Digital Backpropagation Based on Binary Logarithmic Step Size Distribution for Fiber Nonlinearity Compensation

Albert Dede[1], Emmanuel Akowuah[1], and Shyqyri Haxha[2]

[1]Kwame Nkrumah University of Science and Technology
[2]Royal Holloway University of London

May 24, 2023

Abstract

Capacity crunch has become critical in recent years as commercial communication systems approach their theoretical data rate limits. This work presents a low-complexity digital backpropagation (DBP) implementation approach based on step size distribution that uses a binary logarithmic step size method to achieve high data rate optical transmission. The proposed scheme shows performance improvements ([?]Q) of 2.36 dB, 1.19 dB, and 0.71 dB over linear compensation, constant step size DBP, and logarithmic step size DBP techniques in a 2400 km 112 Gbit/s DP-16QAM system, respectively. At 13 dBm, a high performance (Q) of 10.9 dB (BER = 2.25×10-4) is achieved, above the 3.80×10-3 hard-decision forward error correction (HD-FEC) limit, using the proposed scheme. Also, the allowable transmission distance is extended by 960 km at the HD-FEC limit over the linear compensation technique. The optimization achieves a 38% savings in the number of DBP calculation steps compared to the constant step size DBP, which considerably reduces the computational cost since a few steps are required for effective nonlinearity compensation.

Digital Backpropagation Based on Binary Logarithmic Step Size Distribution for Fiber Nonlinearity Compensation

Albert Dede[1], Emmanuel Kofi Akowuah[1], Shyqyri Haxha[2]

[1]Department of Computer Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.

[2]Department of Electronic Engineering, School of Engineering, Physical and Mathematical Sciences, Royal Holloway, University of London, Egham, Surrey, TW20 0EX, United Kingdom

CorrespondenceAlbert Dede, Department of Computer Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.

Email:*adede4@st.knust.edu.gh*

Abstract

Capacity crunch has become critical in recent years as commercial communication systems approach their theoretical data rate limits. This work presents a low-complexity digital backpropagation (DBP) implementation approach based on step size distribution that uses a binary logarithmic step size method to achieve high data rate optical transmission. The proposed scheme shows performance improvements ([?]Q) of 2.36 dB, 1.19 dB, and 0.71 dB over linear compensation, constant step size DBP, and logarithmic step size DBP techniques in a 2400 km 112 Gbit/s DP-16QAM system, respectively. At 13 dBm, a high performance (Q) of 10.9 dB (BER = 2.25×10^{-4}) is achieved, above the 3.80×10^{-3} hard-decision forward error correction (HD-FEC) limit, using the proposed scheme. Also, the allowable transmission distance is extended by 960 km at

the HD-FEC limit over the linear compensation technique. The optimization achieves a 38% savings in the number of DBP calculation steps compared to the constant step size DBP, which considerably reduces the computational cost since a few steps are required for effective nonlinearity compensation.

Keywords

Fiber nonlinearity, digital backpropagation, split-step Fourier method, long-haul transmission, optical fiber communication, fiber impairment compensation

Introduction

The demand for data traffic has surged in the past decade, owing to the development of bandwidth-consuming applications such as big data, video over IP applications, Internet Protocol Television (IPTV), and cloud computing, among others. The proliferation of 'smart devices' supported by the Internet of Things (IoT) technology also contributes substantially to the increasing demand for data traffic. The International Telecommunication Union (ITU) has predicted data traffic to exceed 55 zettabytes (ZB) per month by 2030 (1). This expected growth has driven research efforts in recent years toward protracting the data rate of next-generation transmissions to 100 Gbit/s and beyond. While there has been significant research into extending the total capacity of wavelength division multiplexing (WDM) in the region of a few terabits per second (2), a more trending area of research is to design optical transceivers capable of employing high-order modulation formats and symbol rates to achieve high data rate per channel. The 6th generation (6G) technology has recently been investigated as a key enabler for ultra-high-speed and reliable communication in the range of 100 Gbit/s – 1 Tbit/s (3,4). Ultra-high data rate in 6G communication networks is achieved by exploiting the sub-terahertz (THz) and THz spectrum (0.1 – 10 THz), which uses advanced optical fiber communication technologies such as optical heterodyning and high-order modulation schemes for faster and high-capacity data transfer (5,6). Again, the benefits of radio-over-fiber (RoF) technology has been demonstrated for scaling 5G and 6G networks by implementing their full integration with existing access network technologies (3,7).

However, high and ultra-high data rate optical transmissions employing high-order modulation schemes are critically affected by optical fiber impairments, limiting their reach (8,9). Digital signal processing (DSP) techniques and coherent detection have become popular for compensating fiber impairments and promise to be a potential solution for designing high data rate transmissions. Subsequently, several techniques presented for nonlinear impairments compensation include digital backpropagation (DBP) (10,11), Volterra series nonlinear equalizer (VSNLE) (12), phase-conjugate twin waves (PCTW) (13), machine learning-based techniques (14), nonlinear Fourier transform (NFT) (De Koster & Wahls, 2020), among others. The DBP technique has been demonstrated to be more promising and effective for jointly mitigating fiber dispersion and nonlinear distortions, becoming a benchmark for evaluating other compensation techniques (16). This technique is implemented by using the split-step Fourier method (SSFM) to determine a numerical solution of the inverse nonlinear Schrödinger equation (NLSE) (10). In DBP, a signal is propagated through a virtual fiber divided into small-sized steps per fiber span and characterized by the inverted properties of a typical fiber. However, the effective implementation of DBP depends considerably on the number of steps and the selection of step sizes, as it can become computationally complex with an increasing number of steps per span (17). Thus, there is a trade-off between accuracy and computational cost. Several advanced DBP techniques with low implementation computational overhead have been proposed (18–21).

