

# Curvature of a regular black hole in Loop quantum gravity model using the RVB method

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## Abstract

This paper attempted to use the RVB method to find that there is a constant that cannot be crossed out in the calculation of the curvature term representing the geometric invariant. The constant may be a parameter item.

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**Keywords:** RVB method, Loop quantum gravity, Hawking temperature

## 1. INTRODUCTION

Loop quantum gravity[1], also translated loop quantum gravity, quantum geometry; by Abe Ashtika, Lee Smolin, Carlo Lowe. The quantum gravity theory developed by Leigh et al., along with string theory, is the most successful theory for quantizing gravity today. The theory that uses the perturbation theory of quantum field theory to realize the quantization of gravity theory cannot be renormalized. If we claim that space-time has only four dimensions and start with general relativity, the result can be transformed into a theory similar to gauge field theory. The basic regular variable is the Ashtika-Barbero connection instead of the metric tensor, and then the connection is defined as The translation operator (holonomy) and the flux variable is the basic variable to achieve quantization. Under this theory, the space-time description is background-independent, and the space-time geometry is paved by the spin network woven by relational loops. The length of each edge in the network is the Planck length. Loops do not exist in space-time but define the geometry of space-time in a kink of loops. At the Planck scale, space-time geometry is full of random quantum fluctuations, so spin networks are also called spin bubbles. Under this theory, space-time is discrete.

In the classical view, a black hole has always been considered an extreme object from which nothing can escape. Hawking and Bekenstein discovered that black holes can have temperature and entropy, and a black hole system can be thought of as a thermodynamic system. The topological properties of black holes can be characterized by the topologically invariant Euler characteristic [2–9]. Black holes have some important characteristics that can be more easily studied by calculating Euler characteristics. For example, black hole entropy has been discussed before [9, 10]. While Padmanabhan et al. The importance of the topological properties of the event horizon temperature [11, 12] is emphasized. Recently, Robson, Villari, and Biancalana [12–14] showed that the Hawking temperature of a black hole is closely related to its topology. In addition, a topological method related to the Euler property is proposed to obtain the Hawking temperature, which has been successfully applied to Schwarzschild-like or charged black holes such as four-dimensional Schwarzschild black holes and anti-de Sitter black holes. Based on the work of Robson, Villari, and Biancalana [12–14], Liu et al. using this topological method [15], the Hawking temperature of a charged spinning BTZ black hole is accurately calculated. Chen et al.[16] attempted to apply the RVB method for calculating the Hawking temperatures of black holes under  $f(R)$  gravity. In calculating the Hawking temperature, we found a difference in the integration constant between the RVB and general methods.

Previous literature applied the RVB method to various black holes under general relativity and under  $f(R)$  gravity [12–16]. Therefore, for many special black holes, whether the complex coordinate system of the RVB makes its temperature difficult to calculate, it is found that the Hawking temperature of the black hole under loop quantum gravity can be easily obtained by the RVB method. After using the RVB method in this paper, we find that there is a constant that cannot be crossed out in the calculation of the curvature term representing the geometric invariant. The constant may be a parameter item.

This paper is organized as follows. The second part introduces the main formula studied in this paper, that is, a new expression for the Hawking temperature of two-dimensional black hole systems. In the **3** section, we discussed the effective loop quantum gravity under the black hole model. In Section 4, we list the curvature term expression and calculate the value of the term by the RVB method. We summarize our results in the **5** section.

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## 2. LOOP QUANTUM GRAVITY AND RVB METHOD FOR CALCULATING THE TEMPERATURE OF A BLACK HOLE

Loop quantum gravity [1] can be derived from the ADM notation of general relativity. The regular variables of ADM notation are the metric tensor  $q_{ab}$  of the three-dimensional space and its regular momentum  $P^{ab}$ . Using the Dirac constraint processing method, the ADM representation has two constraints of the first kind:

1. Differential homograph constraint:  $-2q^{-\frac{1}{2}} \nabla_a P_b^a = 0$ .
2. Hamiltonian constraint:  $-{}^3R + q^{-1} (P^{ab} P_{ab} - \frac{1}{2} P^2) = 0$ .

Ashitika-Barbero Liaison At this time, the geometric method of the card is used, and the metric tensor is represented by the triad linear form,  $e_a^i e_b^j \delta_{ij} = q_{ab}$ .

Assuming that the connection compatible with the tripod is  $\Gamma_a^i$  (called a spin connection), the external curvature tensor of the three-dimensional space is  $K_a^i$ ,  $\gamma$  is any real number, define a new connection  $A_a^i = \Gamma_a^i + \gamma K_a^i$  Namely the Ashitika-Barbero liaison. Its canonical momentum is  $E_i^a = \det(e) e_i^a$ . Three first-class constraints are obtained using the Dirac constraint processing method:

1. Gaussian Constraint:  $D_a E_i^a = 0$ .
2. Differential homeomorphism constraint:  $F_{ab}^i E_i^a = 0$ .
3. Hamiltonian constraint:  $\frac{1}{\sqrt{|\det(E)|}} F_{ab}^i E_j^a E_k^b \epsilon_{ijk} + \dots = 0$ .

