Conformable mathematical modeling of the COVID-19 transmission dynamics: A more general study

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Conformable mathematical modeling of the COVID-19 transmission dynamics: A more general study

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Abstract

Many challenges are still faced in bridging the gap between Mathematical modeling and biological sciences. Measuring population immunity to assess the epidemiology of health and disease is a challenging task and is currently an active area of research. However, to meet these challenges, mathematical modeling is an effective technique in shaping the population dynamics that can help disease control. In this paper, we introduce a Susceptible-Infected-Recovered (SIR) model and a Susceptible-Infected-Recovered-Exposed-Deceased (SEIRD) model based on conformable space-time PDEs for the Coronavirus Disease 2019 (COVID-19) pandemic. As efficient analytical tools, we present new modifications based on the fractional exponential rational function method (ERFM) and an analytical technique based on the Adomian decomposition method for obtaining the solutions for the proposed models. These analytical approaches are more efficious for obtaining analytical solutions for nonlinear systems of partial differential equations (PDEs) with conformable derivatives. The interesting result of this paper is that it yields new exact and approximate solutions to the proposed COVID-19 pandemic models with conformable space-time partial derivatives.

KEYWORDS:
Coronavirus disease 2019, Conformable derivatives, Fractional exponential rational function method, Mathematical modeling, New analytical Technique, Partial differential equations

1 | INTRODUCTION

Several problems in biology, ecology, ontology, and epidemiology can be modeled in terms of local and nonlocal diffusion through mathematical modeling. Mathematical modeling is implemented to compute and evaluate parameters that are substantial for a dynamical understanding of epidemic transmission. However, mathematical modeling plays a significant role in epidemiology. Results of mathematical modeling for epidemiological studies help to attract health interventions for effective disease control. Although epidemiology is a descriptive science, evaluating epidemiological data is not often possible because of the high complexity of epidemic observations and fighting against diseases. Fighting against infectious diseases, however, is like targeting a moving objective. Although diverse infection control strategies, still create large-scale continuous pathogens and generate renewed challenges to infectious disease control. Many infectious diseases such as malaria, tuberculosis, SARS, and COVID-19, have been and maintain a global threat to human health [1–3]. COVID-19 is a spread infectious disease that has been a threat to humans for a long time. Most people infected with malaria, tuberculosis, SARS, or the COVID-19 virus will experience mild to moderate respiratory illness and recover without requiring special treatment. Although many factors for reduced activation rates of COVID-19 such as the development of antibiotics and vaccines, some infectious diseases are drug-susceptible and resistant. Consequently, some significant factors must be taken into account to limit the evolution of the infectious disease, for example, improved diagnostic procedures and changes in host potential population size. However, many diverse challenges are still faced in bridging the gap between immunology and epidemiology. Measuring population immunity to assess the epidemiology of health and disease is a challenging task and is currently an active area of research. Therefore, modeling the potential host population and developing mathematical techniques can help in the evolution and reduction of infectious diseases and inform public health interventions. However, mathematical modeling plays a significant role in epidemiology. Results of mathematical modeling for epidemiological studies help to attract health interventions for effective disease control. Mathematical modeling, which refers to the process of using mathematics to solve problems, is implemented to compute and evaluate essential parameters [4–6]. Modeling infectious diseases with non-local diffusion equations through mathematical modeling provides a better fit due to their non-local characters since the diffusion of a density function at a given point does not only depend on the value of that function at the given point but all the values of the function in a neighborhood of the given point. Moreover, non-local diffusion differential equations have a memory effect and they can capture long-distance interactions during the process that occur in many epidemiological models [7–10]. In addition, time series modeling based on statistical methodology has been widely used to model, analyze, forecast, and addressed various problems of epidemiological science. However, many challenges are still faced in bridging the gap between mathematical modeling and epidemiology. Jyoti Mishra [11] investigated, by a developed mathematical model, the spreading behavior of COVID-19 among humans using different differential and integral operators. Further, Shabir Ahmad et al. [12] presented a mathematical model with different compartments for the transmission dynamics of COVID-19 under the fractional-order derivative. Furthermore, Mohammed A. Aba Oud et al. [13] formulated a fractional epidemic model in the Caputo sense with the consideration of quarantine, isolation, and environmental impacts to examine the dynamics of the COVID-19 outbreak. These fractional models are quite useful for understanding better the disease epidemics as well as capturing the memory and nonlocality effects.

Over the last decades, it was observed that some nonlinear phenomena could not be described due to their complex behavior. Therefore, modeling such phenomena with conformable derivatives gives a better understanding due to their non-local characters. However, some properties of conformable derivatives were not satisfied. Consequently, a new significant contribution with the so-called conformable derivative was introduced by Khalil et al. [14]. Moreover, throughout this contribution, several works were devoted [15–19]. Further, with the conformable derivatives, it has been proved the product rule, the mean value theorem with fractional order, and solved some differential equations of conformable derivatives. However, some functions could not be represented or their integral transforms can not be calculated, but it is possible to do so with the help of conformable calculus theory. Therefore, the conformable calculus theory is still an active area of research. Recently, Hayman Thabet and Subhash Kendre [17] introduced a conformable differential transform for solving nonlinear conformable PDEs, Yücel Çenesiz et al. [20] studied PDEs with conformable derivative using the first integral method, Thabet et al [21] obtained analytical solutions for nonlinear wave equations of conformable derivatives, K. Hosseini et al. [22] applied the Kudryashov method for Klein–Gordon equations with conformable derivatives, Yaslan [23] used the method to solve the conformable Kawahara equation, and Khodadad et al. [24] introduced the of method sub equation to solve the Zakharov-Kuznetsov equation with conformable derivatives. However, several methods have been introduced to solve conformable PDEs, most of which provide approximate solutions, exact solutions, however, are rarely available. Therefore, obtaining exact solutions for a complex system, by analytical methods, is a changing problem and is currently a very active area of research. One of the most powerful methods for obtaining an exact solution is the exponential rational function method (ERFM). The ERFM has been widely used to obtain a series of exact solutions for higher-dimensional nonlinear PDEs (see, for example,[25–31]). Exact solutions, however, are vitally important in the proper understanding of the qualitative features of the concerned phenomena and processes. In this paper, we introduce effective modifications of ERFM. Further, we apply these effective modifications for solving conformable space-time nonlinear PDEs. The rest of the sections are organized as follows. In Section 2, we introduce some properties of the conformable calculus theory that are needed in this paper. Section 3 introduces effective modifications of ERFM for solving nonlinear systems of conformable space-time PDEs. In Section 5, we obtain new exact solutions of the conformable space-time SIR model describing the COVID-19 pandemic. In Section 6, we introduce a discussion and graphical representations.

2 | BASIC RESULTS AND DEFINITIONS

There are several results on conformable calculus available in the literature. We present here some of these results which can be found in [32–36] and among other references.

Definition 1. For $\varphi : \mathbb{R}\times[a,\infty) \to \mathbb{R}$, the partial derivative of conformable order α with respect to t for φ is defined as follows:

$$ {}_a\mathcal{T}_t^{\alpha}\varphi(x,t) = \lim_{\xi\to 0}\frac{\varphi(x,t+\xi(t-b)^{1-\alpha})-\varphi(x,t)}{\xi},\ t>b,\ 0<\alpha\leq 1. \tag{1} $$

Theorem 1. Let $\alpha \in (0,1]$ and the functions $\varphi(x,t), \phi(x,t)$ be α-differentiable. Then

$$ \begin{aligned} &1)\ \mathcal{T}_t^{\alpha}(a\varphi \pm b\phi) = a\mathcal{T}_t^{\alpha}\varphi \pm a\mathcal{T}_t^{\alpha}\phi,\ 2)\ \mathcal{T}_t^{\alpha}(t^p) = pt^{p-\alpha}, \forall p\in\mathbb{R},\\ &3)\ \mathcal{T}_t^{\alpha}(\varphi) = 0, \text{for all } \varphi(x,t)=c,\ 4)\ \mathcal{T}_t^{\alpha}(\varphi\phi) = \varphi\mathcal{T}_t^{\alpha}\phi + \phi\mathcal{T}_t^{\alpha}\varphi,\\ &5)\ \mathcal{T}_t^{\alpha}(\varphi/\phi) = \frac{\phi(\mathcal{T}_t^{\alpha}\varphi)-\phi(\mathcal{T}_t^{\alpha}\phi)}{\phi^2},\\ &6)\ If\ \varphi \text{ is differentiable, then }\ \mathcal{T}_t^{\alpha}\varphi(x,t) = t^{1-\alpha}\frac{\partial\varphi(x,t)}{\partial t}. \end{aligned} $$

Theorem 2. Let $\alpha \in (0,1]$ and φ be α-differentiable. Then

$$ \begin{aligned} &1)\ {}_t\mathcal{T}_{\alpha}^{c}((t-c)^r) = r(t-c)^{r-\alpha}\text{for all } r\in\mathbb{R},\ 2)\ {}_t\mathcal{T}_{\alpha}^{ac}(e^{\lambda(\frac{(t-c)^{\alpha}}{\alpha}+x)}) = \lambda e^{\lambda(\frac{(t-a)^{\alpha}}{\alpha}+x)},\\ &3)\ {}_t\mathcal{T}_{\alpha}^{a}(\frac{(t-a)^{\alpha}}{\alpha}) = 1,\ 4)\ If\ \varphi \text{ is differentiable then, } {}_t\mathcal{T}_{\alpha}^{a}\varphi(x,t) = (t-a)^{1-\alpha}\frac{\partial\varphi(x,t)}{\partial t}. \end{aligned} $$

Theorem 3. Let a function $\varphi(x,t)$ defined on $\mathbb{R}\times(0,\infty)$ be $m-$times differentiable at $(x,t_0)\in$. Then, for $0<\alpha\leq 0$, the $m-$times conformable partial derivative of order α with respect to t for a function φ is given by

$$ {}_{t_0}\mathcal{T}_t^{m\alpha}\varphi(x,t) = (t-t_0)^{m-m\alpha}\frac{\partial^m\varphi(x,t)}{\partial t^m}|_{(x,t_0)}\ \text{ if }\ \frac{\partial^r\varphi(x,t)}{\partial t^r}|_{(x,t_0)} = 0, \tag{2} $$

for $r=1,2,\ldots,m-1$ where ${}_{t_0}\mathcal{T}_t^{m\alpha}\varphi(x,t) = \underbrace{{}_{t_0}\mathcal{T}_t^{\alpha}\,{}_{t_0}\mathcal{T}_t^{\alpha}\cdots{}_{t_0}\mathcal{T}_t^{\alpha}}_{m-times}\varphi(x,t)$.

Definition 2. Let a function $\varphi(x,t)$ defined on $\mathbb{R}\times(t_0,\infty)$. The conformable integral of order α with respect to t for a function φ is defined by

$$ {}_{t_0}\mathcal{J}_t^{\alpha}\varphi(x,t) = \int_{t_0}^{t}(\tau-t_0)^{\alpha-1}\varphi(x,\tau)d\tau,\ 0<\alpha\leq 1. \tag{3} $$

Theorem 4. Let $u(x,t) : \mathbb{R}\times(t_0,\infty)\to\mathbb{R}$ be k,h-differentiable. Then, for all $x\geq x_0, t\geq t_0$, we have

$$ \begin{cases} {}_{x_0}\mathcal{J}_x^{k\alpha}\,{}_{x_0}\mathcal{T}_x^{k\alpha}u(x,t) = u(x,t) - \displaystyle\sum_{i'=0}^{k-1}\frac{x^{i'}}{i'!}\frac{\partial^{i'}u(x,t)}{\partial x^{i'}}|_{(x_0,t)},\\ {}_{t_0}\mathcal{J}_t^{h\beta}\,{}_{t_0}\mathcal{T}_t^{h\beta}u(x,t) = u(x,t) - \displaystyle\sum_{j'=0}^{h-1}\frac{t^{j'}}{j'!}\frac{\partial^{j'}u(x,t)}{\partial t^{j'}}|_{(x,t_0)}. \end{cases} \tag{4} $$

3 | MODIFIED ERFM FOR SOLVING NONLINEAR SYSTEMS OF CONFORMABLE PDES

In this section, we introduce effective modifications of ERFM for solving nonlinear PDEs with conformable derivatives of the following type:

$$ \mathcal{T}_t^{k\alpha}u_i(x,t) + Li(u(x,t)) + N_i(u(x,t)) = 0,\ k-1<\alpha\leq k,\ i=1,2,\ldots,m \tag{5} $$

where L_j are linear operators and N_i are nonlinear operators of functions u_i and their conformable derivatives, and $\mathcal{T}_t^{k\alpha}u_i(x,t)$ are the $k-$times conformable derivatives of orders α_i of the functions $u_i(x,t)$ for $i=1,2,\ldots,m$.

3.1 | Fractional ERFM

In this section, we introduce new tools based on ERFM for solving nonlinear systems of PDEs with conformable derivatives of arbitrary orders of the form (5). To apply these tools, we assume that the system (5) has solutions of the form

$$u_i(x,t) = u_i(\xi), \;\; \xi^\alpha = e^{l_i x^\alpha + k_i t^\alpha}, \; i = 1, 2, \dots, m. \tag{6}$$

for $l_j, k_j \in \mathbb{R}$ are nonzero arbitrary constants to be determined later. From the system (6), we have

$$\begin{cases} \mathcal{T}_t^\alpha u_i(x,t) = \mathcal{T}_t^\alpha u_i(\xi), \quad \xi^\alpha = e^{l_i x^\alpha + k_i t^\alpha}, \; i = 1, 2, \dots, m, \\ \qquad\qquad\quad = t^{1-\alpha} u'(\xi) \dfrac{d\xi}{dt}, \; \dfrac{d\xi}{dt} = k_i \xi t^{\alpha-1}, \; k_i \neq 0, \\ \mathcal{T}_t^\alpha u_i(x,t) = k_i \xi u'(\xi), \; k_i \neq 0, i = 1, 2, \dots, m, \\ \mathcal{T}_t^{2\alpha} u_i(x,t) = k_i^2 (\xi^2 u''(\xi) + \xi u'(\xi)), \; k_i \neq 0, i = 1, 2, \dots, m, \\ \qquad\qquad \vdots \end{cases} \tag{7}$$

Similarly, we obtain

$$\begin{cases} \mathcal{T}_x^\alpha u_i(x,t) = \mathcal{T}_x^\alpha u_i(\xi), \quad \xi^\alpha = e^{l_i x^\alpha + k_i t^\alpha}, \; i = 1, 2, \dots, m, \\ \qquad\qquad\quad = x^{1-\alpha} u'(\xi) \dfrac{d\xi}{dx}, \; \dfrac{d\xi}{dt} = l_i \xi x^{\alpha-1}, \; k_i, \; l_i \neq 0, \\ \mathcal{T}_x^\alpha u_i(x,t) = l_i \xi u'(\xi), \; l_i \neq 0, i = 1, 2, \dots, m, \\ \mathcal{T}_x^{2\alpha} u_i(x,t) = l_i^2 (\xi^2 u''(\xi) + \xi u'(\xi)), \; l_i \neq 0, i = 1, 2, \dots, m, \\ \qquad\qquad \vdots \end{cases} \tag{8}$$

However, by substituting the systems (7) and (8) into equation (5), we obtain a reduced system of ODEs. Further, we assume that the reduced system of ODEs has a solution given by

$$u_i(\xi) = \sum_{j=0}^{M} \frac{b_{ij}}{(1+\xi^\alpha)^j}, i = 1, 2, \dots, m, \; j = 0, 2, \dots, M. \tag{9}$$

where $b_{ij} \in \mathbb{R}, j = 1, ..., M$ are arbitrary constants to be determined ($b_{iM} \neq 0$). By a similar argument, after that, the balancing number M is obtained from the reduced equation (13) by using the homogeneous balance principle. When we substitute equation (9) into equation (13), and collect the coefficients of each power of ξ^α, we obtain a polynomial of ξ^α. By equating all the coefficients of ξ^α to zero to obtain a system of algebraic equations. Then, we solve the obtained system of algebraic equations, we obtain the values of $b_{ij} (j = 1, ..., M)$, k_i, and l_i Subsequently, by inserting these values into equation (14), we find a series of exact solutions for equation (5).