Notably, the low complexity correlated DBP (C-DBP) has been proposed based on perturbation-based techniques to reduce the number of steps in a 3200 km 28 GBaud 32-QAM transmission system by a factor of 10 (21). A 60% reduction in complexity has been realized for a 21-channel 32 GBaud WDM PM-16QAM by the time-domain DBP (TD-DBP) using a randomly distributed step size (18). The work reported that sensitivity to quantization noise from FIR filter coefficients was significantly improved by randomizing the step sizes. In (19), an 80% reduction in required steps per fiber span is reported by the weighted DBP (WDBP), which uses a weighted-average approach to consider the correlation between adjacent symbols as well as an optimization of the position of the nonlinear operator calculation point. The algorithm is demonstrated in a 1600 km 112 Gbit/s PM-QPSK transmission link. Also, a low complexity DBP using a single step DBP per fiber span is experimentally demonstrated in a 112 Gbit/s PM-QPSK over 3200 km, reducing power consumption and complexity by a factor of 16 and the computation time by a factor of 20 (20).

In most of these techniques, the constant step size is used for DBP backpropagation. However, this does not provide an optimal solution in the presence of fiber attenuation. The nonlinear phase shift accumulated in each step decreases exponentially with distance due to fiber attenuation (22). Subsequently, various adaptive step size distribution DBP algorithms such as the local error method (23), logarithmic step size distribution method (24), and nonlinear phase-rotation method (25) have been presented.

In this work, we propose a step size optimization scheme of the split-step Fourier method for effective and low-complexity implementation of the DBP technique for fiber impairment compensation. The conventional logarithmic step size distribution scheme is modified using the binary logarithm to generate optimum adaptive step sizes for accurate DBP calculation. The algorithm is demonstrated in a 112 Gbit/s DP-16QAM system over a 2400 km transmission link. The simulation results show that implementing the binary logarithmic step size distribution for step size optimization outperforms traditional logarithmic and constant step size distribution techniques.

Variable step size

When a signal is launched into an optical fiber, the signal power decreases exponentially along the fiber span in the presence of fiber attenuation. As the instantaneous power determines the nonlinear phase shift, its accumulation similarly follows an exponential profile. The power along a fiber of length L is given by 1:

11* MERGEFORMAT ()

where a is the attenuation coefficient. The accumulated nonlinear phase shift is expressed as:

22* MERGEFORMAT ()

where is the nonlinear coefficient of the virtual link. The nonlinear impairment is observed to be larger at the beginning of the propagation through the span. For backward propagation along the virtual fiber, this works reversely. Since the nonlinear impairment varies along a span, a smaller step size is required at larger powers for accurate DBP estimation of signal distortion, as illustrated by Fig. 1(c).

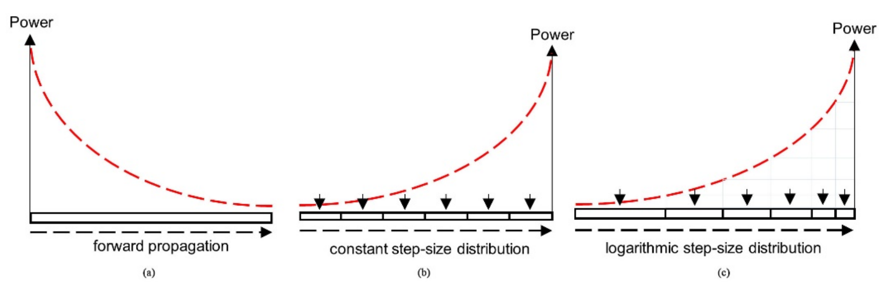

Fig. 1. (a) Power distribution along a fiber span in forward propagation, (b) constant step size distribution backpropagation, and (c) logarithmic step size distribution backpropagation

In a constant step size (CSS) distribution, the step size is uniform for all steps across the span of the virtual fiber. For such schemes, it is required to choose a very small step size, typically less than 100m, to control the generation of fictitious four-wave mixing (FWM) artifacts (24). Alternatively, by evaluating the smallest step, which allows a 10% accuracy on all in-band FWM artifacts, a constant step size of up to 400m can be chosen (26). The selection of such a small constant step size significantly compromises the computational speed of the SSFM calculation. On the other hand, a large constant step size generates fictitious FWM tones during SSFM calculation, degrading the accuracy of nonlinear phase shift compensation.

Logarithmic Step size SSFM

Variable step sizes where the step size increases and power decreases are usually preferred to the constant step size. The logarithmic step size (LSS) distribution is an emerging non-uniform distribution technique proposed to accurately estimate the nonlinear distortion in fewer steps and suppress the generation of fictitious FWM tones. This method aims to keep the average nonlinear phase shift after each step constant (27). For a fiber span of length L , attenuation coefficient a , the nth step size is given by (24).