$D_a$  is the covariant derivative defined by the Ashitika-Barbero connection, and  $F_{ab}^i$  is the curvature tensor defined by the Ashitika-Barbero connection. Due to the Gaussian constraint, loop quantum gravity is a theory similar to gauge field theory.

Black hole systems are various, and stationary or rotating black holes have simple metrics, thus the temperature can be easily deduced. However, the complex coordinate system makes their temperature difficult to calculate for many special black holes. Therefore, the RVB method coordinates the Hawking temperature of a black hole to the Euler characteristic  $\chi$ , which is very useful in calculating the Hawking temperature in any coordinate system. Note: Since we are between the cosmological and event horizons, the inner horizon temperature cannot be observed. Below we discuss only the observable Hawking temperature.

The Euler characteristic [1–5] can be described:

$$\chi = \int_{r_0} \Pi - \int_{r_H} \Pi - \int_{r_0} \Pi = - \int_{r_H} \Pi. \quad (1)$$

In a word, in the calculation of the Euler characteristic, the outer boundary is always canceled out. Therefore, the integral should only be related to the Killing horizon.

According to references [12–19], the Hawking temperature of a two-dimensional black hole can be obtained by the topological formula:

$$T_H = \frac{\hbar c}{4\pi \chi k_B} \sum_{j \leq \chi} \int_{r_{H_j}} \sqrt{|g|} R dr, \quad (2)$$

among them,  $\hbar$  is Planck's constant,  $c$  is the speed of light,  $k_B$  is the Boltzmann constant,  $g$  is the determinant of the metric,  $R$  is the Ricci scalar, and  $r_{H_j}$  is the location of the Killing horizon. In this paper, we use the natural unit system  $\hbar = c = k_B = 1$ . The  $\chi$ , which depends on the spatial coordinate  $r$ , is an Euler characteristic, representing the Killing level in Euclidean geometry. The symbol Eq.(4) represents the summation associated with the horizon. Through transformation, in this paper  $|g|=1$ .

We now go back to the four dimensions and deduce the  $\chi$  of the loop quantum gravity space, which directly leads to its thermal equilibrium temperature. Now the general formula for the Euler property of a four-dimensional Riemannian manifold given in terms of local coordinates is:

$$\chi = \frac{1}{32\pi^2} \int d^4x \sqrt{g} (K_1 - 4R_{ab}R^{ab} + R^2) \quad (3)$$

where  $K_1 \equiv R_{abcd}R^{abcd}$  is the Kretschmann invariant and  $R_{ab}$  is the Ricci tensor.

## 3. BLACK HOLE IN LOOP QUANTUM GRAVITY

The common property of BHs in loop quantum gravity is that the singularities within them are altered by a transition surface that connects a BH to a white hole, and spacetime is precise everywhere. After solving the effective

equation of loop quantum gravity and redefining the two new coordinates, the static spherical symmetry metric can be defined as:[20, 21]

$$ds^2 = -g(r)dt^2 + h(r)dr^2 + C(r) (d\theta^2 + \sin^2 \theta d\phi^2), \quad (4)$$

where

$$g(r) = \frac{1}{h(r)} = \frac{\sqrt{8AM^2 + r^2} (\sqrt{8AM^2 + r^2} - 2M)}{2AM^2 + r^2}, \quad (5)$$

$$C(r) = 2AM^2 + r^2, \quad (6)$$

where  $A$  is the dimensionless non-negative parameter.