3.2 | Inverse fractional ERFM

In this section, we introduce the inverse fractional ERFM for solving nonlinear systems of PDEs with conformable derivatives of arbitrary orders of the form (5). However, to find exact solutions for the nonlinear system of conformable PDEs of the form (5) by using fractional ERFM, we assume that the solutions of (5) are given by

$$u_i(x,t) = u_i(\tau), \;\; \tau^\alpha = l_i x^\alpha + k_i t^\alpha, \; l_i, k_i \neq 0, \; i = 1, 2, \dots, m, \tag{10}$$

From the system (10), we have

$$\begin{aligned} \mathcal{T}_t^\alpha u_i(x,t) &= \mathcal{T}_t^\alpha u_i(\tau), \quad \tau^\alpha = l_i x^\alpha + k_i t^\alpha, \; i = 1, 2, \dots, m, \\ &= t^{1-\alpha} u_i'(\tau) \frac{d\tau}{dt}, \; \tau^{\alpha-1} \frac{d\tau}{dt} = k_i t^{\alpha-1}, \; k_i \neq 0, \\ \mathcal{T}_t^\alpha u_i(x,t) &= k_i \mathcal{T}^\alpha u_i(\tau), \; k_i \neq 0, i = 1, 2, \dots, m, \\ &\vdots \\ \mathcal{T}_t^{k\alpha} u_i(x,t) &= k_i^k \mathcal{T}^{k\alpha} u_i(\tau), \; k = 1, 2, \dots, \; k_i \neq 0, \; i = 1, 2, \dots, m, \end{aligned} \tag{11}$$

Similarly, we may have

$$\begin{cases} \mathcal{T}_x^{\alpha} u_i(x,t) = \mathcal{T}_x^{\alpha} u_i(\tau),\ \tau^{\alpha} = l_i x^{\alpha} + k_i t^{\alpha},\ i = 1,2,\dots,m, \\ \qquad\qquad = x^{1-\alpha} u_i'(\tau)\dfrac{d\tau}{dx},\ \tau^{\alpha-1}\dfrac{d\tau}{dt} = l_i \tau x^{\alpha-1},\ l_i \neq 0, \\ \mathcal{T}_x^{\alpha} u_i(x,t) = l_i \mathcal{T}_x^{k\alpha} u_i(\tau),\ l_i \neq 0, i = 1,2,\dots,m, \\ \qquad\qquad \vdots \\ \mathcal{T}_x^{k\alpha} u_i(x,t) = l_i^k \mathcal{T}_x^{k\alpha} u_i(\tau),\ k = 1,2,\dots,\ l_i \neq 0,\ i = 1,2,\dots,m, \end{cases} \tag{12}$$

for $l_j, k_j \in \mathbb{R}$ are nonzero constants that should be evaluated. Therefore, by substituting (10), (11), and (12) into equation (5), we obtain the following system of conformable ordinary differential equations

$$\mathcal{T}_t^{M\alpha} u_i(\tau) + L_\tau(\bar{u}(\tau)) + N_\tau(\bar{u}(\tau)) = 0,\ \bar{u} = (u_1, u_2, \dots, u_m),\ i = 1,2\dots,m. \tag{13}$$

Next, we assume that the system (13) of ODEs has solutions given by

$$u_i(\tau) = \sum_{j=0}^{M} \frac{a_{ij}}{(1+e^{\tau^{\alpha}})^j}, \quad \tau^{\alpha} = l_i x^{\alpha} + k_i t^{\alpha},\ l_i,\ k_i, \neq 0,\ i = 1,2,\dots,m. \tag{14}$$

where $a_{ij} \in \mathbb{R}, j = 1, ..., M$ are arbitrary constants to be determined ($a_{iM} \neq 0$). After that, the balancing number M is obtained from the reduced equation (13) by using the homogeneous balance principle. When we substitute equation (14) into equation (13), and collect the coefficients of each power of $e^{\tau^{\alpha}}$, we obtain a polynomial of $e^{\tau^{\alpha}}$. Then equate all the coefficients of $e^{\tau^{\alpha}}$ to zero to obtain a system of algebraic equations. By solving this system, we find the values of $a_{ij}(j = 1, ..., M)$, l_i, and k_i. Therefore, by inserting the obtained values of a_{ij} into equation (14), we obtain a series of exact solutions for equation (5).

4 | ANALYTICAL TECHNIQUE FOR SOLVING NONLINEAR SYSTEM OF CONFORMABLE PDES

In this section, we introduce an approximate analytical method to solve a nonlinear system of variable time-fractional order partial differential equations of the following form:

$$\begin{cases} \mathcal{T}_t^{\alpha} u_j(x,t) = f_j(x,t) + L_j \bar{u}(x,t) + N_j \bar{u}(x,t),\ k-1 < \alpha \le k, \\ \dfrac{\partial^i u_j(x,t_0)}{\partial t^i} = f_j(x),\ t_0 \ge 0,\ i = 0,1,\dots,k-1,\ j = 1,2,\dots,m, \end{cases} \tag{15}$$

where $\bar{u}(x,t) = (u_1(x,t), u_2(x,t), \dots, u_n(x,t))$, and L_j, N_j are linear and nonlinear operators respectively of $\bar{u} = \bar{u}(x,t)$ and its derivatives which might include other fractional derivatives of orders other than α, and $f_j(x,t)$ and $f_j(x),\ j = 1,2,\dots,m$, are known analytic functions and $\mathcal{T}^{\alpha}$ is the conformable time fractional partial derivative of variable order $q(t)$. In the case of $f_j(x,t) = 0$, the system (15) becomes in the homogeneous form.

In order to solve system (15), we assume that the solution function $u_j(barx, t)$ can be written as $u_j(x,t) = f_j(x)g_j(t)$, where $f_j(x)$ and $g_j(t)$ are analytical functions.

Theorem 5. Let a function $\bar{u}(x,t)$ be $\bar{u}(x,t) : \mathbb{R} \times (0,\infty) \to \mathbb{R}$. For $\bar{u}(x,t) = \bar{f}(x)\bar{g}(t)$, with assumption that $\bar{f}(x)$ and $\bar{g}(t)$ are analytical functions, then the nonlinear operator $N_j\bar{u}(x,t)$ has the following conformable power series:

$$N_j \bar{u}(x,t) = \sum_{n=0}^{\infty} \frac{(t-t_0)^{n\alpha}}{\alpha^n \Gamma(n+1)} \mathcal{T}_t^{n\alpha} N_j(\bar{u}(x,t))|_{(x,t_0)}, \tag{16}$$

for $j = 1,2,\dots,m$.

Theorem 6. Let a function $\bar{u}(x,t)$ be $\bar{u}(x,t) : \mathbb{R} \times (0,\infty) \to \mathbb{R}$. For $\bar{u} = \sum_{k=0}^{\infty} \lambda^{k\alpha(x)} \bar{u}_k(x,t)$, the nonlinear operators $N_j\bar{u}(x,t)$ are satisfied the following property:

$$N_j \bar{u} = N_j \sum_{k=0}^{\infty} \lambda^{kq(t)} \bar{u}_k = \sum_{n=0}^{\infty} \left[\frac{1}{\alpha^n \Gamma(n+1)} \mathcal{T}_\lambda^{n\alpha} \left[N_j \sum_{k=0}^{n} \lambda^{k\alpha} \bar{u}_k \right]_{\lambda=0} \right] \lambda^{n\alpha}, \tag{17}$$

for $0 < q(t) < 1$ and $i = 1,2,\dots,m$.

Proof. The proof of Theorem 6 is similar to the proof of Theorem 3 in [37]. □

Definition 3. The polynomials $P_{in} = P_{in}(u_{i0}, u_{i1}, \dots, u_{in})$, for $i = 1, 2, \dots m$, are defined as

$$P_{in} = \frac{1}{\alpha^n \Gamma(n+1)} \mathcal{T}_\lambda^{n\alpha} \big[N_j \sum_{k=0}^{n} \lambda^{k\alpha} \bar{u}_k\big]_{\lambda=0}, \tag{18}$$

Remark 1. Let $P_{in} = P_{in}(u_{i0}, u_{i1}, \dots, u_{in})$, by using Theorem 5 and Definition 3, the nonlinear operators $N_j \bar{u}_\lambda$ can be expressed in terms of P_{in} as

$$N_j \bar{u} = \sum_{n=0}^{\infty} P_{in}, \ i = 1, 2, \dots, n. \tag{19}$$

Let the solutions series $u_j(\bar{x}, t)$ of the system (15) be as in the following form:

$$u_j(\bar{x}, t) = \sum_{r=0}^{\infty} u_{ir}(\bar{x}, t), \ i = 1, 2, \dots, m. \tag{20}$$

By an analytical technique, (see, [37]), the components of the series (20) can be obtained as follows:

$$\begin{cases} u_{i0}(x,t) = \displaystyle\sum_{j=0}^{k-1} \frac{t^j}{j!} f_{ij}(x), \\ u_{i1}(x,t) = f_{it}^{(-\alpha)}(x,t) + L_{it}^{(-\alpha)} \bar{u}_0 + P_{i0t}^{(-\alpha)}, \\ u_{ir}(x,t) = L_{it}^{(-\alpha)} \bar{u}_{(r-1)} + P_{i(r-1)t}^{(-\alpha)}, \end{cases} \tag{21}$$

for $r = 2, 3, \dots,$ $i = 1, 2, \dots, m$, where $f_{it}^{(-\alpha)}(x,t)$, $L_{it}^{(-\alpha)} = \mathcal{I}_t^{-\alpha} L_j \bar{u}$, and $P_{it}^{(-\alpha)} = \mathcal{I}_t^{(-\alpha)} P_i(u_{i0}, u_{i1}, \dots, u_{ir})$ denote the time conformable partial integral of order α for $f_j(x,t)$, $L_j \bar{u}$ and $P_i(u_{i0}, u_{i1}, \dots, u_{ir})$ respectively.

Theorem 7. Let $\mathcal{B}$ be a Banach space. Then the series solution given by (21) converges to $S_j \in \mathcal{B}$ for $j = 1, 2, \dots, m$, if there exists λ_i, $0 \le \lambda_i < 1$ such that, $\|u_{in}\| \le \lambda_i \|u_{i(n-1)}\|$ for $\forall n \in \mathbb{N}$.

Proof. See [37]. □

5 | ANALYTICAL SOLUTIONS OF COVID-19 PANDEMIC CONFORMABLE MODELS

5.1 | Exact solutions for a SIR model

In this section, we apply the advances we introduce in the previous section to obtain exact analytical solutions for the following conformable SIR model of dispersion using diffusion describing the COVID-19 pandemic:

$$\begin{cases} \mathcal{T}_t^\alpha s(x,t) = \phi_b n(x,t) + \beta s(x,t) i(x,t) - \mu s(x,t) + \nu_s \mathcal{T}_x^{2\alpha} s(x,t), \\ \mathcal{T}_t^\alpha i(x,t) = \beta s(x,t) i(x,t) - (\zeta + \mu) i(x,t) + \nu_i \mathcal{T}_x^{2\alpha} i(x,t), \\ \mathcal{T}_t^\alpha r(x,t) = \zeta i(x,t) - \mu r(x,t) + \nu_r \mathcal{T}_x^{2\alpha} r(x,t), \end{cases} \tag{22}$$

where ϕ_b is the birth rate, μ is the general (non-COVID-19) mortality rate, β is the infection rate, ζ is the recovery rate, and ν_s, ν_i and ν_r are diffusion parameters respectively corresponding to the different population groups. Here $s(x,t), e(x,t), i(x,t)$, and $r(x,t)$ denote the densities of the susceptible, infected, and recovered populations respectively, such that

$$n(x,t) = s(x,t) + i(x,t) + r(x,t), \tag{23}$$

where $n(x,t)$ denotes the sum of the host population. In this paper, we use the system (22) to illustrate the dynamics of the spatiotemporal dispersal of the COVID-19 epidemic. The model 22 is represented by the diagram in Figure 1 Further, we assume that $n(x,t) = 1$ and we consider the following reduced system:

$$\begin{cases} \mathcal{T}_t^\alpha s(x,t) = \beta s(x,t) i(x,t) - \mu s(x,t) + \nu_s \mathcal{T}_x^{2\alpha} s(x,t), \\ \mathcal{T}_t^\alpha i(x,t) = \beta s(x,t) i(x,t) - (\zeta + \mu) i(x,t) + \nu_i \mathcal{T}_x^{2\alpha} i(x,t). \end{cases} \tag{24}$$

The size of recovered host population $r(x,t)$ can be obtained using the solutions of system (24) and equation (23) with the assumption that $n(x,t) = 1$.

To solve the system (24) by using the fractional ERFM, we assume the following transformations

$$s(x,t)=s(\xi),\ i(x,t)=i(\xi),\ \xi^{\alpha}=e^{l_i x^{\alpha}+k_i t^{\alpha}},\ i=1,2. \tag{25}$$

From (25) with the help of 7 and 8, we obtain

$$\begin{cases}\mathcal{T}_t^{\alpha}s(x,t)=k_1\xi s'(\xi),\ \mathcal{T}_x^{2\alpha}s(x,t)=l_1^2\xi s'(\xi)+l_1^2\xi^2 s''(\xi),\\ \mathcal{T}_t^{\alpha}i(x,t)=k_2\xi i'(\xi),\ \mathcal{T}_x^{2\alpha}i(x,t)=l_2^2\xi i'(\xi)+l_2^2\xi^2 i''(\xi).\end{cases} \tag{26}$$

By substituting systems (25) and (26) into system (22), we obtain

$$\begin{cases}k_1\xi s'(\xi)=\beta s(\xi)i(\xi)-\mu s(\xi)+v_s(l_1^2\xi s'(\xi)+l_1^2\xi^2 s''(\xi)),\\ k_2\xi i'(\xi)=\beta s(\xi)i(\xi)-(\zeta+\mu)i(\xi)+v_i(l_2^2\xi i'(\xi)+l_2^2\xi^2 i''(\xi)),\end{cases} \tag{27}$$

By doing some simplifications, system (27) can be rewritten as

$$\begin{cases}(k_1-v_s l_1^2)\xi s'(\xi)=(\beta i(\xi)-\mu)s(\xi)+v_s l_1^2\xi^2 s''(\xi),\\ (k_2-v_i l_2^2)\xi i'(\xi)=(\beta s(\xi)-\zeta-\mu)i(\xi)-v_i l_2^2\xi^2 i''(\xi),\end{cases} \tag{28}$$

By equating the highest order derivatives in system (28) to the highest order of the nonlinear terms, we find the balancing numbers as $m_1=m_2=m_3=2$. Therefore, we assume that the system (28) has solutions of the form

$$\begin{cases}s(\xi)=a_0+\dfrac{a_1}{1+\xi^{\alpha}}+\dfrac{a_2}{(1+\xi^{\alpha})^2},\ \xi^{\alpha}=e^{l_1x^{\alpha}+k_1t^{\alpha}},\ l_1,k_1\neq 0,\\ i(\xi)=b_0+\dfrac{b_1}{1+\xi^{\alpha}}+\dfrac{b_2}{(1+\xi^{\alpha})^2},\ \xi^{\alpha}=e^{l_2x^{\alpha}+k_2t^{\alpha}},\ l_2,k_2\neq 0.\end{cases} \tag{29}$$