33* MERGEFORMAT ()

where N is the number of steps per span for SSFM calculation.

In (28), the slope coefficient of the conventional logarithmic distribution has been chosen as 1 to reduce the relative global error. A modified logarithmic step size (MLSS) has also been introduced. The slope coefficient is modified as an optimized attenuation adjusting factor to control the difference in step sizes of adjacent steps for optimal performance (29). A generalized logarithmic step size distribution scheme has been proposed to consider high symbol rates and the number of steps by optimizing the base for the step size calculation and the nonlinear coefficient scale factor (27).

Principle of Binary Logarithmic Step size

For a small number of steps, the conventional logarithmic step-size method, which computes sizes using the natural logarithm (ln), results in a significant variation of the signal waveform over one step and does not produce optimal SSFM calculation. We present a binary logarithmic step size (BLSS) for implementing the DBP, where the step size selection of the conventional LSS is optimized using the binary logarithm ($\log_2$). Also, the optimized factor introduced by (29) for adjusting the attenuation coefficient is considered. The proposed BLSS algorithm for SSFM step size selection is given by 4:

44* MERGEFORMAT ()

where k is the attenuation adjusting factor.

The modification using the binary logarithm better approximates the nonlinear distortions for compensation with higher accuracy than the natural logarithmic step size technique. The proposed technique assumes that the optimised target function is highly multimodal, with a large number of local optima in terms of step size. This can make it difficult for traditional optimization methods to converge to the global optimum. To address this, the technique utilizes a binary logarithmic step size distribution. The binary aspect allows the algorithm to explore the complex search space efficiently by allowing for coarse and fine-grained searches. On the other hand, the logarithmic scale enables the algorithm to move swiftly across the search space, which helps to overcome the issue of getting stuck in local optima.

This approach of controlling the variation of the signal waveform over one step using a binary logarithmic step size is particularly useful in digital backpropagation. Here, when a small number of steps per span is used, and the attenuation factor k is optimized, it helps to ensure that the difference between one step and the next is not too substantial for nonlinear compensation. The modification produces optimal step size distribution while maintaining a low computational overhead.

Fig. 2 compares the variation of adjacent step sizes for different compensation techniques. We use the optimized adjusting factor k= 0.4 determined by . For an 80 km fiber span, the step sizes for 10 steps using different selection techniques are given in Table 1.

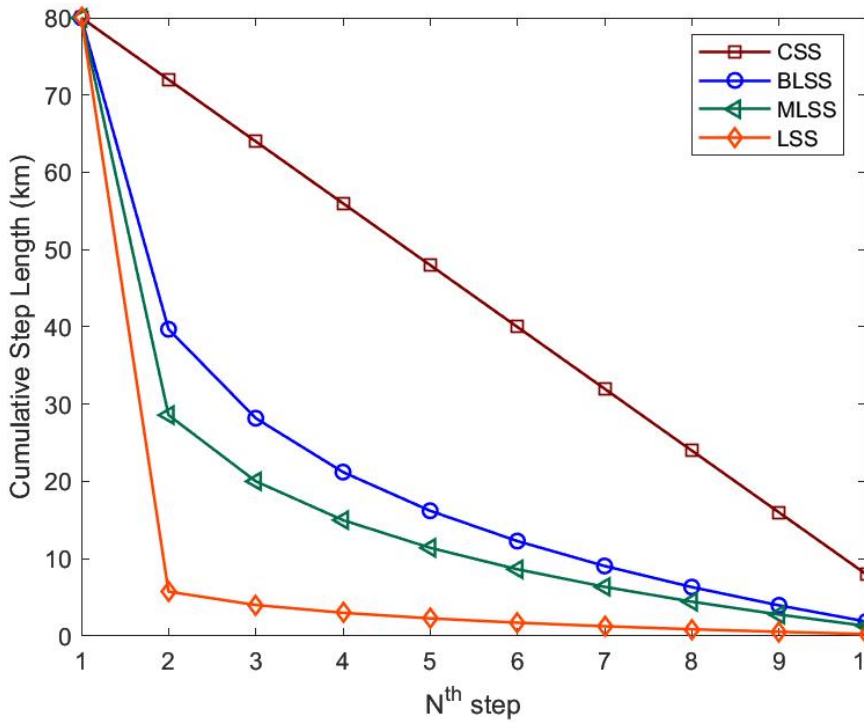

Fig. 2. Comparison of step-to-step size variation among constant (CSS), logarithmic (LSS), and binary logarithmic (BLSS) step size selection algorithms

The algorithm has been investigated in a 2400 km single-mode fiber (SMF) DP-16QAM transmission system without inline dispersion compensation. Again, it has been demonstrated for different system capacity and bandwidth-efficiency configurations with 100 Gbit/s – 12 GBaud, 112 Gbit/s – 14 GBaud, 168 Gbit/s – 21 GBaud, and 224 Gbit/s – 28 GBaud. Performance comparison of the algorithm is made with different compensation techniques in terms of quality factor (Q) and bit-error rate (BER), calculated from the error vector magnitude (EVM) of received symbols.