#### 4. THE CURVATURE TERM EXPRESSION AND CALCULATE THE VALUE OF THE TERM BY THE RVB METHOD

$$K_1 - 4R_{ab}R^{ab} + R^2 = \quad (7)$$

$$\begin{aligned} & \left( 16M^2 \left( 6291456A^{10}M^{18} + 262144A^9M^{16} \left( 448M^2 - 138r^2 - 55M\sqrt{8AM^2 + r^2} \right) + 32768A^8M^{14} \left( 6496M^4 - 26256M^2r^2 - 411r^4 - 2128M^3\sqrt{8AM^2 + r^2} + 2868Mr^2\sqrt{8AM^2 + r^2} \right) + \right. \\ & 8192A^7M^{12} \left( 8960M^6 - 256928M^4r^2 - 20960M^2r^4 + 300r^6 - 6608M^5\sqrt{8AM^2 + r^2} + 72296M^3r^2\sqrt{8AM^2 + r^2} + 2519Mr^4\sqrt{8AM^2 + r^2} \right) + 512A^6M^{10} \left( 7936M^8 - \right. \\ & 2137856M^6r^2 - 131712M^4r^4 + 239360M^2r^6 + 4125r^8 - 15616M^7\sqrt{8AM^2 + r^2} + 1265600M^5r^2\sqrt{8AM^2 + r^2} + 59856M^3r^4\sqrt{8AM^2 + r^2} - 21884Mr^6\sqrt{8AM^2 + r^2} \right) + \\ & 3r^{10} \left( 256M^8 + 1792M^6r^2 + 1120M^4r^4 + 112M^2r^6 + r^8 - 1024M^7\sqrt{8AM^2 + r^2} - 1792M^5r^2\sqrt{8AM^2 + r^2} - 448M^3r^4\sqrt{8AM^2 + r^2} - 16Mr^6\sqrt{8AM^2 + r^2} \right) + \\ & 4AMr^8 \left( 6912M^9 + 67328M^7r^2 + 58016M^5r^4 + 8624M^3r^6 + 179Mr^8 - 29696M^8\sqrt{8AM^2 + r^2} - 73472M^6r^2\sqrt{8AM^2 + r^2} - 26432M^4r^4\sqrt{8AM^2 + r^2} - \right. \\ & 1616M^2r^6\sqrt{8AM^2 + r^2} - 8r^8\sqrt{8AM^2 + r^2} \left. \right) + 8A^4M^6r^2 \left( -172032M^{10} + 4500224M^8r^2 + 29659392M^6r^4 + 16138528M^4r^6 + 1215664M^2r^8 + 6267r^{10} + \right. \\ & 851968M^9\sqrt{8AM^2 + r^2} - 5522432M^7r^2\sqrt{8AM^2 + r^2} - 15950592M^5r^4\sqrt{8AM^2 + r^2} - 3941824M^3r^6\sqrt{8AM^2 + r^2} - 106576Mr^8\sqrt{8AM^2 + r^2} \left. \right) + \\ & 4A^3M^4r^4 \left( 159744M^{10} + 6629632M^8r^2 + 13818112M^6r^4 + 4332384M^4r^6 + 213472M^2r^8 + 681r^{10} - 754176M^9\sqrt{8AM^2 + r^2} - \right. \\ & 8008192M^7r^2\sqrt{8AM^2 + r^2} - 7182784M^5r^4\sqrt{8AM^2 + r^2} - 1005984M^3r^6\sqrt{8AM^2 + r^2} - 17074Mr^8\sqrt{8AM^2 + r^2} \left. \right) + \\ & 4A^2M^2r^6 \left( 68352M^{10} + 1081344M^8r^2 + 1349152M^6r^4 + 285488M^4r^6 + 9483M^2r^8 + 15r^{10} - 310016M^9\sqrt{8AM^2 + r^2} - 1251840M^7r^2\sqrt{8AM^2 + r^2} - \right. \\ & 665056M^5r^4\sqrt{8AM^2 + r^2} - 61184M^3r^6\sqrt{8AM^2 + r^2} - 639Mr^8\sqrt{8AM^2 + r^2} \left. \right) - 64A^5M^8 \left( 1890816M^8r^2 - 2272256M^6r^4 - 6297536M^4r^6 - 875520M^2r^8 - \right. \\ & 7278r^{10} + 1792M^9\sqrt{8AM^2 + r^2} - 2492160M^7r^2\sqrt{8AM^2 + r^2} + 1143968M^5r^4\sqrt{8AM^2 + r^2} + 1573520M^3r^6\sqrt{8AM^2 + r^2} + 80679Mr^8\sqrt{8AM^2 + r^2} \left. \right) \Big) \Big/ \\ & \left( (2AM^2 + r^2)^6 \sqrt{8AM^2 + r^2} \left( -2M + \sqrt{8AM^2 + r^2} \right)^5 \left( 8AM^2 + r^2 - 2M\sqrt{8AM^2 + r^2} \right)^3 \right) \end{aligned}$$

We see that when  $r$  tends to 0, the term takes the form of  $a^*X(r)+b$ , where  $a$  and  $b$  are parameters, and  $X$  is a function of  $r$ . At the origin of  $r=0$ ,  $b$  is a function that cannot be crossed out constant. The Euler characteristic at that time was 4.

#### 5. CONCLUSION AND DISCUSSION

Previous literature applied the RVB method to general relativity and various black holes under  $f(R)$  gravity [12–16]. Therefore, for many special black holes, whether the complex RVB coordinate system makes it difficult to calculate the temperature, we find that the Hawking temperature of the black hole under loop quantum gravity can be easily

obtained by using the RVB method. After using the RVB method in this paper, we found that there is a constant that cannot be crossed out in the calculation of the curvature term representing the geometric invariant. Constants can be parameter terms.

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