Next, we substitute equation (29) into equation (28) and equating the coefficients with identical powers of ξ^{α} to zero, we obtain the following systems of algebraic equations

$$\begin{cases}\xi^{4\alpha}\ :\ a_0\beta b_0-a_0\mu,\\ \xi^{3\alpha}\ :\ 4a_0\beta b_0+a_1\beta b_0+a_0\beta b_1+\alpha a_1k_1+\alpha^2a_1l_1^2v_s-4a_0\mu-a_1\mu,\\ \xi^{2\alpha}\ :\ 6a_0\beta b_0+3a_1\beta b_0+a_2\beta b_0+3a_0\beta b_1+a_1\beta b_1+a_0\beta b_2+2\alpha a_1k_1+2\alpha a_2k_1+4\alpha^2a_2l_1^2v_s-6a_0\mu\\ \qquad -3a_1\mu-a_2\mu,\\ \xi^{\alpha}\ :\ 4a_0\beta b_0+3a_1\beta b_0+2a_2\beta b_0+3a_0\beta b_1+2a_1\beta b_1+a_2\beta b_1+2a_0\beta b_2+a_1\beta b_2+\alpha a_1k_1+2\alpha a_2k_1\\ \qquad -\alpha^2a_1l_1^2v_s-2\alpha^2a_2l_1^2v_s-4a_0\mu-3a_1\mu-2a_2\mu,\\ \xi^{0\alpha}\ :\ a_0\beta b_0+a_0\beta b_1+a_0\beta b_2+a_1\beta b_0+a_2\beta b_0+a_1\beta b_1+a_2\beta b_1+a_1\beta b_2+a_2\beta b_2+a_0(-\mu)\\ \qquad -a_1\mu-a_2\mu,\end{cases} \tag{30}$$

and

$$\begin{cases}\xi^{4\alpha}\ :\ a_0\beta b_0+b_0(-\zeta)-b_0\mu,\\ \xi^{3\alpha}\ :\ 4a_0\beta b_0+a_1\beta b_0+a_0\beta b_1-4b_0\zeta-b_1\zeta+\alpha b_1k_2+\alpha^2b_1v_il_2^2-4b_0\mu-b_1\mu,\\ \xi^{2\alpha}\ :\ 6a_0\beta b_0+3a_1\beta b_0+a_2\beta b_0+3a_0\beta b_1+a_1\beta b_1+a_0\beta b_2-6b_0\zeta-3b_1\zeta-b_2\zeta+2\alpha b_1k_2+2\alpha b_2k_2\\ \qquad +4\alpha^2b_2v_il_2^2-6b_0\mu-3b_1\mu-b_2\mu,\\ \xi^{\alpha}\ :\ 4a_0\beta b_0+3a_1\beta b_0+2a_2\beta b_0+3a_0\beta b_1+2a_1\beta b_1+a_2\beta b_1+2a_0\beta b_2+a_1\beta b_2-4b_0\zeta-3b_1\zeta\\ \qquad -2b_2\zeta+\alpha b_1k_2+2\alpha b_2k_2-\alpha^2b_1v_il_2^2-2\alpha^2b_2v_il_2^2-4b_0\mu-3b_1\mu-2b_2\mu,\\ \xi^{0\alpha}\ :\ a_0\beta b_0+a_1\beta b_0+a_2\beta b_0+a_0\beta b_1+a_1\beta b_1+a_2\beta b_1+a_0\beta b_2+a_1\beta b_2+a_2\beta b_2+b_0(-\zeta)-b_1\zeta\\ \qquad -b_2\zeta-b_0\mu-b_1\mu-b_2\mu,\end{cases} \tag{31}$$

Next, we solve systems (30) and (31) of algebraic equations in $\mathbb{R}$ with the help of Mathematica and Maple, we have the following two cases:

Case 1:

$$\begin{cases}a_0=\dfrac{\zeta+\mu}{\beta},a_1=0,a_2=-\dfrac{\zeta+\mu}{\beta},b_0=\dfrac{\mu}{\beta},b_1=0,b_2=-\dfrac{\mu}{\beta},\\ k_1=-\dfrac{5\mu}{6\alpha},k_2=-\dfrac{5(\zeta+\mu)}{6\alpha},\ l_1=\pm\dfrac{\sqrt{\mu}}{\alpha\sqrt{6v_s}},\ l_2=\pm\dfrac{\sqrt{\zeta+\mu}}{\alpha\sqrt{6v_i}}.\end{cases} \tag{32}$$

By using systems (32), (29), (25), and (23), we obtain the following exact solutions of (22):

$$\begin{cases} s_1(x,t) = \dfrac{\zeta+\mu}{\beta} - \dfrac{\frac{\zeta+\mu}{\beta}}{\left(1+e^{\pm\frac{\sqrt{6\mu}}{6\alpha\sqrt{v_s}}x^\alpha - 5/6\,\frac{\mu}{\alpha}t^\alpha}\right)^2}, \\ i_1(x,t) = \dfrac{\mu}{\beta} - \dfrac{\frac{\mu}{\beta}}{\left(1+e^{\pm\frac{\sqrt{\zeta+\mu}}{\alpha\sqrt{6v_i}}x^\alpha - 5/6\,\frac{\zeta+\mu}{\alpha}t^\alpha}\right)^2}, \\ r_1(x,t) = 1 - \dfrac{\zeta+2\mu}{\beta} - \dfrac{\frac{\zeta+\mu}{\beta}}{\left(1+e^{\pm\frac{\sqrt{6\mu}}{6\alpha\sqrt{v_s}}x^\alpha - 5/6\,\frac{\mu}{\alpha}t^\alpha}\right)^2} - \dfrac{\frac{\mu}{\beta}}{\left(1+e^{\pm\frac{\sqrt{\zeta+\mu}}{\alpha\sqrt{6v_i}}x^\alpha - 5/6\,\frac{\zeta+\mu}{\alpha}t^\alpha}\right)^2}. \end{cases} \tag{33}$$

Case 2:

$$\begin{cases} a_0 = 0, a_1 = \dfrac{2(\zeta+\mu)}{\beta}, a_2 = -\dfrac{\zeta+\mu}{\beta}, b_0 = 0, b_1 = \dfrac{2\mu}{\beta}, b_2 = -\dfrac{\mu}{\beta}, \\ k_1 = \dfrac{5\mu}{6\alpha}, k_2 = \dfrac{5(\zeta+\mu)}{6\alpha}, l_1 = \pm\dfrac{\sqrt{6\mu}}{6\alpha\sqrt{v_s}}, l_2 = \pm\dfrac{\sqrt{\zeta+\mu}}{\alpha\sqrt{6v_i}}. \end{cases} \tag{34}$$

By using the systems (34), (29), and (25), we obtain the following exact traveling solutions of (22):

$$\begin{cases} s_2(x,t) = \dfrac{2(\zeta+\mu)}{\beta\left(e^{\frac{(5\mu)t^\alpha}{6\alpha}\pm\frac{\sqrt{6\mu}x^\alpha}{6\alpha\sqrt{v_s}}}+1\right)} - \dfrac{\zeta+\mu}{\beta\left(e^{\frac{(5\mu)t^\alpha}{6\alpha}\pm\frac{\sqrt{6\mu}x^\alpha}{6\alpha\sqrt{v_s}}}+1\right)^2}, \\ i_2(x,t) = \dfrac{2\mu}{\beta\left(e^{\frac{(5(\zeta+\mu))t^\alpha}{6\alpha}\pm\frac{\sqrt{\zeta+\mu}x^\alpha}{\alpha\sqrt{6v_i}}}+1\right)} - \dfrac{\mu}{\beta\left(e^{\frac{(5(\zeta+\mu))t^\alpha}{6\alpha}\pm\frac{\sqrt{\zeta+\mu}x^\alpha}{\alpha\sqrt{6v_i}}}+1\right)^2}, \\ r_2(x,t) = 1 - \dfrac{2(\zeta+\mu)}{\beta\left(e^{\frac{(5\mu)t^\alpha}{6\alpha}\pm\frac{\sqrt{6\mu}x^\alpha}{6\alpha\sqrt{v_s}}}+1\right)} + \dfrac{\zeta+\mu}{\beta\left(e^{\frac{(5\mu)t^\alpha}{6\alpha}\pm\frac{\sqrt{6\mu}x^\alpha}{6\alpha\sqrt{v_s}}}+1\right)^2} \\ \qquad - \dfrac{2\mu}{\beta\left(e^{\frac{(5(\zeta+\mu))t^\alpha}{6\alpha}\pm\frac{\sqrt{\zeta+\mu}x^\alpha}{\alpha\sqrt{6v_i}}}+1\right)} + \dfrac{\mu}{\beta\left(e^{\frac{(5(\zeta+\mu))t^\alpha}{6\alpha}\pm\frac{\sqrt{\zeta+\mu}x^\alpha}{\alpha\sqrt{6v_i}}}+1\right)^2}. \end{cases} \tag{35}$$

Next, we solve the system (24) by using the inverse conformable ERFM, we assume the following

$$s(x,t) = s(\tau),\ i(x,t) = i(\tau),\ \tau^\alpha = l_i x^\alpha + k_i t^\alpha,\ k_i, l_i \neq 0,\ i = 1,2. \tag{36}$$

By using the conformable derivatives in (36), we have

$$\begin{cases} \mathcal{T}_t^\alpha s(x,t) = k_1 \mathcal{T}^\alpha s(\tau),\ \mathcal{T}_x^{2\alpha} s(x,t) = l_1^2 \mathcal{T}^{2\alpha} s(\tau), \\ \mathcal{T}_t^\alpha i(x,t) = k_2 \mathcal{T}^\alpha s(\tau),\ \mathcal{T}_x^{2\alpha} i(x,t) = l_2^2 \mathcal{T}^{2\alpha} i(\tau). \end{cases} \tag{37}$$

By using (37) into (22), we obtain the following system

$$\begin{cases} k_1 \mathcal{T}^\alpha s(\tau) = \beta s(\tau) i(\tau) - \mu s(\tau) + v_s l_1^2 \mathcal{T}^{2\alpha} s(\tau), \\ k_2 \mathcal{T}^\alpha i(\tau) = \beta s(\tau) i(\tau) - (\zeta+\mu) i(\tau) + v_i l_2^2 \mathcal{T}^{2\alpha} i(\tau). \end{cases} \tag{38}$$

By equating the highest order derivatives in system (28) to the highest order of the nonlinear terms, we find the balancing numbers as $m_1 = m_2 = m_3 = 2$. Therefore, we assume that the system (28) has solutions of the form

$$\begin{cases} s(\tau) = a_0 + \dfrac{a_1}{1+e^{\tau^\alpha}} + \dfrac{a_2}{(1+e^{\tau^\alpha})^2},\ \tau^\alpha = l_1 x^\alpha + k_1 t^\alpha,\ l_1, k_1 \neq 0, \\ i(\tau) = b_0 + \dfrac{b_1}{1+e^{\tau^\alpha}} + \dfrac{b_2}{(1+e^{\tau^\alpha})^2},\ \tau^\alpha = l_2 x^\alpha + k_2 t^\alpha,\ l_2, k_2 \neq 0. \end{cases} \tag{39}$$

Further, we substitute system (39) into system (38) and collecting the coefficients of each power of e^{τ^α} and set them to zero, we obtain a system of algebraic equations

$$\begin{cases} e^{4\tau^\alpha} : a_0\beta b_0 - a_0\mu, \\ e^{3\tau^\alpha} : 4a_0\beta b_0 + a_1\beta b_0 + a_0\beta b_1 + \alpha a_1 k_1 + \alpha^2 a_1 l_1^2 v_s - 4a_0\mu - a_1\mu, \\ e^{2\tau^\alpha} : 6a_0\beta b_0 + 3a_1\beta b_0 + a_2\beta b_0 + 3a_0\beta b_1 + a_1\beta b_1 + a_0\beta b_2 + 2\alpha a_1 k_1 + 2\alpha a_2 k_1 + 4\alpha^2 a_2 l_1^2 v_s \\ \qquad - 6a_0\mu - 3a_1\mu - a_2\mu, \\ e^{\tau^\alpha} : 4a_0\beta b_0 + 3a_1\beta b_0 + 2a_2\beta b_0 + 3a_0\beta b_1 + 2a_1\beta b_1 + a_2\beta b_1 + 2a_0\beta b_2 + a_1\beta b_2 + \alpha a_1 k_1 + 2\alpha a_2 k_1 \\ \qquad - \alpha^2 a_1 l_1^2 v_s - 2\alpha^2 a_2 l_1^2 v_s - 4a_0\mu - 3a_1\mu - 2a_2\mu, \\ e^{0\tau^\alpha} : a_0\beta b_0 + a_0\beta b_1 + a_0\beta b_2 + a_1\beta b_0 + a_2\beta b_0 + a_1\beta b_1 + a_2\beta b_1 + a_1\beta b_2 + a_2\beta b_2 + a_0(-\mu) \\ \qquad - a_1\mu - a_2\mu, \end{cases} \tag{40}$$

and

$$\begin{cases} e^{4\tau^\alpha} : a_0\beta b_0 - b_0\zeta - b_0\mu, \\ e^{3\tau^\alpha} : 4a_0\beta b_0 + a_1\beta b_0 + a_0\beta b_1 - 4b_0\zeta - b_1\zeta + \alpha b_1 k_2 + \alpha^2 b_1 l_2^2 v_i - 4b_0\mu - b_1\mu, \\ e^{2\tau^\alpha} : 6a_0\beta b_0 + 3a_1\beta b_0 + a_2\beta b_0 + 3a_0\beta b_1 + a_1\beta b_1 + a_0\beta b_2 - 6b_0\zeta - 3b_1\zeta - b_2\zeta + 2\alpha b_1 k_2 + 2\alpha b_2 k_2 \\ \qquad + 4\alpha^2 b_2 l_2^2 v_i - 6b_0\mu - 3b_1\mu - b_2\mu, \\ e^{\tau^\alpha} : 4a_0\beta b_0 + 3a_1\beta b_0 + 2a_2\beta b_0 + 3a_0\beta b_1 + 2a_1\beta b_1 + a_2\beta b_1 + 2a_0\beta b_2 + a_1\beta b_2 - 4b_0\zeta - 3b_1\zeta - 2b_2\zeta \\ \qquad + \alpha b_1 k_2 + 2\alpha b_2 k_2 - \alpha^2 b_1 l_2^2 v_i - 2\alpha^2 b_2 l_2^2 v_i - 4b_0\mu - 3b_1\mu - 2b_2\mu, \\ e^{0\tau^\alpha} : a_0\beta b_0 + a_1\beta b_0 + a_2\beta b_0 + a_0\beta b_1 + a_1\beta b_1 + a_2\beta b_1 + a_0\beta b_2 + a_1\beta b_2 + a_2\beta b_2 + b_0(-\zeta) - b_1\zeta \\ \qquad - b_2\zeta - b_0\mu - b_1\mu - b_2\mu. \end{cases} \tag{41}$$

Next, we solve systems (40) and (41) of algebraic equations in $\mathbb{R}$ with the help of Mathematica and Maple, we obtain the following two cases:

Case 1:

$$\begin{cases} a_0 = 0, a_1 = \dfrac{2(\zeta+\mu)}{\beta},\ a_2 = -\dfrac{\zeta+\mu}{\beta},\ b_0 = 0, b_1 = \dfrac{2\mu}{\beta},\ b_2 = -\dfrac{\mu}{\beta}, \\ k_1 = \dfrac{5\mu}{6\alpha},\ k_2 = \dfrac{5(\zeta+\mu)}{6\alpha},\ l_1 = \pm\dfrac{\sqrt{6\mu}}{6\alpha\sqrt{v_s}}, l_2 = \pm\dfrac{\sqrt{\zeta+\mu}}{\alpha\sqrt{6v_i}}. \end{cases} \tag{42}$$

By using the systems (42), (39), (36), and (23), we obtain the following exact solutions of (22):

$$\begin{aligned} s_3(x,t) &= \frac{2(\zeta+\mu)}{\beta\left(e^{\frac{(5\mu)t^\alpha}{6\alpha} \pm \frac{\sqrt{6\mu}x^\alpha}{6\alpha\sqrt{v_s}}} + 1\right)} - \frac{\zeta+\mu}{\beta\left(e^{\frac{(5\mu)t^\alpha}{6\alpha} \pm \frac{\sqrt{6\mu}x^\alpha}{6\alpha\sqrt{v_s}}} + 1\right)^2}, \\ i_3(x,t) &= \frac{2\mu}{\beta\left(e^{\frac{(5(\zeta+\mu))t^\alpha}{6\alpha} \pm \frac{\sqrt{\zeta+\mu}x^\alpha}{\alpha\sqrt{6v_i}}} + 1\right)} - \frac{\mu}{\beta\left(e^{\frac{(5(\zeta+\mu))t^\alpha}{6\alpha} \pm \frac{\sqrt{\zeta+\mu}x^\alpha}{\alpha\sqrt{6v_i}}} + 1\right)^2}, \\ r_3(x,t) &= 1 - \frac{2(\zeta+\mu)}{\beta\left(e^{\frac{(5\mu)t^\alpha}{6\alpha} \pm \frac{\sqrt{6\mu}x^\alpha}{6\alpha\sqrt{v_s}}} + 1\right)} + \frac{\zeta+\mu}{\beta\left(e^{\frac{(5\mu)t^\alpha}{6\alpha} \pm \frac{\sqrt{6\mu}x^\alpha}{6\alpha\sqrt{v_s}}} + 1\right)^2} \\ &\quad - \frac{2\mu}{\beta\left(e^{\frac{(5(\zeta+\mu))t^\alpha}{6\alpha} \pm \frac{\sqrt{\zeta+\mu}x^\alpha}{\alpha\sqrt{6v_i}}} + 1\right)} + \frac{\mu}{\beta\left(e^{\left(\frac{(5(\zeta+\mu))t^\alpha}{6\alpha} \pm \frac{\sqrt{\zeta+\mu}x^\alpha}{\alpha\sqrt{6v_i}}\right)} + 1\right)^2}. \end{aligned} \tag{43}$$

Case 2:

$$\begin{cases} a_0 = \dfrac{\zeta+\mu}{\beta}, a_1 = 0, a_2 = -\dfrac{\zeta+\mu}{\beta}, b_0 = \dfrac{\mu}{\beta}, b_1 = 0, b_2 = -\dfrac{\mu}{\beta}, \\ k_1 = -\dfrac{5\mu}{6\alpha}, k_2 = -\dfrac{5(\zeta+\mu)}{6\alpha},\ l_1 = \pm\dfrac{\sqrt{\mu}}{\alpha\sqrt{6v_s}},\ l_2 = \pm\dfrac{\sqrt{\zeta+\mu}}{\alpha\sqrt{6v_i}}. \end{cases} \tag{44}$$

By using the systems (44), (39), and (36), we obtain the following exact traveling solutions of (22)

$$\begin{cases} s_4(x,t) = \dfrac{\zeta+\mu}{\beta} - \dfrac{\frac{\zeta+\mu}{\beta}}{(1+e^{\pm\frac{\sqrt{6\mu}}{6\alpha\sqrt{v_s}}x^\alpha+5/6\,\frac{\mu}{\alpha}t^\alpha})^2}, \\ i_4(x,t) = \dfrac{\mu}{\beta} - \dfrac{\frac{\mu}{\beta}}{(1+e^{\pm\frac{\sqrt{\zeta+\mu}}{\alpha\sqrt{6v_i}}x^\alpha+5/6\,\frac{\zeta+\mu}{\alpha}t^\alpha})^2}, \\ r_4(x,t) = 1 - \dfrac{\zeta+2\mu}{\beta} - \dfrac{\frac{\zeta+\mu}{\beta}}{(1+e^{\pm\frac{\sqrt{6\mu}}{6\alpha\sqrt{v_s}}x^\alpha+5/6\,\frac{\mu}{\alpha}t^\alpha})^2} - \dfrac{\frac{\mu}{\beta}}{(1+e^{\pm\frac{\sqrt{\zeta+\mu}}{\alpha\sqrt{6v_i}}x^\alpha+5/6\,\frac{\zeta+\mu}{\alpha}t^\alpha})^2}. \end{cases} \tag{45}$$

However, the exact solutions (43) and (45), obtained by the fractional ERFM are the same as the exact solutions (33) and (35) obtained by the inverse fractional ERFM.

5.2 | Approximate solutions for a SEIRD model

in this section, we present approximate analytical solutions for a SEIRD model with conformable space-time derivatives of the following form:

$$\begin{cases} \mathcal{T}_t^\alpha s(x,t) = \phi_b n(x,t) - \mu s(x,t) - \beta s(x,t)i(x,t) - \beta s(x,t)e(x,t) + {}^c\nabla_x^\alpha \cdot \left(n v_s {}^c\nabla_x^\alpha s\right), \\ \mathcal{T}_t^\alpha e(x,t) = (-\rho-\epsilon-\mu)e(x,t) + \beta s(x,t)i(x,t) + \beta s(x,t)e(x,t) + {}^c\nabla_x^\alpha \cdot \left(n v_e {}^c\nabla_x^\alpha e\right), \\ \mathcal{T}_t^\alpha i(x,t) = \rho e(x,t) + \beta s(x,t)i(x,t) + (-\eta-\zeta-\mu)i(x,t) + {}^c\nabla_x^\alpha \cdot \left(n v_i {}^c\nabla_x^\alpha i\right), \\ \mathcal{T}_t^\alpha r(x,t) = \zeta i(x,t) - \epsilon e(x,t) - \mu r(x,t) + {}^c\nabla_x^\alpha \cdot \left(n v_r {}^c\nabla_x^\alpha r\right), \ \mathcal{T}_t^\alpha d(x,t) = \eta i(x,t), \end{cases} \tag{46}$$

where $s(x,t), e(x,t), i(x,t), r(x,t)$, and $d(x,t)$ refer to the densities of the susceptible, exposed, infected, recovered, and deceased populations respectively, $\mathcal{T}_t^\alpha$ is a conformable derivative of order $0<\alpha<1$ for t, and $(x,t)\in\Omega\times[0,T]$, $\Omega\subset\mathbb{R}^m$, $n(x,t)>0, {}^c\nabla_x^\alpha = \sum_j^m x_j^{1-\alpha}\frac{\partial^j}{\partial x^j}$ and ϕ_b is the birth rate, μ is the general (non-COVID-19) mortality rate, β is the infection rate, ϵ is the asymptomatic recovery, ρ is the inverse of the incubation period, ζ is the infected recovery rate, η is the deceased mortality rate, and v_s, v_e, v_i and v_r are diffusion parameters respectively corresponding to the different population groups. The schematic diagram of the model (46) is presented in Figure 2 We consider $m=1$, and with assumption of $n(x,t) = s(x,t) + i(x,t) + r(x,t) + d(x,t)$. By comparing (46) with (15) for $i=j,2,3,4,5$, we have

$$\begin{cases} \mathcal{T}_t^\alpha s(x,t) = f_1(x,t) + L_1(s,e,i,r,d) + N_1(s,e,i.r,d), \\ \mathcal{T}_t^\alpha e(x,t) = L_2(s,e,i,r,d) + N_2(s,e,i,r,d), \\ \mathcal{T}_t^\alpha i(x,t) = L_3(s,e,i,r,d) + N_3(s,e,i,r,d), \\ \mathcal{T}_t^\alpha r(x,t) = L_4(s,e,i,r,d) + N_4,(s,e,i,r,d), \\ \mathcal{T}_t^\alpha d(x,t) = L_5(s,e,i,r,d) \end{cases} \tag{47}$$

subject to the following initial conditions given in (46) where we assume that

$$\begin{cases} f_1(x,t),\ f_2(x,t),\ldots,f_5(x,t) = 0,\ L_1(s,e,i,r,d) = \phi_b n(x,t) - \mu s(x,t), \\ L_2(s,e,i,r,d) = (-\rho-\epsilon-\mu)e(x,t),\ L_3(s,e,i,r,d) = \rho e(x,t) + (-\eta-\zeta-\mu)i(x,t), \\ L_4(s,e,i,r,d) = \zeta i(x,t) - \epsilon e(x,t) - \mu r(x,t),\ L_5(s,e,i,r,d) = \eta i(x,t), \\ N_1(s,e,i,r,d) = \beta s(x,t)i(x,t) - \beta s(x,t)e(x,t) + {}^c\nabla_x^\alpha \cdot \left(n v_s {}^c\nabla_x^\alpha s\right), \\ N_2(s,e,i,r,d) = \beta s(x,t)i(x,t) + \beta s(x,t)e(x,t) + {}^c\nabla_x^\alpha \cdot \left(n v_e {}^c\nabla_x^\alpha e\right), \\ N_3(s,e,i,r,d) = \beta s(x,t)i(x,t) + {}^c\nabla_x^\alpha \cdot \left(n v_i {}^c\nabla_x^\alpha i\right), \\ N_4(s,e,i,r,d) = {}^c\nabla_x^\alpha \cdot \left(n v_r {}^c\nabla_x^\alpha r\right),\ N_5(s,e,i,r,d) = 0. \end{cases} \tag{48}$$

To solve the system (46), we assume that the solutions of (46) have the following forms:

$$\begin{cases} s(x,t) = \sum_{r=0}^{\infty} s_r(x,t),\ e(x,t) = \sum_{r=0}^{\infty} e_r(x,t),\ i(x,t) = \sum_{r=0}^{\infty} i_r(x,t), \\ r(x,t) = \sum_{r=0}^{\infty} r_r(x,t),\ d(x,t) = \sum_{r=0}^{\infty} d_r(x,t). \end{cases} \tag{49}$$

Thus from the system (21), we obtain

$$\begin{cases} s_0(x,t) = \sum_{j=0}^{k-1} \frac{t^j}{j!} s_j(x,0),\ e_0(x,t) = \sum_{j=0}^{k-1} \frac{t^j}{j!} e_j(x,0),\ i_0(x,t) = \sum_{j=0}^{k-1} \frac{t^j}{j!} i_j(x,0), \\ r_0(x,t) = \sum_{j=0}^{k-1} \frac{t^j}{j!} r_j(x,0),\ d_0(x,t) = \sum_{j=0}^{k-1} \frac{t^j}{j!} d_j(x,0), \end{cases} \tag{50}$$

and

$$\begin{cases} s_1(x,t) = f_1(x,t) + L_{1t}^{(-\alpha)} + P_{1t}^{(-\alpha)}(s_0,e_0,i_0,r_0,d_0),\ e_1(x,t) = f_2(x,t) + L_{2t}^{(-\alpha)} + P_{2t}^{(-\alpha)}(s_0,e_0,i_0,r_0,d_0), \\ i_1(x,t) = f_3(x,t) + L_{3t}^{(-\alpha)} + P_{3t}^{(-\alpha)}(s_0,e_0,i_0,r_0,d_0),\ r_1(x,t) = f_4(x,t) + L_{4t}^{(-\alpha)} + P_{4t}^{(-\alpha)}(s_0,e_0,i_0,r_0,d_0), \\ d_1(x,t) = f_5(x,t) + L_{5t}^{(-\alpha)}(s_0,e_0,i_0,r_0,d_0), \end{cases} \tag{51}$$

and

$$\begin{cases} s_{r'}(x,t) = L_{1t}^{(-\alpha)}(s_{r'-1},e_{r'-1},i_{r'-1},r_{r'-1},d_{r'-1}) + P_{1t}^{(-\alpha)}(\sum_{l=0}^{r'-1} s_l, \sum_{l=0}^{r'-1} e_l, \sum_{l=0}^{r'-1} i_l, \sum_{l=0}^{r'-1} r_l, \sum_{l=0}^{r'-1} d_l), \\ e_{r'}(x,t) = L_{2t}^{(-\alpha)}(s_{r'-1},e_{r'-1},i_{r'-1},r_{r'-1},d_{r'-1}) + P_{2t}^{(-\alpha)}(\sum_{l=0}^{r'-1} s_l, \sum_{l=0}^{r'-1} e_l, \sum_{l=0}^{r'-1} i_l, \sum_{l=0}^{r'-1} r_l, \sum_{l=0}^{r'-1} d_l), \\ i_{r'}(x,t) = L_{3t}^{(-\alpha)}(s_{r'-1},e_{r'-1},i_{r'-1},r_{r'-1},d_{r'-1}) + P_{3t}^{(-\alpha)}(\sum_{l=0}^{r'-1} s_l, \sum_{l=0}^{r'-1} e_l, \sum_{l=0}^{r'-1} i_l, \sum_{l=0}^{r'-1} r_l, \sum_{l=0}^{r'-1} d_l), \\ r_{r'}(x,t) = L_{4t}^{(-\alpha)}(s_{r'-1},e_{r'-1},i_{r'-1},r_{r'-1},d_{r'-1}) + P_{4t}^{(-\alpha)}(\sum_{l=0}^{r'-1} s_l, \sum_{l=0}^{r'-1} e_l, \sum_{l=0}^{r'-1} i_l, \sum_{l=0}^{r'-1} r_l, \sum_{l=0}^{r'-1} d_l), \\ d_{r'}(x,t) = L_{5t}^{(-\alpha)}(s_{r'-1},e_{r'-1},i_{r'-1},r_{r'-1},d_{r'-1}), \end{cases} \tag{52}$$

where $P_{jt}^{(-\alpha)} = \mathcal{I}_t^{(-\alpha)} P_j(s_j, e_j, i_j)$, $j = 0, 1, \ , 5$ are obtained by using Definition 3 and Remark 1.
By evaluating the components of (50)-(52) and substituting the results along with the values from (53) and (48) into (49), we obtain the approximate analytical solutions for the model (22). In particular, we study the model (22) with the initial values

$$\begin{cases} s(x,0) = 1 + \beta x^\alpha/\alpha - e^{-\beta x^\alpha/\alpha},\ r(x,0) = d(x,0) = 0, \\ e(x,0) = i(x,0) = -\beta x^\alpha/\alpha + e^{-\beta x^\alpha/\alpha}, \end{cases} \tag{53}$$