Table 1: DBP step sizes for an 80 km fiber span (10 steps)

nth step	1	2	3	4	5	6	7	8	9	10
CSS	8	8	8	8	8	8	8	8	8	8
LSS	0.26	0.29	0.33	0.39	0.46	0.56	0.72	1.01	1.73	74.25
MLSS	1.31	1.47	1.67	1.92	2.27	2.78	3.58	5.03	8.56	51.40
BLSS	1.88	2.09	2.37	2.73	3.22	3.92	5.01	6.97	11.51	40.3

NUMERICAL SIMULATION

Numerical Model

The numerical model of the DP-16QAM transmission system illustrated in Fig. 3 is implemented with OptiSystem 18. For a single-channel transmission, a symbol rate of 14 GBaud is chosen to transmit at a high bit rate of 112 Gbit/s. Considering a ˜4% ethernet overhead and ˜7% forward error correction (FEC) overhead translates into about 100 Gbit/s net data rate. At the transmitter, a polarization beam splitter (PBS) separates the two polarization states of the output of a continuous-wave (CW) laser operating at a wavelength of 1550 nm and a narrow linewidth of 0.1 MHz. The laser power is swept from 7 dBm to 15 dBm.

Using two identical LiNbO3 Mach-Zehnder Modulators (MZM), the and polarization respectively modulates symbols from two 16-QAM modulators fed by a data stream of 215 − 1 pseudo-random bit sequence (PRBS). A polarization beam combiner (PBC) then recombines the two polarization signals, after which the signal is transmitted over a standard single-mode fiber of up to 4,000 km (50 spans 80 km).

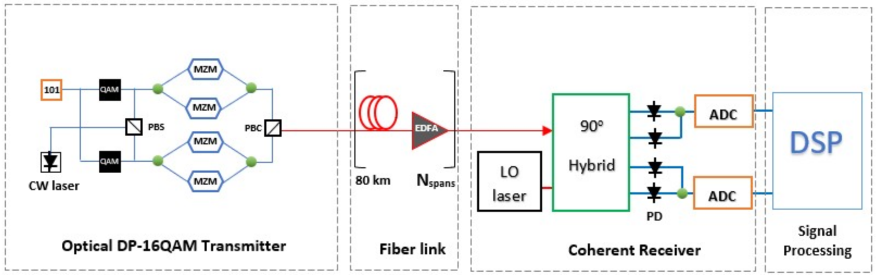

Fig. 3. Numerical model of a long-haul coherent transmission system implementing DSP

The physical properties characterize the S-SMF; attenuation α = 0.2 dB, dispersion D = 16.75 ps/nm/km, dispersion slope = 0.075 ps/nm2/km, and nonlinearity coefficient γ = 1.2 km-1W-1. A 16 dB gain Erbium-doped fiber amplifier (EDFA) with a noise figure of 4 dB is used for optical amplification to compensate for power loss. On the receiver side, the incoming in-phase and quadrature-phase signals in the x and y polarization states are detected by a homodyne coherent polarization and phase diversity receiver. The receiver comprises two 90° optical hybrids and four pairs of balanced PIN photodetectors. The local oscillator (LO) laser for coherent detection operates at a frequency, power, and linewidth of 1550 nm, 10 dB, and 0.1 MHz, respectively. After coherent detection, the received signal undergoes analog-to-digital conversion by four samples per symbol down-sampling method at the reference symbol rate. The DSP module processes it after that.

Digital Signal Processing Module

The DSP module, implemented with MATLAB, comprises segmental blocks for polarization multiplexing, carrier phase estimation, linear dispersion compensation, and nonlinearity compensation. The constant modulus phase algorithm implements an adaptive finite impulse response (FIR) butterfly equalizer to demultiplex the dual-polarized signal. The Viterbi-and-Viterbi joint-phase estimation algorithm is adopted to account for relative phase error between the transmitter laser and the LO laser, which arises due to their non-zero laser linewidth (Viterbi & Viterbi, 1983). The DBP-based nonlinearity techniques are implemented by the SSFM method based on the Weiner-Hammerstein model where the nonlinear part is computed at the midpoint of each step with two surrounding virtual linear blocks . We use the step sizes given in Table 1 for the 10-step SSFM calculation. The DSP linear blocks have been characterized in Table 2.

Table 2: Characterization of DSP linear blocks

DSP	Algorithm	Parameter	Value
Downsampling	Analog to Digital Conversion	Symbol rate	14 GBaud
		Number of symbols	Sequence length/8
		Samples per symbol	4
Linear Dispersion Compensation	Savory algorithm (33)	Channel Wavelength	1550 nm
		Dispersion	16.75 ps/nm/km
		Dispersion Slope	0.075 ps/nm2/km
Polarization Demultiplexing	Constant modulus algorithm (CMA) (30)	Step	0.11
		Initial value	1
		FIR filter order	3
Carrier Phase Estimation	Viterbi and Viterbi algorithm (31)	Coupling factor	0

DSP	Algorithm	Parameter	Value
		FIR filter order	3

SIMULATION RESULTS AND DISCUSSION

Simulation results of various configurations over a sweep of signal launch power have been given. Generally, the DBP algorithms are more efficient for mitigating the effects of dispersion and nonlinearities and thus demonstrate improved system performance compared to linear dispersion compensation (LDC) methods. At high launch powers, the system performance is critically limited by amplified spontaneous emission (ASE) noise introduced by optical amplifiers. The peak performance is observed at an optimum launch power, called the nonlinear threshold point (NLT). The Q-factor curves in this work follow the NLT phenomenon and align with the literature on the limitation of the DBP algorithm for impairments mitigation. That is, the inability of DBP to account for non-deterministic distortions such as ASE noise. As a result, it is observed that at launch powers higher than the NLT, accumulated ASE nonlinearities degrade the performance of DBP mitigation.

