}]/[73e^{\frac{19t}{6}+x}[2041t^2+t(2560y -2016x+1134z)+9(16x-9z)^2+1024y^2]+55296e^{\frac{2y}{3}+3z}]. \tag{12}$$

The dynamic properties of Eq. (12) are shown in Fig. 1. In Fig. 1(a), we can observe that a soliton and a lump wave propagate forward respectively. In Fig. 1(b), the soliton and the lump wave slowly close together, and the amplitude of the lump wave begins to decrease. Until Fig. 1(c), the soliton and the lump wave gradually converge, and the amplitude of the lump wave becomes smaller. It can be seen that the interaction between soliton and lump wave is an inelastic collision, and the energy is consumed.

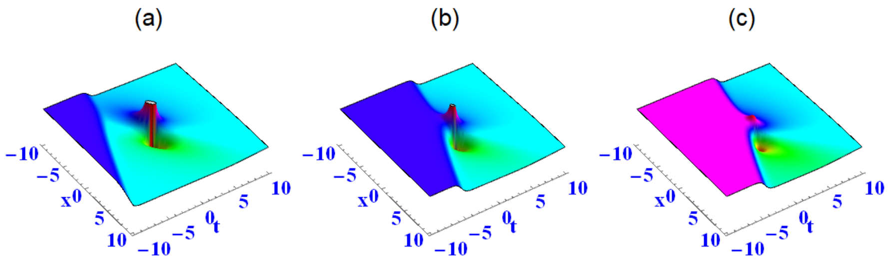

Fig. 1. $\mathcal{G}_2=\alpha=\beta=\delta=\gamma=1$, $\mathcal{G}_1=-4$, $k_1=6$, $\mathcal{G}_4=\mathcal{G}_{12}=3$, $y=0$, $\mathcal{G}_{10}=-1$, (a) $z=-5$, (b) $z=0$, (c) $z=1$.

3 Lump-2-soliton solution

Next, we want to further consider the interaction between lump wave and double solitons. Therefore, we assume

$$\begin{aligned}\xi &= (\mathcal{G}_4t+\mathcal{G}_1x+\mathcal{G}_2y+\mathcal{G}_3z)^2+(\mathcal{G}_8t+\mathcal{G}_5x+\mathcal{G}_6y+\mathcal{G}_7z)^2\\ &+\mathcal{G}_9+k_1\cosh(\mathcal{G}_{13}t+\mathcal{G}_{10}x+\mathcal{G}_{11}y+\mathcal{G}_{12}z).\end{aligned} \tag{13}$$

Substituting Eq. (14) into Eq. (6), we derive

$$\begin{aligned}(I)\quad \mathcal{G}_3 &= \frac{\alpha\left(-\mathcal{G}_2\mathcal{G}_1^2-2\mathcal{G}_5\mathcal{G}_6\mathcal{G}_1+\mathcal{G}_2\mathcal{G}_5^2\right)}{\delta\left(\mathcal{G}_2^2+\mathcal{G}_6^2\right)}-\frac{\beta\mathcal{G}_2+\gamma\mathcal{G}_1+\mathcal{G}_4}{\delta},\\ \mathcal{G}_8 &= -[\mathcal{G}_6\left(\alpha\mathcal{G}_5^2+\beta\mathcal{G}_6^2+\gamma\mathcal{G}_6\mathcal{G}_5+\delta\mathcal{G}_6\mathcal{G}_7\right)-\alpha\mathcal{G}_6\mathcal{G}_1^2+2\alpha\mathcal{G}_2\mathcal{G}_5\mathcal{G}_1\\ &+\mathcal{G}_2^2\left(\beta\mathcal{G}_6+\gamma\mathcal{G}_5+\delta\mathcal{G}_7\right)]/(\mathcal{G}_2^2+\mathcal{G}_6^2),\\ \mathcal{G}_9 &= -\frac{3\left(\mathcal{G}_2^2+\mathcal{G}_6^2\right)\left(4\mathcal{G}_2\mathcal{G}_1^3+4\mathcal{G}_5\mathcal{G}_6\mathcal{G}_1^2+4\mathcal{G}_2\mathcal{G}_5^2\mathcal{G}_1+4\mathcal{G}_5^3\mathcal{G}_6+\mathcal{G}_{10}^3\mathcal{G}_{11}k_1^2\right)}{4\alpha\left(\mathcal{G}_2\mathcal{G}_5-\mathcal{G}_1\mathcal{G}_6\right)^2},\end{aligned}$$