for $0 < \alpha < 1$.
Therefore, from the systems (50) and (53), we obtain and

$$\begin{aligned} s_1(x,t) = {} & \frac{\phi_b t^\alpha}{\alpha} - \frac{\beta^2 v_s t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} - \frac{\mu t^\alpha}{\alpha} + \frac{2\beta^3 t^\alpha x^{2\alpha}}{\alpha^3} + \frac{2\beta^2 t^\alpha x^\alpha}{\alpha^2} - \frac{4\beta^2 t^\alpha x^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha^2} \\ & - \frac{\beta\mu t^\alpha x^\alpha}{\alpha^2} + \frac{\mu t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} + \frac{2\beta t^\alpha e^{-\frac{2\beta x^\alpha}{\alpha}}}{\alpha} - \frac{2\beta t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha}, \\ e_1(x,t) = {} & \alpha\beta^2 v_e t^\alpha e^{\beta(-x^\alpha)} - \frac{2\beta^3 t^\alpha x^{2\alpha}}{\alpha} + \frac{\beta\mu t^\alpha x^\alpha}{\alpha^2} + \frac{\beta\rho t^\alpha x^\alpha}{\alpha^2} + \frac{\beta\epsilon t^\alpha x^\alpha}{\alpha^2} + \frac{4\beta^2 t^\alpha x^\alpha e^{\beta(-x^\alpha)}}{\alpha} \\ & - \frac{2\beta^2 t^\alpha x^\alpha}{\alpha} - \frac{\mu t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} - \frac{\rho t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} - \frac{\epsilon t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} + \frac{2\beta t^\alpha e^{\beta(-x^\alpha)}}{\alpha} - \frac{2\beta t^\alpha e^{-2\beta x^\alpha}}{\alpha}, \\ i_1(x,t) = {} & \alpha\beta^2 v_i t^\alpha e^{\beta(-x^\alpha)} - \frac{\beta^3 t^\alpha x^{2\alpha}}{\alpha} + \frac{\beta\zeta t^\alpha x^\alpha}{\alpha^2} + \frac{\beta\eta t^\alpha x^\alpha}{\alpha^2} + \frac{\beta\mu t^\alpha x^\alpha}{\alpha^2} - \frac{\beta\rho t^\alpha x^\alpha}{\alpha^2} + \frac{2\beta^2 t^\alpha x^\alpha e^{\beta(-x^\alpha)}}{\alpha} \end{aligned}$$

$$
\begin{aligned}
&-\frac{\beta^2 t^\alpha x^\alpha}{\alpha} - \frac{\zeta t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} - \frac{\eta t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} - \frac{\mu t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} + \frac{\rho t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} + \frac{\beta t^\alpha e^{\beta(-x^\alpha)}}{\alpha} - \frac{\beta t^\alpha e^{-2\beta x^\alpha}}{\alpha},\\
r_1(x,t) &= -\frac{\beta\zeta t^\alpha x^\alpha}{\alpha^2} + \frac{\beta\epsilon t^\alpha x^\alpha}{\alpha^2} + \frac{\zeta t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} - \frac{\epsilon t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha},\\
d_1(x,t) &= \frac{\eta t^\alpha e^{-\frac{\beta x^\alpha}{\alpha}}}{\alpha} - \frac{\beta\eta t^\alpha x^\alpha}{\alpha^2},\\
s_2(x,t) &= \frac{t^{2\alpha}}{2\alpha^5}\Bigg(e^{-\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Bigg(\alpha^2\Big(-e^{\beta x^\alpha}\Big)\Bigg(\phi_b\Big(2\alpha\beta - 2\beta^2 x^\alpha e^{\frac{\beta x^\alpha}{\alpha}} + \alpha\mu e^{\frac{\beta x^\alpha}{\alpha}}\Big)\\
&\quad + \alpha\beta^4 \nu_s^2 + 2\beta^2\nu_s\Bigg(3\beta^2 x^\alpha - \alpha\Bigg(3\beta + \mu - 3\beta\sinh\left(\frac{\beta x^\alpha}{\alpha}\right) + 7\beta\cosh\left(\frac{\beta x^\alpha}{\alpha}\right)\Bigg)\Bigg)\Bigg)\\
&\quad - \alpha^4\beta^3\nu_i\Big(\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}} + \alpha\Big(e^{\frac{\beta x^\alpha}{\alpha}} - 1\Big)\Big)\Bigg) + \alpha^4\beta^3\nu_e e^{-\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha + \beta x^\alpha\Big(-e^{\frac{\beta x^\alpha}{\alpha}}\Big) - \alpha e^{\frac{\beta x^\alpha}{\alpha}}\Big)\\
&\quad + e^{-\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\Big(\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}} + \alpha\Big(e^{\frac{\beta x^\alpha}{\alpha}} - 1\Big)\Big)\Big(\alpha^2\Big(\beta^2\Big(-3e^{\beta x^\alpha} + 4e^{\frac{2(\alpha-1)\beta x^\alpha}{\alpha}} + 3\Big) + \mu^2 e^{2\beta x^\alpha}\\
&\quad + \beta e^{\frac{(2\alpha-1)\beta x^\alpha}{\alpha}}(\zeta + \eta + 6\mu + \epsilon)\Big) - \alpha\beta^2 x^\alpha\Big(e^{2\beta x^\alpha}(-3\alpha\beta + \zeta + \eta + 6\mu + \epsilon) + 8\beta e^{\frac{(2\alpha-1)\beta x^\alpha}{\alpha}} + 6\alpha\beta e^{\beta x^\alpha}\Big)\\
&\quad + \Big(3\alpha^2 + 4\Big)\beta^4 x^{2\alpha} e^{2\beta x^\alpha}\Big)\Bigg),\\
e_2(x,t) &= \frac{t^{2\alpha} e^{-\frac{(2\alpha+3)\beta x^\alpha}{\alpha}}}{2\alpha^5}\Bigg(\alpha^2\beta\Big(2e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha - \beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}\Big)\Big(\phi_b e^{\frac{\beta x^\alpha}{\alpha}} - \beta^2\nu_s\Big) + \alpha^5\beta^3\nu_e^2 e^{\frac{(\alpha+3)\beta x^\alpha}{\alpha}}\\
&\quad + \alpha\beta\nu_e\Big(\alpha(4\alpha+1)\beta^2 x^\alpha e^{\frac{(\alpha+3)\beta x^\alpha}{\alpha}} - e^{\frac{2\beta x^\alpha}{\alpha}}\Big(\alpha^2 e^{\frac{(\alpha+1)\beta x^\alpha}{\alpha}}(5\beta + \mu + \rho + \epsilon) + 8\alpha^2\beta e^{\frac{\beta x^\alpha}{\alpha}} + 4\alpha^2\beta e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\\
&\quad + \alpha^2\beta e^{\beta x^\alpha} + e^{2\beta x^\alpha}(\mu + \rho + \epsilon)\Big)\Big) + \alpha^2\beta^2\nu_i e^{\frac{(\alpha+2)\beta x^\alpha}{\alpha}}\Big(\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}} + \alpha\Big(e^{\frac{\beta x^\alpha}{\alpha}} - 1\Big)\Big)\Big) + 3\alpha^3\beta^2 e^{\frac{2\beta x^\alpha}{\alpha}}\\
&\quad + 4\alpha^3\beta^2 e^{2\beta x^\alpha} - 3\alpha^3\beta^2 e^{\frac{(\alpha+2)\beta x^\alpha}{\alpha}}\Big(2\beta x^\alpha + 1\Big) - \alpha^2\beta e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha(4\beta - \zeta - \eta - 4\mu - \epsilon) + 12\beta^2 x^\alpha\Big)\\
&\quad - \alpha^2\beta e^{\frac{3\beta x^\alpha}{\alpha}}\Big(3\alpha\beta - 2\alpha(\mu + \rho + \epsilon) + 3\beta^2 x^\alpha\Big) + \alpha^2\beta e^{\frac{(\alpha+3)\beta x^\alpha}{\alpha}}\Big(2\beta x^\alpha + 1\Big)\Big(3\alpha\beta - 2\alpha(\mu + \rho + \epsilon) + 3\beta^2 x^\alpha\Big)\\
&\quad + \beta x^\alpha e^{\frac{(2\alpha+3)\beta x^\alpha}{\alpha}}\Big(-\alpha^2\Big(\alpha\beta(3\beta - 2(\mu + \rho + \epsilon)) - \beta(\zeta + \eta + 4\mu + \epsilon) + (\mu + \rho + \epsilon)^2\Big) - \Big(3\alpha^2 + 4\Big)\beta^4 x^{2\alpha}\\
&\quad - \alpha\beta^2 x^\alpha\Big(-2\alpha^2(\mu + \rho + \epsilon) + (3\alpha(\alpha+1) + 4)\beta - \zeta - \eta - 4\mu - \epsilon\Big)\Big) + \alpha e^{\frac{2(\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha^2\Big((\mu + \rho + \epsilon)^2\\
&\quad - \beta(\zeta + \eta + 4\mu + \epsilon)\Big) + 3\Big(\alpha^2 + 4\Big)\beta^4 x^{2\alpha} + \alpha\beta^2 x^\alpha((3\alpha + 8)\beta - 2(\zeta + \eta + 4\mu + \epsilon))\Big)\Bigg),\\
i_2(x,t) &= -\frac{t^{2\alpha}}{2\alpha^5}\Bigg(\alpha^2\beta e^{-\frac{3(\alpha+1)\beta x^\alpha}{\alpha}}\Big(e^{\frac{(3\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha - \beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}\Big)\Big(\beta^2\nu_s - \phi_b e^{\frac{\beta x^\alpha}{\alpha}}\Big) - \alpha^3\beta\rho\nu_e e^{\frac{(2\alpha+3)\beta x^\alpha}{\alpha}}\\
&\quad + \alpha^5\beta^3\nu_i^2\Big(-e^{\frac{(2\alpha+3)\beta x^\alpha}{\alpha}}\Big) + \alpha\beta\nu_i e^{\frac{(\alpha+2)\beta x^\alpha}{\alpha}}\Big(4\alpha^2\beta e^{\frac{\beta x^\alpha}{\alpha}} + 2\alpha^2\beta e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}} + \alpha^2\beta e^{\beta x^\alpha}\\
&\quad - \alpha e^{\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big((2\alpha+1)\beta^2 x^\alpha - \alpha(2\beta + \zeta + \eta + \mu)\Big) + e^{2\beta x^\alpha}(\zeta + \eta + \mu - \rho)\Big)\Big) - 2\alpha^3\beta^2 e^{-\frac{3\beta x^\alpha}{\alpha}}\\
&\quad - \alpha^3\beta^2 e^{-\frac{(2\alpha+1)\beta x^\alpha}{\alpha}} + \alpha^3\beta^2 e^{-\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big(2\beta x^\alpha + 1\Big) + \alpha^2\beta e^{-\frac{2\beta x^\alpha}{\alpha}}\Big(\alpha(2\beta - \zeta - \eta - 2\mu + \rho) + 6\beta^2 x^\alpha\Big)\\
&\quad + \alpha^2\beta e^{-2\beta x^\alpha}\Big(\alpha(\beta - \zeta - \eta - \mu + 2\rho) + \beta^2 x^\alpha\Big) - \alpha^2\beta e^{\beta\left(-x^\alpha\right)}\Big(2\beta x^\alpha + 1\Big)\Big(\alpha(\beta - \zeta - \eta - \mu + 2\rho) + \beta^2 x^\alpha\Big)\\
&\quad + x^\alpha\Big(\alpha^2\beta\Big(\alpha\beta(\beta - \zeta - \eta - \mu + 2\rho) - \beta(\zeta + \eta + 2\mu - \rho) + (\zeta + \eta + \mu)^2 - \rho^2 - \rho(\zeta + \eta + 2\mu + \epsilon)\Big)\\
&\quad + \Big(\alpha^2 + 2\Big)\beta^5 x^{2\alpha} + \alpha\beta^3 x^\alpha\Big(\Big(\alpha^2 + \alpha + 2\Big)\beta - \Big(\alpha^2(\zeta + \eta + \mu - 2\rho)\Big) - \zeta - \eta - 2\mu + \rho\Big)\Big)\\
&\quad + \alpha e^{-\frac{\beta x^\alpha}{\alpha}}\Big(\alpha^2\Big(\beta(\zeta + \eta + 2\mu - \rho) - (\zeta + \eta + \mu)^2 + \rho^2 + \rho(\zeta + \eta + 2\mu + \epsilon)\Big) - \Big(\alpha^2 + 6\Big)\beta^4 x^{2\alpha}\\
&\quad - \alpha\beta^2 x^\alpha((\alpha + 4)\beta - 2(\zeta + \eta + 2\mu - \rho))\Big)\Bigg),\\
r_2(x,t) &= \frac{t^{2\alpha} e^{-\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}}{2\alpha^3}\Big(\alpha\beta^2\Big(\alpha^2\Big(-e^{\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big)\Big(\epsilon\nu_e - \zeta\nu_i\Big) - \nu_r(\epsilon - \zeta)e^{2\beta x^\alpha}\Big)\\
&\quad + \beta x^\alpha e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\Big(\zeta(\zeta + \eta + 2\mu - \rho) + \alpha\beta(2\epsilon - \zeta)\Big(\beta x^\alpha + 1\Big) - \epsilon^2 - \epsilon(2\mu + \rho)\Big)
\end{aligned}
$$

$$+\alpha e^{2\beta x^\alpha}\Big(-\zeta(\zeta+\eta+2\mu-\rho)+\epsilon^2+\epsilon(2\mu+\rho)\Big)+\alpha\beta(2\epsilon-\zeta)e^{\frac{\beta x^\alpha}{\alpha}}-\alpha\beta(2\epsilon-\zeta)e^{\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big(2\beta x^\alpha+1\Big)\Big),$$

$$d_2(x,t)=\frac{\eta t^{2\alpha}}{2\alpha^3}\Big(\alpha\beta e^{-2\beta x^\alpha}\Big(\alpha^2\beta v_i e^{\beta x^\alpha}-\Big(\beta x^\alpha e^{\beta x^\alpha}-1\Big)\Big(e^{\beta x^\alpha}\Big(\beta x^\alpha+1\Big)-1\Big)\Big)$$
$$-\Big((\zeta+\eta+\mu)\Big(\alpha e^{-\frac{\beta x^\alpha}{\alpha}}-\beta x^\alpha\Big)\Big)+\rho\Big(\alpha e^{-\frac{\beta x^\alpha}{\alpha}}-\beta x^\alpha\Big)\Big), \tag{54}$$

and so on. Therefore, the approximate analytical solutions for the system (46) are

$$s(x,t)=1+\beta x^\alpha/\alpha-e^{-\beta x^\alpha/\alpha}+\frac{t^\alpha}{\alpha^3}\Big(\alpha^2\Big(\phi_b-\beta^2 v_s e^{-\frac{\beta x^\alpha}{\alpha}}\Big)-e^{-\frac{2\beta x^\alpha}{\alpha}}\Big(\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}+\alpha\Big(e^{\frac{\beta x^\alpha}{\alpha}}-1\Big)\Big)$$
$$\times\Big(2\alpha\beta-2\beta^2x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}+\alpha\mu e^{\frac{\beta x^\alpha}{\alpha}}\Big)\Big)+\frac{t^{2\alpha}}{2\alpha^5}\Big(e^{-\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha^2\Big(-e^{\beta x^\alpha}\Big)\Big(\phi_b\Big(2\alpha\beta-2\beta^2x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}+\alpha\mu e^{\frac{\beta x^\alpha}{\alpha}}\Big)$$
$$+\alpha\beta^4 v_s^2+2\beta^2 v_s\Big(3\beta^2x^\alpha-\alpha\Big(3\beta+\mu-3\beta\sinh\Big(\frac{\beta x^\alpha}{\alpha}\Big)+7\beta\cosh\Big(\frac{\beta x^\alpha}{\alpha}\Big)\Big)\Big)\Big)$$
$$-\alpha^4\beta^3 v_i\Big(\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}+\alpha\Big(e^{\frac{\beta x^\alpha}{\alpha}}-1\Big)\Big)\Big)+\alpha^4\beta^3 v_e e^{-\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha+\beta x^\alpha\Big(-e^{\frac{\beta x^\alpha}{\alpha}}\Big)-\alpha e^{\frac{\beta x^\alpha}{\alpha}}\Big)$$
$$+e^{-\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\Big(\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}+\alpha\Big(e^{\frac{\beta x^\alpha}{\alpha}}-1\Big)\Big)\Big(\alpha^2\Big(\beta^2\Big(-3e^{\beta x^\alpha}+4e^{\frac{2(\alpha-1)\beta x^\alpha}{\alpha}}+3\Big)+\mu^2e^{2\beta x^\alpha}$$
$$+\beta e^{\frac{(2\alpha-1)\beta x^\alpha}{\alpha}}(\zeta+\eta+6\mu+\epsilon)\Big)-\alpha\beta^2x^\alpha\Big(e^{2\beta x^\alpha}(-3\alpha\beta+\zeta+\eta+6\mu+\epsilon)+8\beta e^{\frac{(2\alpha-1)\beta x^\alpha}{\alpha}}+6\alpha\beta e^{\beta x^\alpha}\Big)$$
$$+\Big(3\alpha^2+4\Big)\beta^4x^{2\alpha}e^{2\beta x^\alpha}\Big)\Big), \tag{55}$$