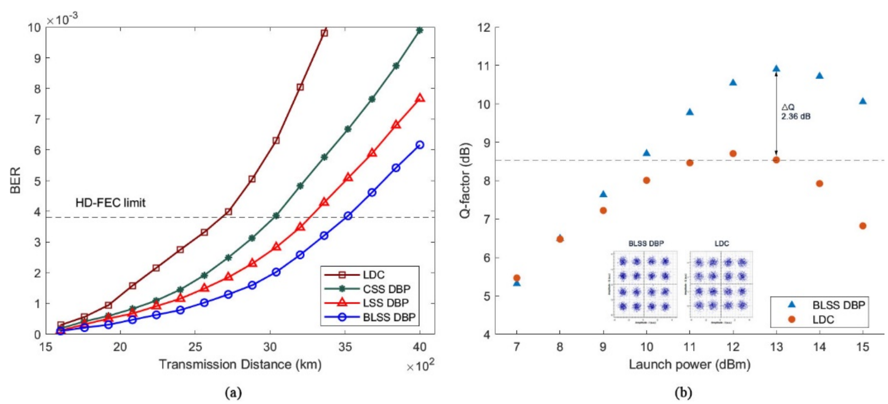

Fig. 4. (a) Comparison of the impact of transmission distance (km) on performance (in BER) using LDC and different DBP techniques; (b) Performance improvement of 112 Gbit/s DP-16QAM transmission with LDC and 10 steps/span BLSS DBP over 2400 km

Firstly, the impact of increasing the transmission distance for different signal processing techniques on overall system performance is examined. The DSP-based LDC, constant step size (CSS DBP), conventional LSS DBP, and the proposed BLSS DBP algorithms for the single-channel transmission are considered. The time-domain FIR filter implements the LDC. The number of spans controlled by a loop counter is increased serially to vary the transmission distance. Fig. 4(a) shows the BER against the transmission distance. It is observed that generally, the DBP algorithms extend the transmission distance by several kilometers as they cancel the distortions associated which long-haul communication. Specifically, at the ~10^{-3}hard-decision forward error correction (HD-FEC) threshold, the distance is extended from 2560 km to 3040 km, 3200 km, and 3520 km by implementing the CSS, LSS, and BLSS DBP algorithms, respectively. As expected, the logarithmic step size distribution shows better performance when compared to the constant step size distribution. Also, the proposed BLSS DBP algorithm is more effective for compensating intra-channel nonlinear self-phase modulation. For a 2400 km transmission, the BLSS DBP algorithm achieves a Q-factor improvement ([?]Q) of 2.36 dB at 13 dBm signal launch power, as shown in Fig. 4(b). The inset shows the respective constellation diagrams.

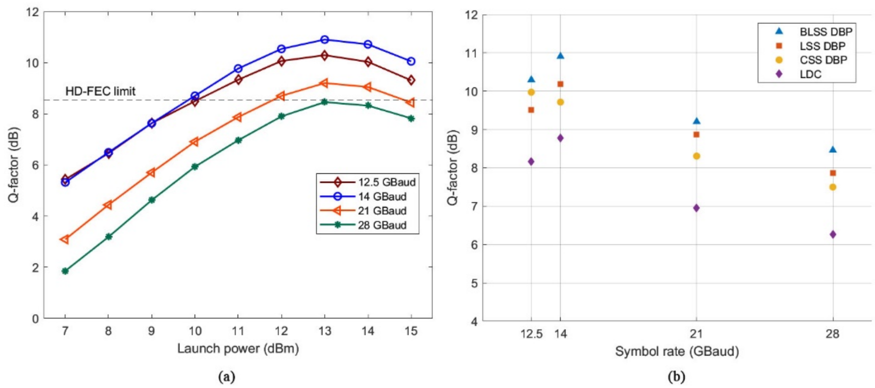

Fig. 5. (a) Comparison of the performance (Q) of transmissions at different symbol rates implementing the BLSS DBP technique; (b) Performance benefits (Q) of LDC, CSS DBP, LSS DBP, and BLSS at NLT for 12.5 GBaud, 14 GBaud, 21 GBaud, and 28 GBaud over 2400 km

As demonstrated by Fig. 5, the performance benefit of the logarithmic step size distribution is dependent on the signal bandwidth, which is indicated by the transmission symbol rate, also known as the Baud rate. Fig. 5(a) shows that by increasing the symbol rate, the effectiveness of the optimized BLSS techniques declines for the same number of calculation steps/span. The best performance is demonstrated at a 14 GBaud symbol rate with a Q-factor of 10.9 dB (BER ˜ 2.25×10^{-4}) using 10 calculation steps/span. The observation can explain this trend that at small steps/span, the signal waveform changes substantially over one step as the symbol rate is increased due to the impact of increased dispersion. It is inferred that more calculation steps are required for large data rates for optimum performance of the logarithmic step size, indicating a trade-off between data rate and complexity. The performance of different compensation techniques as a function of symbol rate is shown in Fig. 5(b).