$$\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12},\mathcal{G}_{11}=\epsilon_1\frac{\sqrt{\mathcal{G}_2^2+\mathcal{G}_6^2}\mathcal{G}_{10}}{\sqrt{\mathcal{G}_1^2+\mathcal{G}_5^2}},$$

$$\mathcal{G}_{10}=\epsilon_2\frac{\sqrt{\frac{2}{3}}\sqrt{\alpha\mathcal{G}_1\mathcal{G}_2+\alpha\mathcal{G}_5\mathcal{G}_6+\epsilon_1\alpha\sqrt{\mathcal{G}_1^2+\mathcal{G}_5^2}\sqrt{\mathcal{G}_2^2+\mathcal{G}_6^2}}}{\sqrt{\mathcal{G}_2^2+\mathcal{G}_6^2}}. \tag{14}$$

$$\begin{aligned}(II)\quad &\mathcal{G}_3=\frac{\mathcal{G}_1\mathcal{G}_7}{\mathcal{G}_5},\mathcal{G}_6=-\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5},\mathcal{G}_7=\frac{\alpha\mathcal{G}_5^3-\mathcal{G}_2\mathcal{G}_5\left(\beta\mathcal{G}_2+\gamma\mathcal{G}_1+\mathcal{G}_4\right)}{\delta\mathcal{G}_1\mathcal{G}_2},\\ &\mathcal{G}_{11}=\pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5},\mathcal{G}_8=\frac{\mathcal{G}_1^2\left(\beta\mathcal{G}_2^2-\alpha\mathcal{G}_5^2\right)-\alpha\mathcal{G}_5^4+\mathcal{G}_2\mathcal{G}_5^2\left(\beta\mathcal{G}_2+\mathcal{G}_4\right)}{\mathcal{G}_1\mathcal{G}_2\mathcal{G}_5},\\ &\mathcal{G}_9=\frac{2\alpha^2\mathcal{G}_{10}^2k_1^2}{4\alpha^2\mathcal{G}_1^2+9\mathcal{G}_2^2\mathcal{G}_{10}^4},\mathcal{G}_5=\mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},\\ &\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12}.\end{aligned} \tag{15}$$

$$\begin{aligned}(III)\quad &\mathcal{G}_3=-\frac{\mathcal{G}_5\mathcal{G}_7}{\mathcal{G}_1},\mathcal{G}_6=-\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5},\mathcal{G}_7=\frac{\mathcal{G}_1\left(\mathcal{G}_2\left(\beta\mathcal{G}_2+\gamma\mathcal{G}_1+\mathcal{G}_4\right)-\alpha\mathcal{G}_5^2\right)}{\delta\mathcal{G}_2\mathcal{G}_5},\\ &\mathcal{G}_8=-\frac{\mathcal{G}_1\left(\gamma\mathcal{G}_1+\mathcal{G}_4\right)}{\mathcal{G}_5}-\gamma\mathcal{G}_5,\mathcal{G}_9=\frac{2\alpha^2\mathcal{G}_{10}^2k_1^2}{4\alpha^2\mathcal{G}_1^2+9\mathcal{G}_2^2\mathcal{G}_{10}^4},\mathcal{G}_5=\mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},\\ &\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12},\mathcal{G}_{11}=\pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}.\end{aligned} \tag{16}$$