$$e(x,t)=-\beta x^\alpha/\alpha+e^{-\beta x^\alpha/\alpha}+\alpha\beta^2 v_e t^\alpha e^{\beta(-x^\alpha)}-\frac{t^\alpha}{\alpha}\Big(\beta e^{-2\beta x^\alpha}\Big(\alpha^2\beta v_e e^{\beta x^\alpha}-2\Big(\beta x^\alpha e^{\beta x^\alpha}-1\Big)$$
$$\times\Big(e^{\beta x^\alpha}\Big(\beta x^\alpha+1\Big)-1\Big)\Big)+\Big(\frac{\beta x^\alpha}{\alpha}-e^{-\frac{\beta x^\alpha}{\alpha}}\Big)(\mu+\rho+\epsilon)\Big)$$
$$+\frac{t^{2\alpha}e^{-\frac{(2\alpha+3)\beta x^\alpha}{\alpha}}}{2\alpha^5}\Big(\alpha^2\beta\Big(2e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha-\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}\Big)\Big(\phi_b e^{\frac{\beta x^\alpha}{\alpha}}-\beta^2 v_s\Big)+\alpha^5\beta^3 v_e^2 e^{\frac{(\alpha+3)\beta x^\alpha}{\alpha}}$$
$$+\alpha\beta v_e\Big(\alpha(4\alpha+1)\beta^2x^\alpha e^{\frac{(\alpha+3)\beta x^\alpha}{\alpha}}-e^{\frac{2\beta x^\alpha}{\alpha}}\Big(\alpha^2e^{\frac{(\alpha+1)\beta x^\alpha}{\alpha}}(5\beta+\mu+\rho+\epsilon)+8\alpha^2\beta e^{\frac{\beta x^\alpha}{\alpha}}+4\alpha^2\beta e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}$$
$$+\alpha^2\beta e^{\beta x^\alpha}+e^{2\beta x^\alpha}(\mu+\rho+\epsilon)\Big)\Big)+\alpha^2\beta^2 v_i e^{\frac{(\alpha+2)\beta x^\alpha}{\alpha}}\Big(\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}+\alpha\Big(e^{\frac{\beta x^\alpha}{\alpha}}-1\Big)\Big)\Big)+3\alpha^3\beta^2e^{\frac{2\beta x^\alpha}{\alpha}}$$
$$+4\alpha^3\beta^2e^{2\beta x^\alpha}-3\alpha^3\beta^2e^{\frac{(\alpha+2)\beta x^\alpha}{\alpha}}\Big(2\beta x^\alpha+1\Big)-\alpha^2\beta e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha(4\beta-\zeta-\eta-4\mu-\epsilon)+12\beta^2x^\alpha\Big)$$
$$-\alpha^2\beta e^{\frac{3\beta x^\alpha}{\alpha}}\Big(3\alpha\beta-2\alpha(\mu+\rho+\epsilon)+3\beta^2x^\alpha\Big)+\alpha^2\beta e^{\frac{(\alpha+3)\beta x^\alpha}{\alpha}}\Big(2\beta x^\alpha+1\Big)\Big(3\alpha\beta-2\alpha(\mu+\rho+\epsilon)+3\beta^2x^\alpha\Big)$$
$$+\beta x^\alpha e^{\frac{(2\alpha+3)\beta x^\alpha}{\alpha}}\Big(-\alpha^2\Big(\alpha\beta(3\beta-2(\mu+\rho+\epsilon))-\beta(\zeta+\eta+4\mu+\epsilon)+(\mu+\rho+\epsilon)^2\Big)-\Big(3\alpha^2+4\Big)\beta^4x^{2\alpha}$$
$$-\alpha\beta^2x^\alpha\Big(-2\alpha^2(\mu+\rho+\epsilon)+(3\alpha(\alpha+1)+4)\beta-\zeta-\eta-4\mu-\epsilon\Big)\Big)+\alpha e^{\frac{2(\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha^2\Big((\mu+\rho+\epsilon)^2$$
$$-\beta(\zeta+\eta+4\mu+\epsilon)\Big)+3\Big(\alpha^2+4\Big)\beta^4x^{2\alpha}+\alpha\beta^2x^\alpha((3\alpha+8)\beta-2(\zeta+\eta+4\mu+\epsilon))\Big)\Big),$$

$$i(x,t)=-\beta x^\alpha/\alpha+e^{-\beta x^\alpha/\alpha}+\frac{t^\alpha}{\alpha^2}\Big(\alpha\beta e^{-2\beta x^\alpha}\Big(\alpha^2\beta v_i e^{\beta x^\alpha}-\Big(\beta x^\alpha e^{\beta x^\alpha}-1\Big)\Big(e^{\beta x^\alpha}\Big(\beta x^\alpha+1\Big)-1\Big)\Big)$$
$$-\Big((\zeta+\eta+\mu)\Big(\alpha e^{-\frac{\beta x^\alpha}{\alpha}}-\beta x^\alpha\Big)\Big)+\rho\Big(\alpha e^{-\frac{\beta x^\alpha}{\alpha}}-\beta x^\alpha\Big)\Big)$$
$$-\frac{t^{2\alpha}}{2\alpha^5}\Big(\alpha^2\beta e^{-\frac{3(\alpha+1)\beta x^\alpha}{\alpha}}\Big(e^{\frac{(3\alpha+1)\beta x^\alpha}{\alpha}}\Big(\alpha-\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}\Big)\Big(\beta^2 v_s-\phi_b e^{\frac{\beta x^\alpha}{\alpha}}\Big)-\alpha^3\beta\rho v_e e^{\frac{(2\alpha+3)\beta x^\alpha}{\alpha}}$$
$$+\alpha^5\beta^3 v_i^2\Big(-e^{\frac{(2\alpha+3)\beta x^\alpha}{\alpha}}\Big)+\alpha\beta v_i e^{\frac{(\alpha+2)\beta x^\alpha}{\alpha}}\Big(4\alpha^2\beta e^{\frac{\beta x^\alpha}{\alpha}}+2\alpha^2\beta e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}+\alpha^2\beta e^{\beta x^\alpha}$$
$$-\alpha e^{\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big((2\alpha+1)\beta^2x^\alpha-\alpha(2\beta+\zeta+\eta+\mu)\Big)+e^{2\beta x^\alpha}(\zeta+\eta+\mu-\rho)\Big)\Big)-2\alpha^3\beta^2e^{-\frac{3\beta x^\alpha}{\alpha}}$$
$$-\alpha^3\beta^2e^{-\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}+\alpha^3\beta^2e^{-\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big(2\beta x^\alpha+1\Big)+\alpha^2\beta e^{-\frac{2\beta x^\alpha}{\alpha}}\Big(\alpha(2\beta-\zeta-\eta-2\mu$$
$$+\rho)+6\beta^2x^\alpha\Big)+\alpha^2\beta e^{-2\beta x^\alpha}\Big(\alpha(\beta-\zeta-\eta-\mu+2\rho)+\beta^2x^\alpha\Big)-\alpha^2\beta e^{\beta\left(-x^\alpha\right)}\Big(2\beta x^\alpha+1\Big)\Big(\alpha(\beta-\zeta$$

$$-\eta-\mu+2\rho)+\beta^2x^\alpha\Big)+x^\alpha\Big(\alpha^2\beta\Big(\alpha\beta(\beta-\zeta-\eta-\mu+2\rho)-\beta(\zeta+\eta+2\mu-\rho)+(\zeta+\eta+\mu)^2-\rho^2$$
$$-\rho(\zeta+\eta+2\mu+\epsilon)\Big)+\Big(\alpha^2+2\Big)\beta^5x^{2\alpha}+\alpha\beta^3x^\alpha\Big(\Big(\alpha^2+\alpha+2\Big)\beta-\Big(\alpha^2(\zeta+\eta+\mu-2\rho)\Big)-\zeta-\eta$$
$$-2\mu+\rho\Big)\Big)+\alpha e^{-\frac{\beta x^\alpha}{\alpha}}\Big(\alpha^2\Big(\beta(\zeta+\eta+2\mu-\rho)-(\zeta+\eta+\mu)^2+\rho^2+\rho(\zeta+\eta+2\mu+\epsilon)\Big)$$
$$-\Big(\alpha^2+6\Big)\beta^4x^{2\alpha}-\alpha\beta^2x^\alpha((\alpha+4)\beta-2(\zeta+\eta+2\mu-\rho))\Big)\Big),$$

$$r(x,t)=\frac{t^\alpha}{\alpha^2}(\epsilon-\zeta)e^{-\frac{\beta x^\alpha}{\alpha}}\Big(\beta x^\alpha e^{\frac{\beta x^\alpha}{\alpha}}-\alpha\Big)+\frac{t^{2\alpha}e^{-\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}}{2\alpha^3}\Big(\alpha\beta^2\Big(\alpha^2\Big(-e^{\frac{(\alpha+1)\beta x^\alpha}{\alpha}}\Big)\Big(\epsilon v_e-\zeta v_i\Big)$$
$$-v_r(\epsilon-\zeta)e^{2\beta x^\alpha}\Big)+\beta x^\alpha e^{\frac{(2\alpha+1)\beta x^\alpha}{\alpha}}\Big(\zeta(\zeta+\eta+2\mu-\rho)+\alpha\beta(2\epsilon-\zeta)\Big(\beta x^\alpha+1\Big)-\epsilon^2-\epsilon(2\mu+\rho)\Big)$$
$$+\alpha e^{2\beta x^\alpha}\Big(-\zeta(\zeta+\eta+2\mu-\rho)+\epsilon^2+\epsilon(2\mu+\rho)\Big)+\alpha\beta(2\epsilon-\zeta)e^{\frac{\beta x^\alpha}{\alpha}}-\alpha\beta(2\epsilon-\zeta)e^{\frac{(\alpha+1)\beta x^\alpha}{\alpha}}$$
$$\times\Big(2\beta x^\alpha+1\Big)\Big),$$
$$d(x,t)=\frac{\eta t^\alpha\Big(\alpha e^{-\frac{\beta x^\alpha}{\alpha}}-\beta x^\alpha\Big)}{\alpha^2}+\frac{\eta t^{2\alpha}}{2\alpha^3}\Big(\alpha\beta e^{-2\beta x^\alpha}\Big(\alpha^2\beta v_i e^{\beta x^\alpha}-\Big(\beta x^\alpha e^{\beta x^\alpha}-1\Big)\Big(e^{\beta x^\alpha}\Big(\beta x^\alpha+1\Big)-1\Big)\Big)$$
$$-\Big((\zeta+\eta+\mu)\Big(\alpha e^{-\frac{\beta x^\alpha}{\alpha}}-\beta x^\alpha\Big)\Big)+\rho\Big(\alpha e^{-\frac{\beta x^\alpha}{\alpha}}-\beta x^\alpha\Big)\Big). \tag{56}$$

In the next section, we obtain the graphical representations of the solutions for the (22) subject to the initial conditions (53) with the following assumed values:

$$\begin{cases}\alpha=\mu=0,\beta=0.0139,\ \rho=0.0714,\epsilon=0.1667,\\ \zeta=0.7348,\ \eta=0.0218,v_s=0.05,\ v_e=0.025,\\ v_i=0.04,\ v_r=0.08,\ q(t)=0.5e^t-0.36t.\end{cases} \tag{57}$$

6 | DISCUSSION AND GRAPHICAL REPRESENTATIONS

The 3D graphical representations of the exact solutions for the conformable SIR model with particular values of parameters are presented to illustrate the physical behavior of the solutions. However, Figure 3 shows the graphs of solution $s_1(x,t)$ for equation 22 for $\mu=0.01,\beta=0.139,\zeta=0.0714,\ v_s,\ v_i=0.05$, and $\alpha=0.5,1$, respectively. Further, Figure 4 shows the graphs of solution $i1(x,t)$ for equation 22 for $\mu=0.01,\beta=0.139,\zeta=0.0714,v_s,v_i=0.05$, and $\alpha=0.5,1$, respectively. We plot the graphs of solution $r1(x,t)$ for equation 22 for $\mu=0.01,\beta=0.139,\zeta=0.0714,v_s,v_i=0.05$, and $\alpha=0.5,1$ respectively in Figure 5 . However, the graphs of solution $s_3(x,t)$ are plotted in Figure 9 for equation 22 for $\mu=0.01,\beta=0.139,\zeta=0.0714,v_s,v_i=0.05$, and $\alpha=0.5,1$ respectively. The Figure 6 shows exact solution $i_3(x,t)$ of (22) for $\mu=0.01,\beta=0.139,\zeta=0.0714,v_s,v_i=0.05$, and $\alpha=0.5,1$ respectively. The Figure 7 shows the graphs of the exact solutions $r_3(x,t)$ for $\mu=0.01,\beta=0.139,\zeta=0.0714,v_s,v_i=0.05$, and $\alpha=0.5,1$ respectively. Moreover, In Figure 8 , we plot the graphs of solutions $s_1(x,t)$, $i_1(x,t)$, and $r_1(x,t)$ respectively at different values of t when x is fixed at $x=10$ for $\mu=0.01,\beta=0.139,\zeta=0.07348,v_s,v_i=0.05$ among various values of α. In Figures 10 , 11 , 12 , 13 , and 14 , we plot the graph of the solutions $s(x,t)$, $e(x,t)$, $i(x,t)$, $r(x,t)$, and $d(x,t)$ respectively according to the initial values of the parameters given in (53).

7 | CONCLUSIONS

This paper introduced effective SIR and SEIRD models, based on conformable space-time PDEs for the COVID-19 pandemic. These models are new and have not been introduced before in their current formulations. Moreover, we introduced two effective modifications based on ERFM and an analytical technique based on the decomposition method for solving nonlinear systems of PDEs with conformable derivatives in a general form. The advantage of the used conformable derivative for the debated models is that it relies upon the characteristics of the entire solution functions (on time and size of selected population for all $\alpha\in(0,1]$) and not just at the values in the vicinity of particular points, which is helpful to enhance the performance for the dynamics of the transmission and evolution of infectious diseases. Further, we applied these analytical approaches to solving the proposed mathematical models of the COVID-19 pandemic. The interesting result of this article is that it yields exact and approximate analytical solutions to the proposed models which can help significantly to predict and control the transmission of infectious diseases. However, several analytical solutions of SIR and SEIRD models obtained by different analytical methods are available in the literature. By comparing our solutions with those obtained previously, our analytical solutions obtained in this paper are new and potentially helpful to describe the population dynamics of infectious diseases transmission and controlling infectious diseases.

DATA AVAILABILITY

Data sharing does not apply to this article as no datasets were generated or analyzed during the current.

CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this research.

AUTHOR CONTRIBUTIONS

Hayman Thabet and Subhash Kendre contributed substantially to this paper. Hayman Thabet wrote this paper, Subhash Kendre helped in evaluating and editing the paper.