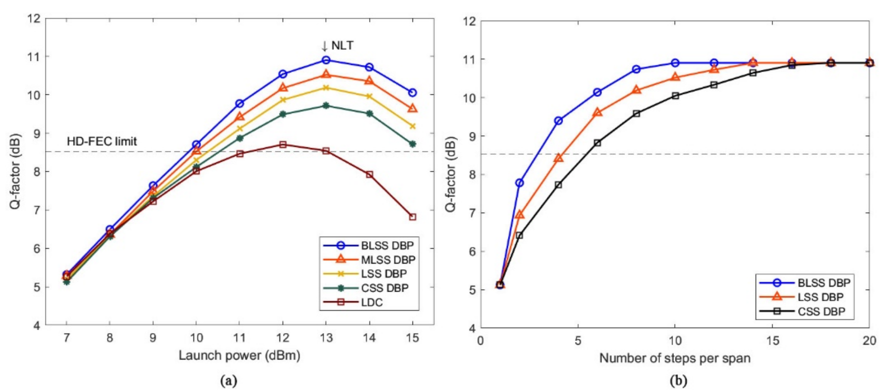

Fig. 6. (a) Performance (QdB) of 14 GBaud DP-16QAM transmissions using LDC, CSS DBP, LSS DBP, and BLSS DBP techniques over varying signal launch powers (dBm); (b) Performance (QdB) of CSS DBP, LSS DBP, and BLSS against the number of calculation steps/span at 13 dBm launch power.

The overall performance benefit of the BLSS DBP over other compensation techniques as a function of signal launch power is analyzed, as shown in Fig. 6(a). Observing the optimum launch power, indicating the NLT, the logarithmic step size-based DBP techniques demonstrate superior performance compared to the constant step size and linear dispersion compensation only. Using the BLSS DBP technique, performance benefits ([?]Q) of 1.19 dB and 0.71 dB are obtained over CSS DBP and LSS DBP techniques, respectively. The results indicate that the BLSS DBP produces optimum step sizes for efficient DBP calculation. At higher powers above the NLT, the performance of DBP is degraded by accumulated stochastic ASE noise.

It is known that the effective implementation of DBP is significantly dependent on the number of calculation steps/span, . However, a high number of steps/spans comes at a high computational cost and are undesirable. Fig. 6(b) shows the accuracy of DBP calculation (in Q-factor) as a function of steps/span for different DBP implementations. It is observed that for N [?] 10, BLSS DBP cancels dispersion and nonlinearity with higher accuracy than LSS DBP and CSS DBP. Specifically, the BLSS DBP reaches optimum performance with N = 10. In contrast, CSS DBP achieves optimum performance with N = 16, reducing the number of calculation steps by 6, corresponding to a reduction in complexity of 38% compared to the conventional CSS DBP. The constellation diagrams for N = 1, 2, 6, and 10 with BLSS DBP are shown in Fig. 7. At a high number of steps (N > 15), optimizing the step sizes becomes less important as the conventional DBP algorithm becomes equally accurate.

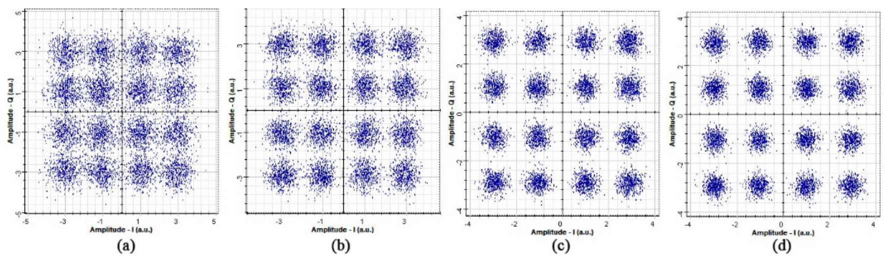

Fig. 7. The 16QAM constellation diagrams after 2400 km transmission using the BLSS DBP technique with: (a) 1 step/span; (b) 2 steps/span; (c) 6 steps/span; (d) and 10 steps/span

CONCLUSION

A binary logarithmic step size optimization technique of the SSFM has been presented to effectively implement the DBP technique for fiber impairment compensation with low complexity. Optimum step sizes are generated using a binary step size distribution for accurate DBP calculation. The numerical results indicate that step size optimization using the binary logarithmic step size distribution brings significant improvement benefits over the conventional logarithmic and constant step size distribution techniques and extends the possible transmission distance over several kilometers. A high performance (Q) of 10.9 dB (BER = 2.25×10^{-4}) is achieved at the HD-FEC limit. The proposed technique can achieve optimum nonlinearity compensation with only 10 steps, whereas the conventional DBP requires 16 steps for optimum performance. Thus, the optimization achieves 38% savings in the number of DBP calculation steps over the constant step size DBP, reducing the computational cost since a few steps are required to mitigate nonlinear distortions effectively. The proposed BLSS is, therefore, a promising technique for high accuracy and low complexity DBP implementation for compensating distortions in high data rate optical transmissions.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

2. Kobayashi T, Nakamura M, Hamaoka F, Nagatani M, Yamazaki H, Nosaka H, et al. Long-haul WDM transmission with over-1-Tb/s channels using electrically synthesized high-symbol-rate signals. In: Optical

Fiber Communication Conference (OFC) 2020 [Internet]. Washington, D.C.: OSA; 2020 [cited 2022 Jun 30]. p. M4K.1. Available from:*https://opg.optica.org/abstract.cfm?URI=OFC-2020-M4K.1*

3. Moon SR, Sung M, Kim ES, Lee JK, Cho SH, Kim J. RoF-based indoor distributed antenna system that can simultaneously support 5G mmWave and 6G terahertz services. Opt Express. 2022 Jan 17;30(2):1521.