By substituting the results of formula (14)-(16) into the transformation (6), we can get the following corresponding interaction solution

$$\Phi=-2\left(\ln\xi\right)_x. \tag{17}$$

In order to understand the dynamic properties of the interaction solutions between lump wave and two solitons, we take Eq. (15) as an example and select special values of parameters (see Fig. 2) to obtain a special solution of the equation as follow

$$\Phi=[96\sinh\left(\frac{19t}{6}+x-\frac{2y}{3}-3z\right)-25(7t+8x-15z)]/[8[\frac{72}{25}+[3t+2x+y-\frac{15z}{4}]^2+\frac{(17t-72x+64y+135z)^2}{2304}-6\cosh\left(\frac{19t}{6}+x-\frac{2y}{3}-3z\right)]]. \tag{18}$$

The dynamic properties of Eq. (18) are described in Fig. 2. In Fig. 2(a), we can see that the lump wave interacts with one of the solitons. In Fig. 2(b), the lump wave begins to move to the middle and interact with another soliton. At this time, the amplitude of the lump wave becomes larger. Until Fig. 2(c), the lump wave shifts to another soliton and its amplitude decreases.

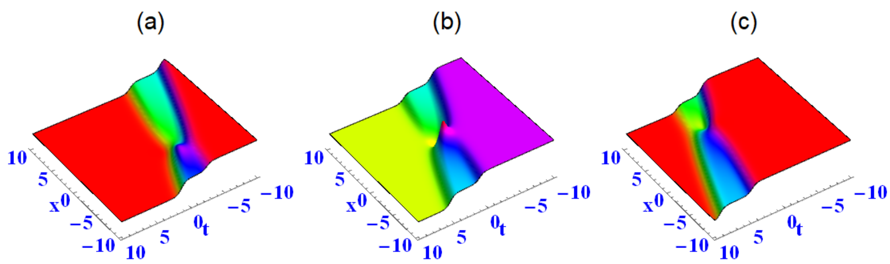

Fig. 2. $\mathcal{G}_2=\alpha=\beta=\delta=\gamma=1$, $\mathcal{G}_1=2$, $k_1=-6$, $\mathcal{G}_4=\mathcal{G}_{12}=3$, $y=0$, $\mathcal{G}_{10}=-1$, (a) $z=-4$, (b) $z=0$, (c) $z=4$.

4 Lump-periodic solution

In this section, we intend to investigate the interaction between lump and periodic wave, we suppose

$$\begin{aligned}\xi=&(\mathcal{G}_4t+\mathcal{G}_1x+\mathcal{G}_2y+\mathcal{G}_3z)^{2}+(\mathcal{G}_8t+\mathcal{G}_5x+\mathcal{G}_6y+\mathcal{G}_7z)^{2}\\&+\mathcal{G}_9+k_1\cos(\mathcal{G}_{13}t+\mathcal{G}_{10}x+\mathcal{G}_{11}y+\mathcal{G}_{12}z).\end{aligned}\tag{19}$$

Substituting Eq. (19) into Eq. (6), we give

$$\begin{aligned}(I)\quad&\mathcal{G}_3=\frac{\mathcal{G}_1\mathcal{G}_7}{\mathcal{G}_5},\mathcal{G}_6=-\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5},\mathcal{G}_7=\frac{\alpha\mathcal{G}_5^3-\mathcal{G}_2\mathcal{G}_5\left(\beta\mathcal{G}_2+\gamma\mathcal{G}_1+\mathcal{G}_4\right)}{\delta\mathcal{G}_1\mathcal{G}_2},\\&\mathcal{G}_{11}=\pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5},\mathcal{G}_8=\frac{\mathcal{G}_1^2\left(\beta\mathcal{G}_2^2-\alpha\mathcal{G}_5^2\right)-\alpha\mathcal{G}_5^4+\mathcal{G}_2\mathcal{G}_5^2\left(\beta\mathcal{G}_2+\mathcal{G}_4\right)}{\mathcal{G}_1\mathcal{G}_2\mathcal{G}_5},\\&\mathcal{G}_9=-\frac{2\alpha^2\mathcal{G}_{10}^2k_1^2}{4\alpha^2\mathcal{G}_1^2+9\mathcal{G}_2^2\mathcal{G}_{10}^4},\mathcal{G}_5=\pm\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},\\&\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12}.\end{aligned}\tag{20}$$