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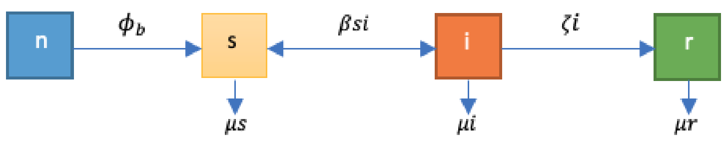

Fig. 1 . The schematic visualization of SIR model for COVID-19.

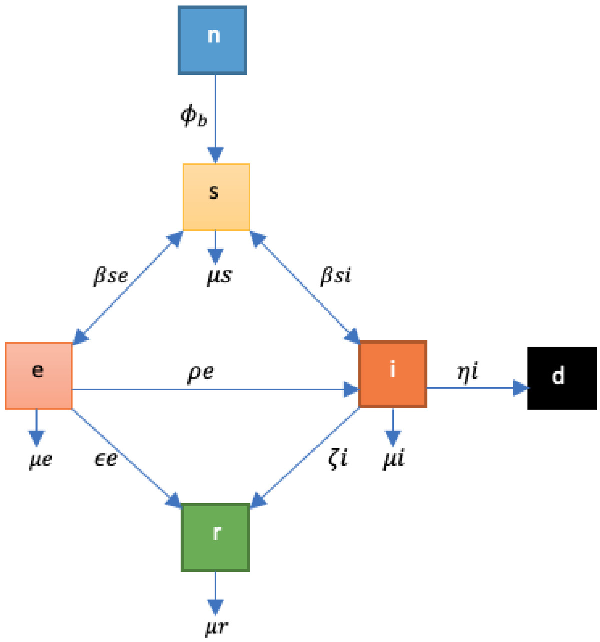

Fig. 2 . The schematic description of SEIRD model for COVID-19 pandemic.

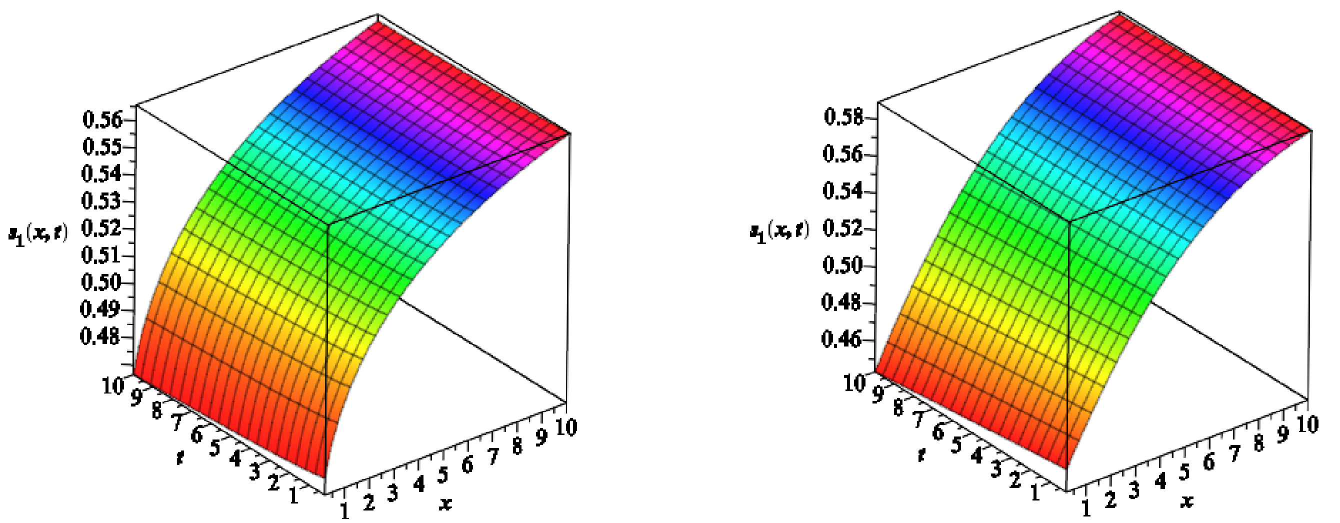

Fig. 3 . The graphical representations of solution $s_1(x,t)$ in system (33) for $\alpha = 0.5$ (on the left) and $\alpha = 1$ (on the right).

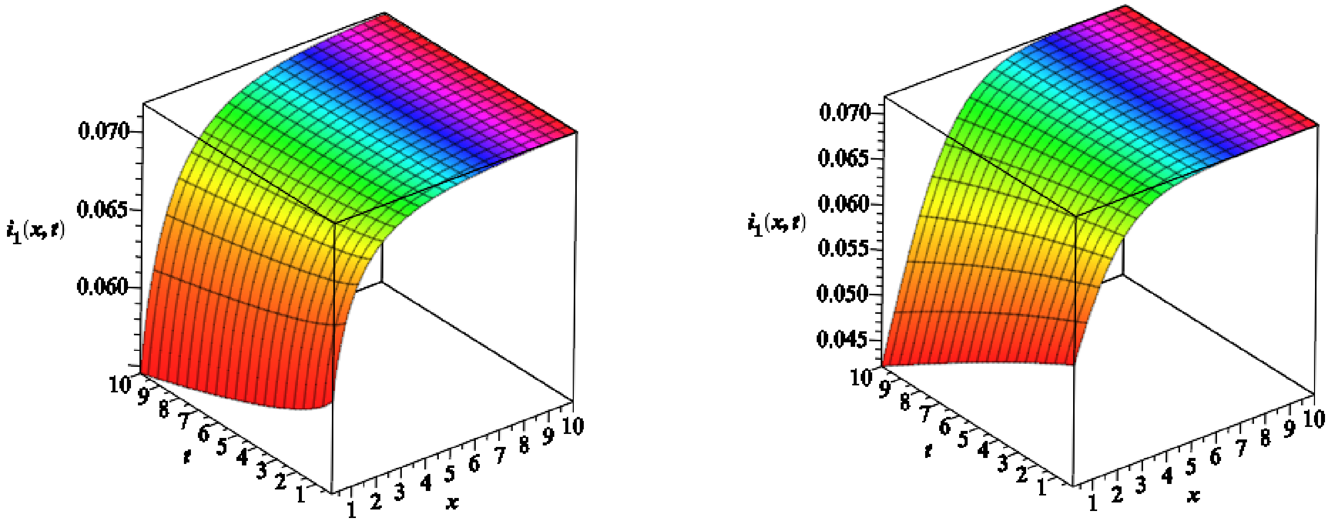

Fig. 4 . The graphical representations of solution $i_1(x,t)$ in system (33) for $\alpha = 0.5$ (on the left) and $\alpha = 1$ (on the right).

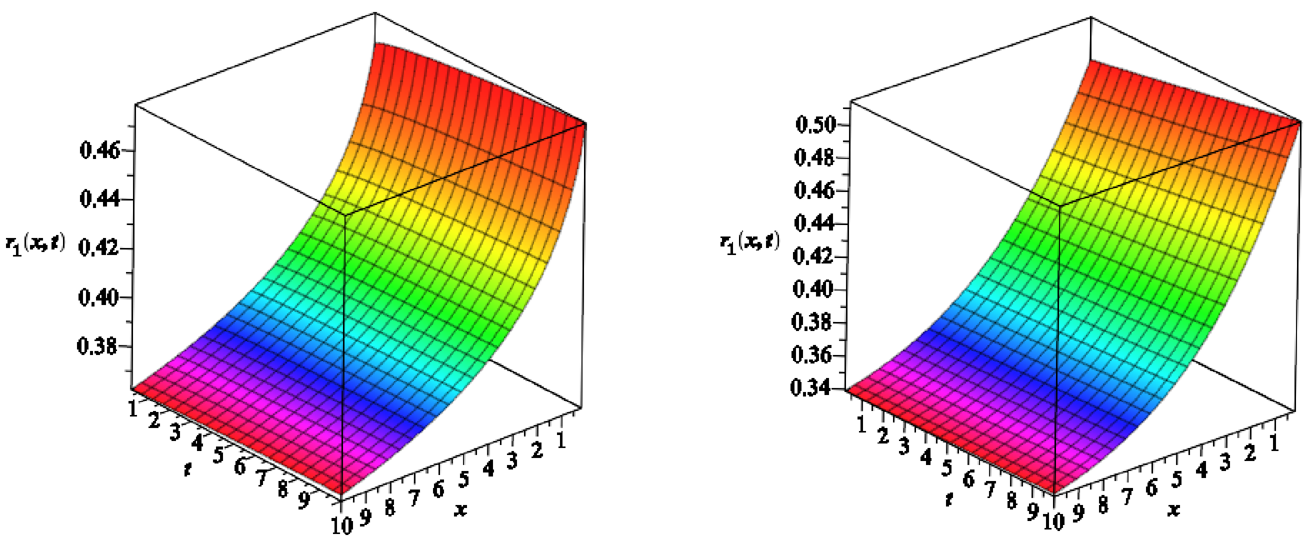

Fig. 5 . The graphical representations of solutions $r_1(x,t)$ in system (33) for $\alpha = 0.5$ (on the left) and $\alpha = 1$ (on the right).

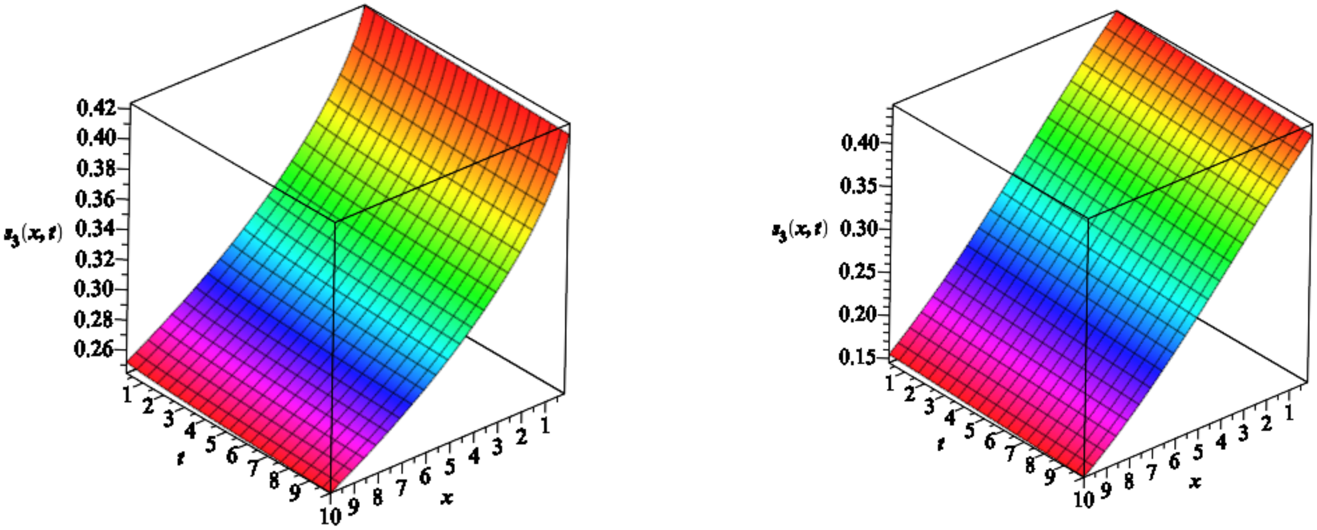

Fig. 6 . The graphical representations of solution $s_3(x, t)$ in system (43) for $\alpha = 0.5$ (on the left) and $\alpha = 1$ (on the right).

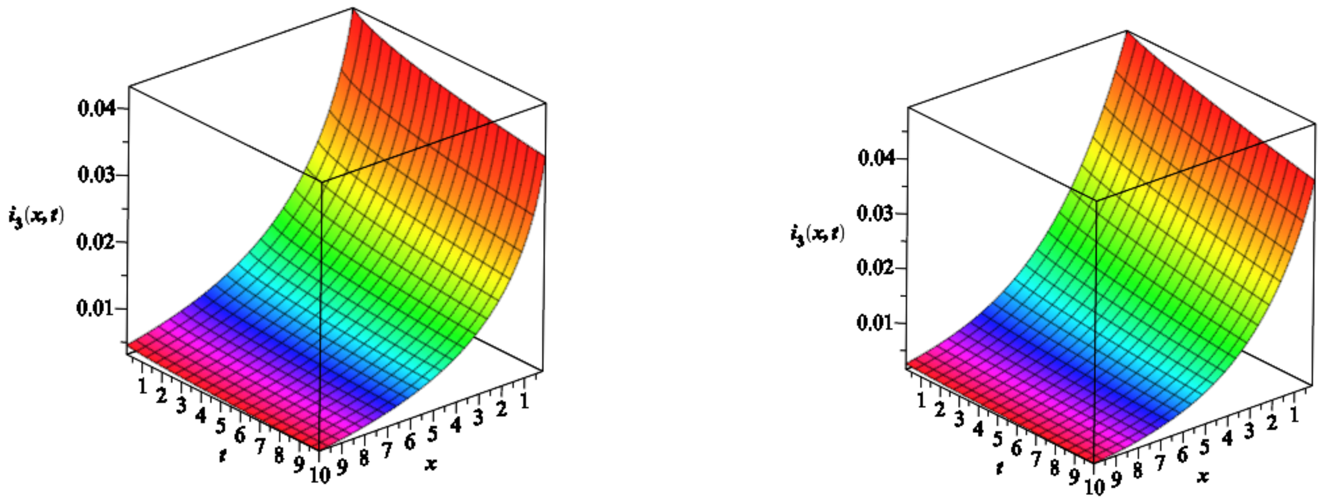

Fig. 7 . The graphical representations of solution $i_3(x, t)$ in system (43) for $\alpha = 0.5$ (on the left) and $\alpha = 1$ (on the right).

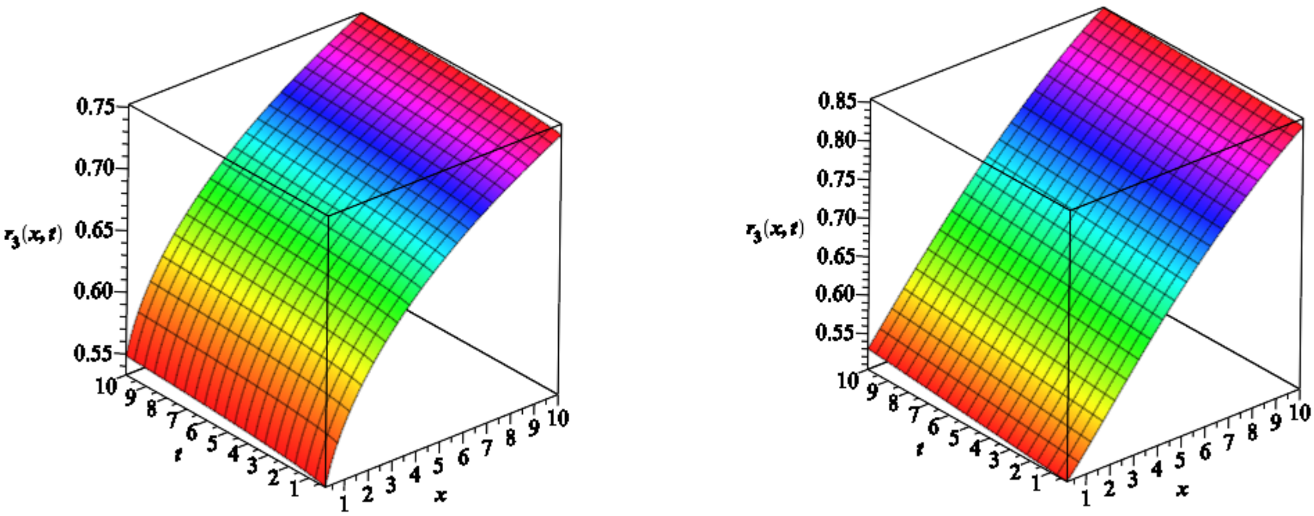

Fig. 8 . The graphical representations of solutions r_3 for system (43) for $\alpha = 0.5$ (on the left) and $\alpha = 1$ (on the right).