4. Alsharif MH, Albreem MAM, Solyman AAA, Kim S. Toward 6G Communication Networks: Terahertz Frequency Challenges and Open Research Issues. Computers, Materials and Continua [Internet]. 2021 [cited 2022 Jul 18];66(3):2831–42. Available from:*http://oak.ulsan.ac.kr/handle/2021.oak/9043*

5. Lee W, Han S, Moon SR, Park J, Yoo S, Park H, et al. Coherent terahertz wireless communication using dual-parallel MZM-based silicon photonic integrated circuits. Opt Express. 2022 Jan 17;30(2):2547.

6. Seeds AJ, Shams H, Fice MJ, Renaud CC. TeraHertz Photonics for Wireless Communications. J Lightwave Technol [Internet]. 2015 Feb;33(3):579–87. Available from:*http://opg.optica.org/jlt/abstract.cfm?URI=jlt-33-3-579*

7. Meshkov IK, Gizatulin AR, Vinogradova IL, Meshkova AG, Sultanov AKh, Bagmanov VKh, et al. Usage of SDM technology in radio-over-fiber (RoF) transmission systems in high-speed scalable 6G wireless networks. In: Andreev VA, Bourdine A v, Burdin VA, Morozov OG, Sultanov ACh, editors. Optical Technologies for Telecommunications 2020 [Internet]. SPIE; 2021. p. 414 – 419. Available from:*https://doi.org/10.1117/12.2593036*

8. Zhao J, Liu Y, Xu T. Advanced DSP for Coherent Optical Fiber Communication. Applied Sciences [Internet]. 2019 Oct 8 [cited 2022 Jun 30];9(19):4192. Available from:*https://www.mdpi.com/2076-3417/9/19/4192*

9. Irfan M, Ali F, Muhammad F, Habib U, Alwadie AS, Glowacz A, et al. DSP-Assisted Nonlinear Impairments Tolerant 100 Gbps Optical Backhaul Network for Long-Haul Transmission. Entropy [Internet]. 2020 Sep 22 [cited 2022 Jun 30];22(9):1062. Available from:*https://www.mdpi.com/1099-4300/22/9/1062*

10. Shao J, Liang X, Kumar S. Comparison of Split-Step Fourier Schemes for Simulating Fiber Optic Communication Systems. IEEE Photonics J. 2014 Aug 1;6(4).

11. Sinkin O v., Holzlöhner R, Zweck J, Menyuk CR. Optimization of the split-step Fourier method in modeling optical-fiber communications systems. Journal of Lightwave Technology. 2003 Jan;21(1):61–8.

12. Mrabet H, Mhatli S. A reduced complexity Volterra-based nonlinear equalizer for up to 100 Gb/s coherent optical communications. Optoelectronics and Advanced Materials-Rapid Communications. 2018;12(4):186–92.

13. Choudhary U, Janyani V, Khan MA. Optical phase conjugation with dual frame OFDM for dispersion and nonlinearity mitigation over multimode fiber. Optical and Quantum Electronics 2021 53:2 [Internet]. 2021 Jan 24 [cited 2022 May 30];53(2):1–11. Available from:*https://link.springer.com/article/10.1007/s11082-021-02736-y*

14. Shakya S. Machine Learning Based Nonlinearity Determination for Optical Fiber Communication - Review. Journal of Ubiquitous Computing and Communication Technologies [Internet]. 2019 Dec 30 [cited 2022 May 30];2019(02):121–7. Available from:*https://www.irojournals.com/jucct/V1/I2/06.pdf*

15. de Koster P, Wahls S. Dispersion and Nonlinearity Identification for Single-Mode Fibers Using the Nonlinear Fourier Transform. Journal of Lightwave Technology. 2020 Jun 15;38(12):3252–60.

16. Amari A, Dobre OA, Venkatesan R, Kumar OSS, Ciblat P, Jaouen Y. A Survey on Fiber Nonlinearity Compensation for 400 Gb/s and Beyond Optical Communication Systems. IEEE Communications Surveys & Tutorials [Internet]. 2017 [cited 2022 May 3];19(4):3097–113. Available from:*http://ieeexplore.ieee.org/document/7959045/*

17. Bosco G, Carena A, Zefreh MR, Poggiolini P, Forghieri F. Advances in Modeling and Mitigation of Nonlinear Effects in Uncompensated Coherent Optical Transmission Systems. In: 2020 European Confe-

rence on Optical Communications (ECOC) [Internet]. IEEE; 2020 [cited 2022 Jun 30]. p. 1–4. Available from:*https://ieeexplore.ieee.org/document/9333258/*