$$\begin{aligned}(II)\quad&\mathcal{G}_3=-\frac{\mathcal{G}_5\mathcal{G}_7}{\mathcal{G}_1},\mathcal{G}_6=-\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5},\mathcal{G}_7=\frac{\mathcal{G}_1\left(\mathcal{G}_2\left(\beta\mathcal{G}_2+\gamma\mathcal{G}_1+\mathcal{G}_4\right)-\alpha\mathcal{G}_5^2\right)}{\delta\mathcal{G}_2\mathcal{G}_5},\\&\mathcal{G}_8=-\frac{\mathcal{G}_1\left(\gamma\mathcal{G}_1+\mathcal{G}_4\right)}{\mathcal{G}_5}-\gamma\mathcal{G}_5,\mathcal{G}_9=-\frac{2\alpha^2\mathcal{G}_{10}^2k_1^2}{4\alpha^2\mathcal{G}_1^2+9\mathcal{G}_2^2\mathcal{G}_{10}^4},\mathcal{G}_5=\mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},\\&\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12},\mathcal{G}_{11}=\pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}.\end{aligned}\tag{21}$$

By substituting the results of formula (20)-(21) into the transformation (6), we can get the following corresponding interaction solution

$$\Phi=-2\,(\ln\xi)_x.\tag{22}$$

In order to understand the dynamic properties of the interaction solutions between lump and periodic wave, we take Eq. (20) as an example and select special values of parameters (see Fig. 3) to obtain a special solution of the equation as follow

$$\begin{aligned}\Phi=&[16\sin\left(\frac{31t}{6}-x-\frac{2y}{3}-3z\right)-25(7t+8x-15z)]/[8[(3t+2x+y-\\&\frac{15z}{4})^2+\frac{(17t-72x+64y+135z)^2}{2304}-\cos\left(\frac{31t}{6}-x-\frac{2y}{3}-3z\right)-\frac{2}{25}]]\end{aligned}\tag{23}$$

The dynamic properties of Eq. (23) are described in Fig. 3. Different from the previous two sections, the lump wave and periodic wave have been entangled and propagated forward.

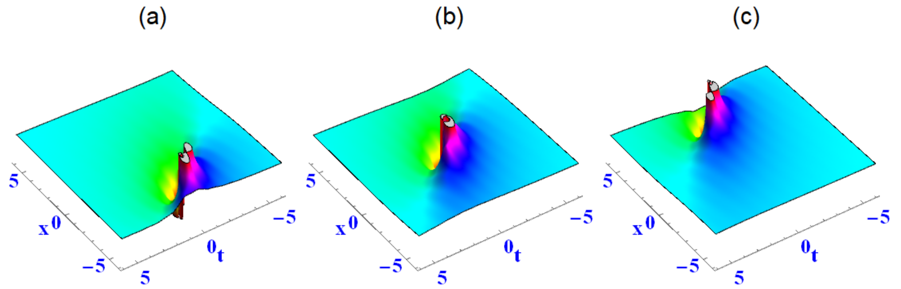

Fig. 3. $\mathcal{G}_2=\alpha=\beta=\delta=\gamma=\mathcal{G}_{10}=1$, $\mathcal{G}_1=2$, $k_1=-1$, $\mathcal{G}_4=\mathcal{G}_{12}=3$, $y=0$, (a) $z=-2$, (b) $z=0$, (c) $z=2$.