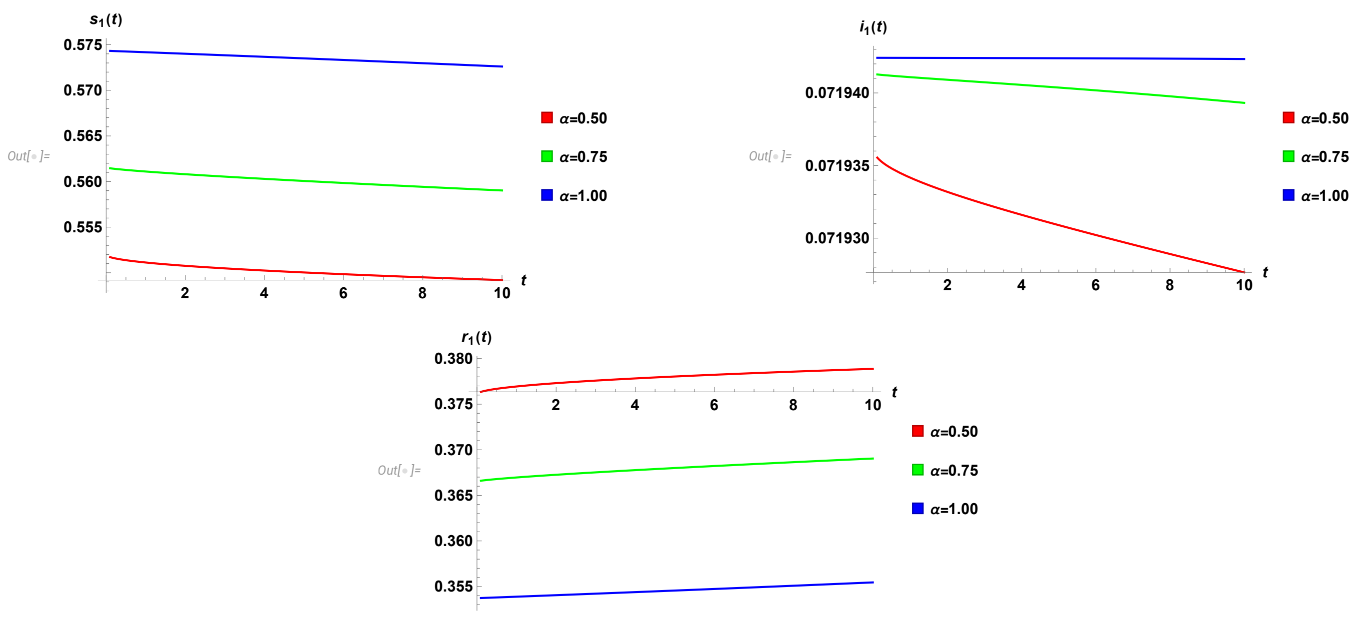

Fig. 9 . The graphical representations of solutions s_1, i_1, and r_1 vs. t when $x = 1$ is fixed for system 33 at different values of α.

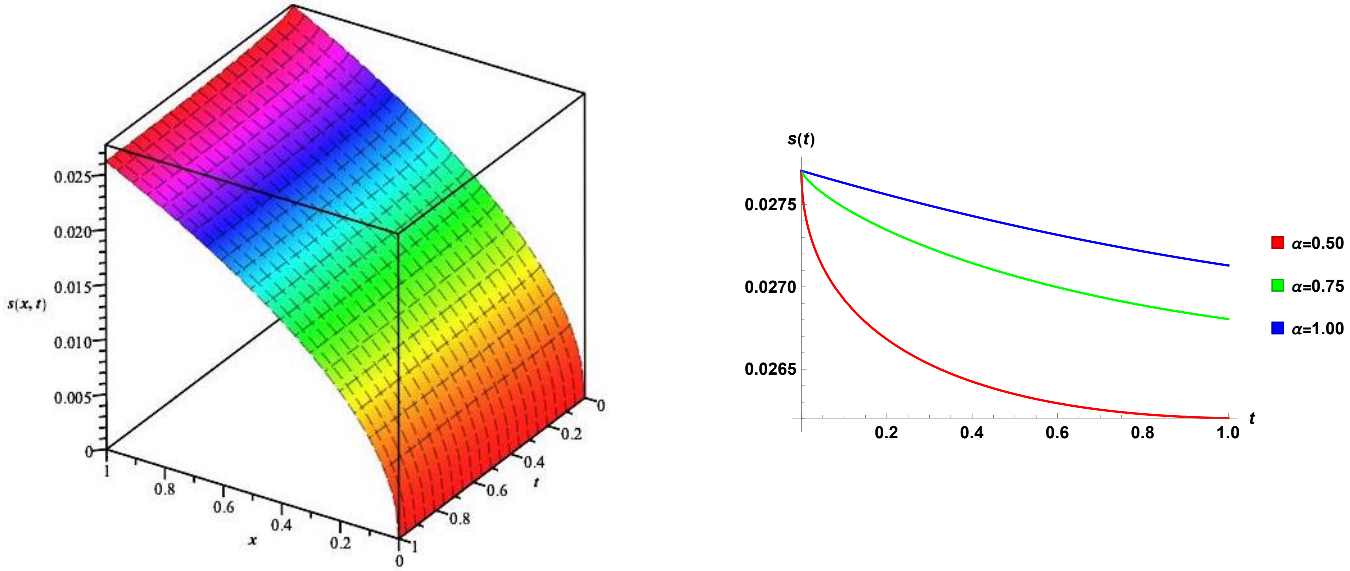

Fig. 10 . The graphical representations of solution $s(x, t)$ (left) and $s(t)$ (right when $x = 1$) subject to the initial values given in (57) for the system (46).

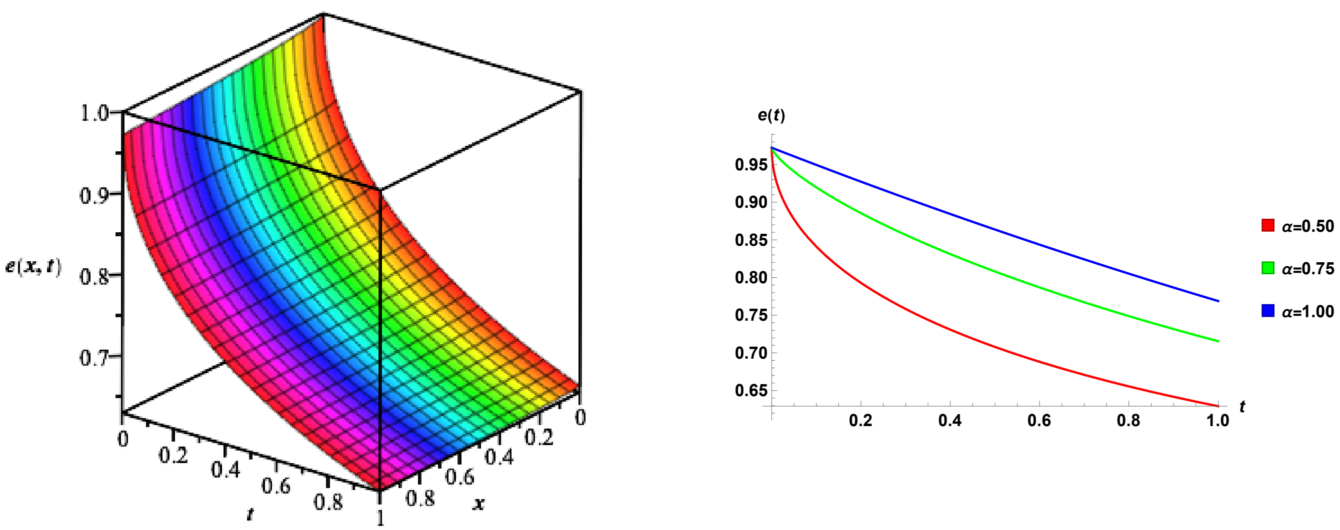

Fig. 11 . The graphical representations of solution $e(x, t)$ (left) and $e(t)$ (right when $x = 1$) subject to the initial values given in (57) for the system (46).

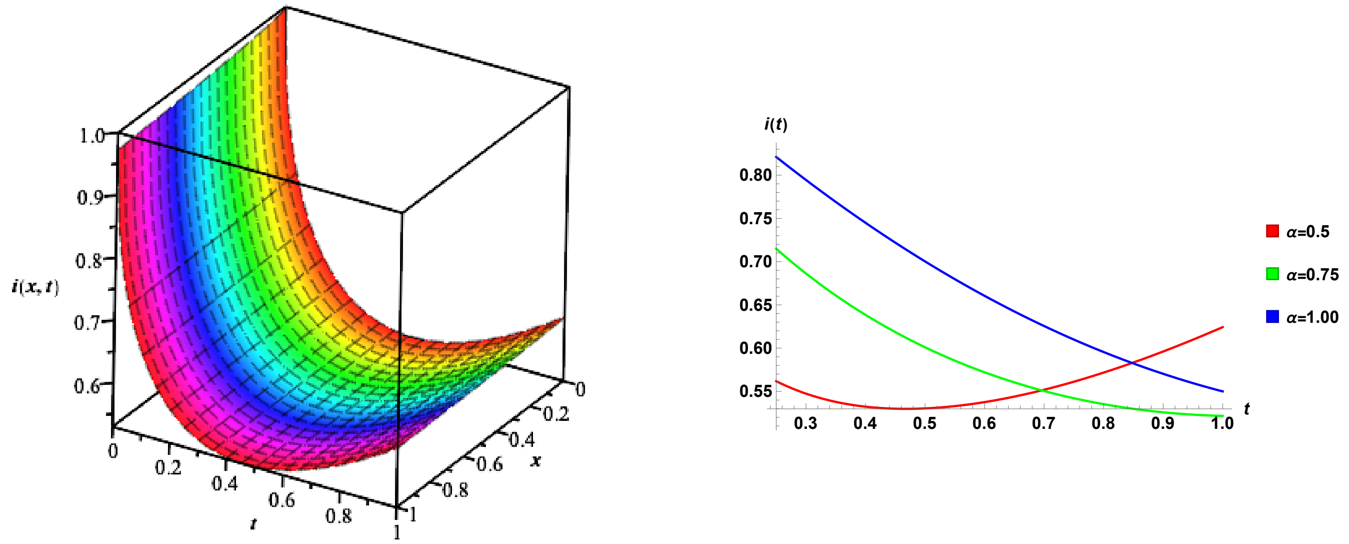

Fig. 12 . The graphical representations of solution $i(x, t)$ (left) and $i(t)$ (right when $x = 1$) subject to the initial values given in (57) for the system (46).

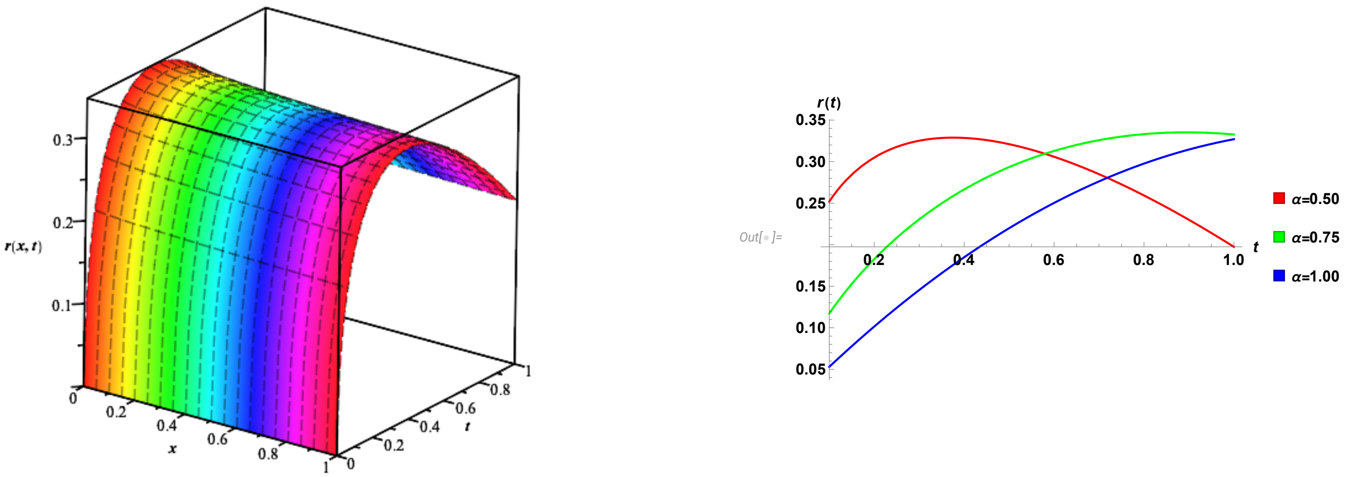

Fig. 13 . The graphical representations of solution $r(x, t)$ (left) and $r(t)$ (right when $x = 1$) subject to the initial values given in (57) for the system (46).

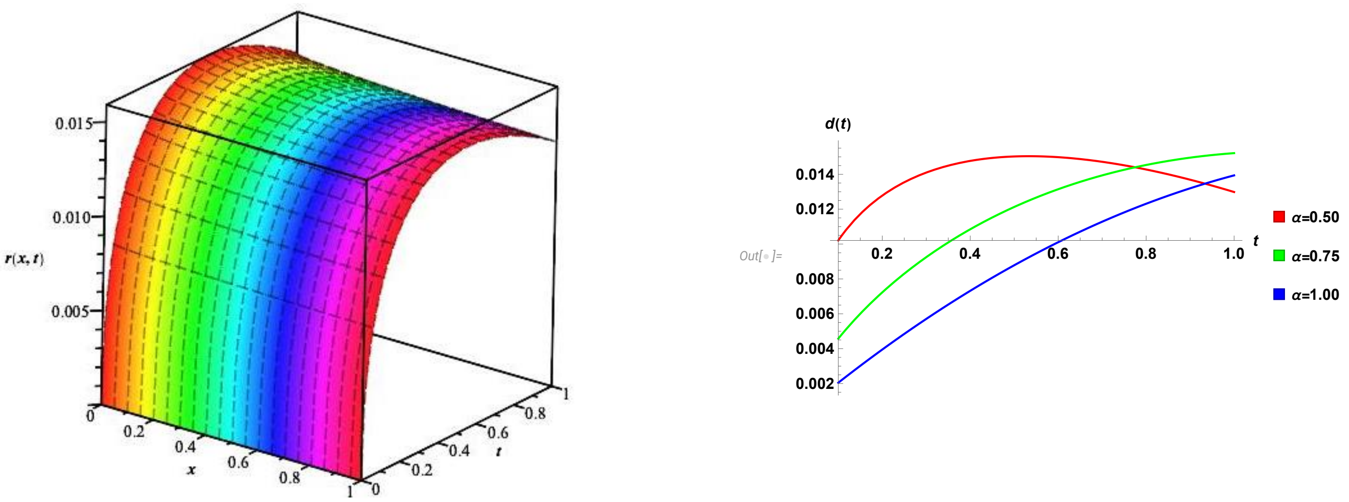

Fig. 14 . The graphical representations of solution $d(x, t)$ (left) and $d(t)$ (right when $x = 1$) subject to the initial values given in (57) for the system (46).