18. Martins CS, Bertignono L, Nespola A, Carena A, Guiomar FP, Pinto AN. Low-Complexity Time-Domain DBP Based on Random Step-Size and Partitioned Quantization. Journal of Lightwave Technology [Internet]. 2018 Jul 15 [cited 2022 Jun 30];36(14):2888–95. Available from:*https://ieeexplore.ieee.org/document/8345692/*

19. Rafique D, Mussolin M, Forzati M, Mårtensson J, Chugtai MN, Ellis AD. Compensation of intra-channel nonlinear fibre impairments using simplified digital back-propagation algorithm. Opt Express [Internet]. 2011 May 9 [cited 2022 Jun 30];19(10):9453. Available from:*https://opg.optica.org/oe/abstract.cfm?uri=oe-19-10-9453*

20. Secondini M, Rommel S, Meloni G, Fresi F, Forestieri E, Potì L. Single-step digital backpropagation for nonlinearity mitigation. Photonic Network Communications [Internet]. 2016 Jun 9 [cited 2022 Jun 30];31(3):493–502. Available from:*http://link.springer.com/10.1007/s11107-015-0586-z*

21. Liang X, Kumar S. Correlated digital back propagation based on perturbation theory. Opt Express [Internet]. 2015 Jun 1 [cited 2022 Jun 30];23(11):14655. Available from:*https://opg.optica.org/abstract.cfm?URI=oe-23-11-14655*

22. Jing Shao, Xiaojun Liang, Kumar S. Comparison of Split-Step Fourier Schemes for Simulating Fiber Optic Communication Systems. IEEE Photonics J [Internet]. 2014 Aug 1 [cited 2022 Jun 30];6(4):1–15. Available from:*http://ieeexplore.ieee.org/document/6860304/*

23. Zhang Q, Hayee MI. Symmetrized Split-Step Fourier Scheme to Control Global Simulation Accuracy in Fiber-Optic Communication Systems. Journal of Lightwave Technology [Internet]. 2008 Jan 15 [cited 2022 Jun 30];26(2):302–16. Available from:*http://ieeexplore.ieee.org/document/4451240/*

24. Bosco G, Carena A, Curri V, Gaudino R, Poggiolini P, Benedetto S. Suppression of spurious tones induced by the split-step method in fiber systems simulation. IEEE Photonics Technology Letters [Internet]. 2000 May [cited 2022 Jun 30];12(5):489–91. Available from:*http://ieeexplore.ieee.org/document/841262/*

25. Sinkin OV, Holzlohner R, Zweck J, Menyuk CR. Optimization of the split-step fourier method in modeling optical-fiber communications systems. Journal of Lightwave Technology [Internet]. 2003 Jan [cited 2022 Jun 30];21(1):61–8. Available from:*http://ieeexplore.ieee.org/document/1190149/*

26. Francia C. Constant step-size analysis in numerical simulation for correct four-wave-mixing power evaluation in optical fiber transmission systems. IEEE Photonics Technology Letters [Internet]. 1999 Jan [cited 2022 Jun 30];11(1):69–71. Available from:*http://ieeexplore.ieee.org/document/736394/*

27. Dien N v., Son LT, Tuan N v., Tien HP, Vien NDN, Hung NL, et al. Digital back-propagation optimization for high-baudrate single-channel optical fiber transmissions. Opt Commun [Internet]. 2021 Jul 15 [cited 2022 Jun 30];491:126913. Available from:*https://linkinghub.elsevier.com/retrieve/pii/S0030401821001632*

28. Asif R, Lin CY, Schmauss B. Efficient compensation of chromatic dispersion and nonlinearities using logarithmic digital backward propagation in N -channel DWDM 1.12Tbit/s DP-QPSK transmission. J Mod Opt [Internet]. 2012 Jan 20 [cited 2022 Jun 30];59(2):95–101. Available from:*http://www.tandfonline.com/doi/abs/10.1080/09500340.2011.631050*

29. Zhang J, Li X, Dong Z. Digital Nonlinear Compensation Based on the Modified Logarithmic Step Size. Journal of Lightwave Technology [Internet]. 2013 Nov [cited 2022 Jun 30];31(22):3546–55. Available from:*http://ieeexplore.ieee.org/document/6632915/*

30. Kazuro Kikuchi. Polarization-demultiplexing algorithm in the digital coherent receiver. In: 2008 Digest of the IEEE/LEOS Summer Topical Meetings [Internet]. IEEE; 2008 [cited 2022 Jun 30]. p. 101–2. Available from:*http://ieeexplore.ieee.org/document/4590509/*

31. Viterbi AJ, Viterbi AM. Nonlinear estimation of PSK-modulated carrier phase with application to burst digital transmission. IEEE Trans Inf Theory [Internet]. 1983 Jul [cited 2022 Jun 30];29(4):543–51. Available from:*https://ieeexplore.ieee.org/document/1056713/*

32. Behrens C, Killey RI, Savory SJ, Chen M, Bayvel P. Nonlinear transmission performance of higher-order modulation formats. IEEE Photonics Technology Letters. 2011;23(6):377–9.

33. Savory SJ. Digital filters for coherent optical receivers. Opt Express [Internet]. 2008 Jan 21 [cited 2022 May 27];16(2):804. Available from:*https://opg.optica.org/oe/abstract.cfm?uri=oe-16-2-804*