5 Lump-periodic-1-soliton solutions

In order to investigate the interaction among lump, periodic and one solitary wave, we suppose

$$\begin{aligned}\xi=&\left(\mathcal{G}_4t+\mathcal{G}_1x+\mathcal{G}_2y+\mathcal{G}_3z\right)^2+\left(\mathcal{G}_8t+\mathcal{G}_5x+\mathcal{G}_6y+\mathcal{G}_7z\right)^2+\mathcal{G}_9\\&+k_2e^{\mathcal{G}_{17}t+\mathcal{G}_{14}x+\mathcal{G}_{15}y+\mathcal{G}_{16}z}+k_1\cos\left(\mathcal{G}_{13}t+\mathcal{G}_{10}x+\mathcal{G}_{11}y+\mathcal{G}_{12}z\right),\end{aligned}\tag{24}$$

where $\mathcal{G}_i(i=14,15,16,17)$ and k_2 are undetermined constants. Substituting Eq. (24) into Eq. (6), we obtain

$$\begin{aligned}(I)\quad&\mathcal{G}_3=\frac{\mathcal{G}_1\mathcal{G}_7}{\mathcal{G}_5},\mathcal{G}_6=-\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5},\mathcal{G}_7=\frac{\alpha\mathcal{G}_5^3-\mathcal{G}_2\mathcal{G}_5\left(\beta\mathcal{G}_2+\gamma\mathcal{G}_1+\mathcal{G}_4\right)}{\delta\mathcal{G}_1\mathcal{G}_2},\\&\mathcal{G}_{11}=\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5},\mathcal{G}_8=\frac{\mathcal{G}_1^2\left(\beta\mathcal{G}_2^2-\alpha\mathcal{G}_5^2\right)-\alpha\mathcal{G}_5^4+\mathcal{G}_2\mathcal{G}_5^2\left(\beta\mathcal{G}_2+\mathcal{G}_4\right)}{\mathcal{G}_1\mathcal{G}_2\mathcal{G}_5},\\&\mathcal{G}_9=-\frac{2\alpha^2\mathcal{G}_{10}^2k_1^2}{4\alpha^2\mathcal{G}_1^2+9\mathcal{G}_2^2\mathcal{G}_{10}^4},\mathcal{G}_5=\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},\\&\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12},\mathcal{G}_{15}=-\frac{2\alpha\mathcal{G}_{14}}{3\mathcal{G}_{10}^2},\\&\mathcal{G}_{17}=-\frac{\alpha\mathcal{G}_{14}^2}{\mathcal{G}_{15}}-\beta\mathcal{G}_{15}-\mathcal{G}_{14}\left(\gamma+\mathcal{G}_{14}^2\right)-\delta\mathcal{G}_{16},\mathcal{G}_{14}=\pm\mathcal{G}_{10}.\end{aligned}\tag{25}$$

$$\begin{aligned}(II)\quad&\mathcal{G}_3=\frac{\mathcal{G}_1\mathcal{G}_7}{\mathcal{G}_5},\mathcal{G}_6=-\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5},\mathcal{G}_7=\frac{\alpha\mathcal{G}_5^3-\mathcal{G}_2\mathcal{G}_5\left(\beta\mathcal{G}_2+\gamma\mathcal{G}_1+\mathcal{G}_4\right)}{\delta\mathcal{G}_1\mathcal{G}_2},\\&\mathcal{G}_{11}=-\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5},\mathcal{G}_8=\frac{\mathcal{G}_1^2\left(\beta\mathcal{G}_2^2-\alpha\mathcal{G}_5^2\right)-\alpha\mathcal{G}_5^4+\mathcal{G}_2\mathcal{G}_5^2\left(\beta\mathcal{G}_2+\mathcal{G}_4\right)}{\mathcal{G}_1\mathcal{G}_2\mathcal{G}_5},\\&\mathcal{G}_9=-\frac{2\alpha^2\mathcal{G}_{10}^2k_1^2}{4\alpha^2\mathcal{G}_1^2+9\mathcal{G}_2^2\mathcal{G}_{10}^4},\mathcal{G}_5=-\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},\end{aligned}$$

$$\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}, \mathcal{G}_{15} = -\frac{2\alpha\mathcal{G}_{14}}{3\mathcal{G}_{10}^2},$$
$$\mathcal{G}_{17} = -\frac{\alpha\mathcal{G}_{14}^2}{\mathcal{G}_{15}} - \beta\mathcal{G}_{15} - \mathcal{G}_{14}\left(\gamma + \mathcal{G}_{14}^2\right) - \delta\mathcal{G}_{16}, \mathcal{G}_{14} = \pm\mathcal{G}_{10}. \quad (26)$$

By substituting the results of formula (25)-(26) into the transformation (6), we can get the following corresponding interaction solution

$$\varPhi = -2\,(\ln\xi)_x. \quad (27)$$

In order to understand the dynamic properties of the interaction solutions among lump, periodic and one solitary wave, we take Eq. (25) as an example and select special values of parameters (see Fig. 4) to obtain a special solution of the equation as follow

$$\varPhi = -[2[3\left(\frac{17t}{48} + \frac{3x}{2} + \frac{4y}{3} - \frac{3z}{16}\right) + 4e^{\frac{13t}{6}+x-\frac{2y}{3}-2z} - 4\left(3t - 2x + y + \frac{z}{4}\right)$$
$$-\sin\left(\frac{31t}{6} - x - \frac{2y}{3} - 3z\right)]]/[\left(\frac{17t}{48} + \frac{3x}{2} + \frac{4y}{3} - \frac{3z}{16}\right)^2 + 4e^{\frac{13t}{6}+x-\frac{2y}{3}-2z}$$
$$+\left(3t - 2x + y + \frac{z}{4}\right)^2 - \cos\left(\frac{31t}{6} - x - \frac{2y}{3} - 3z\right) - \frac{2}{25}]. \quad (28)$$

The dynamic properties of Eq. (28) are described in Fig. 4.

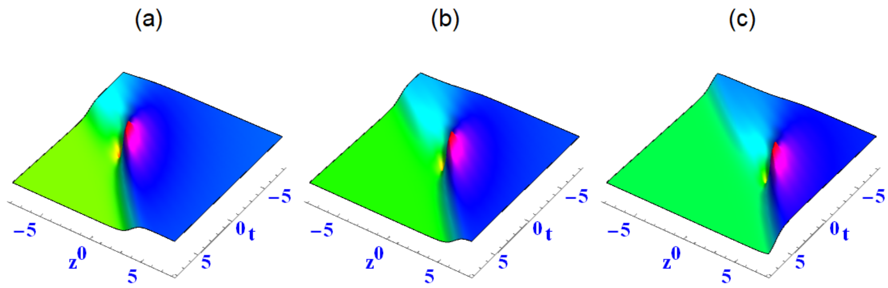

Fig. 4. $\mathcal{G}_2 = \alpha = \beta = \delta = \gamma = \mathcal{G}_{10} = 1$, $\mathcal{G}_1 = \mathcal{G}_{16} = -2$, $k_1 = -1$, $y = 0$, $\mathcal{G}_4 = \mathcal{G}_{12} = 3$, $k_2 = 4$, (a) $x = -5$, (b) $x = 0$, (c) $x = 5$.

6 Conclusion

In this article, we investigate a new extended shallow water wave equation in (3+1) dimensions based on the Hirota bilinear form and symbolic computation [25-29]. The interaction between lump wave and single solitary wave is studied. The interaction between lump wave and two solitary waves, and the interaction between lump wave and periodic wave are also discussed. Finally, we obtain the interaction solutions among lump, periodic and one solitary waves and discribe the dynamic properties of the obtained results in Figs. 1-4.

Data Availability Statements

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Compliance with ethical standards

Conflict of interests The authors declare that there is no conflict of interests regarding the publication of this article.

Ethical standard The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

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