A PDE model approach to formation control of large-scale mobile sensor networks with boundary uncertainties

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Abstract

This paper investigates the consensus problem of an array of large mobile sensor networks. A new framework for the dynamic of mobile sensors as a continuum which described by parabolic system with boundary disturbance is proposed. The communication topology of agents is a chain graph and fixed. Leader feedback laws which is designed in a manner to the boundary control of large mobile sensor networks allow the mobile sensors achieve the formation steadily. By referring to Lyapunov functional method and employing boundary control approach, a new protocol is established to deal with formation problem for the large mobile sensor networks. Finally, numerical examples are given to illustrate the usefulness of the results.

Keywords:
Boundary control, formation control, mobile sensors, parabolic systems, boundary disturbance, consensus.

1. Introduction

During the last few decades, wireless sensor networks have been intensively explored and applied in widely fields[1, 2], including military, surroundings, health and home. A collection of sensing devices which are connected by wireless communication can be built into a wireless sensor network in engineering. In order to adapt to the requirement of flexibility in applications, the sensing nodes of wireless sensor networks can be endowed mobility using mobile devices. Mobile sensing devices reduce power consumption and costs while also improving performance. In the last few years, mobile sensor networks have attracted a lot more interest because they've been successful in a wide range of applications including formation control[3–5], target tracking[6–8], environmental monitoring[9, 10], and other areas. The study of cooperative control of mobile sensing networks has been viewed as a crucial issue. The paper [11] presents a robust control system for mobile sensors moving collaboratively in a distributed environment. A gradient climbing task in which mobile sensor networks can adapt its configuration in response to the measured environment. In [12], three distributed stable deployment algorithms based on flocking in mobile sensor networks are addressed. Also, from the view of swarm, control problem of wireless sensor networks with mobile multi-robots is discussed in [13-14]. Moreover, formation control of mobile sensor networks has been investigated intensively. A distributed control model is given to form the desired formation in [15]. The distributed observers are studied to achieve the formation in [16]. Stabilization of geometric pattern based on Laplacian are considered in [8,9,17]. Deployment onto a desired planar curve is investigated in [18] and [19] using coverage control algorithms and Lie group setting, respectively. In general, the model suggested in [20] or versions of these models are used in the majority of studies concerning mobile sensor networks

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models. The model presented by Reynolds contains 3 heuristic rules. They are alignment, cohesion and separation. Some additional rules such as obstacle avoidance capabilities have been suggested ever since. It is not difficult to see that these models are the particle system essentially. The discrete dynamic characteristics are exhibited in these models. Nevertheless, an increasing number of studies have found that continuity and layered property exist in biological systems,[21] social systems and so on. In [21], the authors proposed the starling swarm coordination for studying the behavior inertia and information transfer of the birds, which could study the cooperative flight of unmanned aerial vehicle. These phenomena show us that the system should be interpreted as a continuum when it is made up of a significant number of mobile sensing devices. That is to say large mobile sensor networks (LMSNs) is a fluid instead of a particle system. Therefore, the large mobile sensor networks should be modeled as partial differential equation (PDE). For the past few years, partial differential equation has been as an effective tool begins to analysis the multi-agent system [22-24]. In particular, [22] used LWR equation for crowd evacuation, [23] analyzes the stability of vehicular platoons based on a unstable hyperbolic system, and [24] models multi-agent as reaction-advection-diffusion equation for deployment onto spatial curves. Furthermore, growing results of experiments such that partial differential equation has gradually become an important way of system modeling. However, it should be pointed out that a PDE-based model for formation control of large mobile sensor networks still has little research attention. It is, therefore, the major purpose of this research is to investigate the formation issue for an array of large-scale mobile sensor networks based on parabolic systems with boundary disturbance. For the PDE model of mobile sensor networks, the boundary control approach is addressed. Such a framework which implies the communication topology of mobile sensors is a chain graph and fixed. Hence, stable formation onto planar curves can be achieved by leader, i.e. the boundary control of the leader such that mobile sensors will reach consensus. By constructing proper Lyapunov functional and employing boundary control approach, some sufficient conditions have been established to cope with the formation issue for large group mobile sensing devices. This study mainly contributes the following innovative points: (1) a framework for the dynamic of larger mobile sensors as a continuum and modeled by parabolic system is presented; (2) the design step about formation of larger mobile sensor networks is proposed; (3) boundary disturbance in the larger group mobile sensing devices is considered. In order to exhibit the utility of the proposed boundary control strategy, numerical simulations are provided.

2. Problem formulation and preliminaries

In this study, a new framework is proposed for the dynamic of a large group of mobile sensors as a continuum by parabolic system and assume that the mobile sensors are moving in 2-dimensional(2-D) planes. The dynamics of mobile sensor networks are described by the following integrator:

$$\frac{\partial \boldsymbol{x}(\theta,t)}{\partial t} = \boldsymbol{v}(\theta,t) \tag{1}$$

where $\boldsymbol{x}(\theta,t) = [x_1(\theta,t), x_2(\theta,t)]^T$ denotes the position of agent θ at time t. The parameter θ as the identity of each mobile sensor in a large mobile sensor network, and $\theta \in [0,1]$. Namely, θ is an mobile sensor's index code and as the spatial variable of mobile sensor network for the group's collective dynamics. $\boldsymbol{v}(\theta,t) = [v_1(\theta,t), v_2(\theta,t)]^T$ is the velocity for the agent θ and the control input $\boldsymbol{v}(\theta,t)$ is determined by

$$\boldsymbol{v}(\theta,t) = D\frac{\partial^2 \boldsymbol{x}(\theta,t)}{\partial \theta^2} \tag{2}$$

with diffusion operator $D = \mathrm{diag}[d_1, d_2]$, constants d_1, d_2 are both of positive.

Remark 1: In recent years, several important results about multi-agent system based on individual have been obtained. Double-integrator dynamics often can be presented as the model of multi-agent systems. The agent dynamics is expressed in the following:

$$\dot{x}_i(t) = \sum_{j\in\mathcal{N}_i}(x_j(t) - x_i(t)), \quad i = 1, 2, \cdots, n,$$

where $\mathcal{N}_i$ is the set of neighbors that communicate with agent i. It is not difficult to obtain an associated continuum model by continuous approximation as follows:

$$\frac{\partial x(\theta, t)}{\partial t} = \frac{\partial^2 x(\theta, t)}{\partial \theta^2},$$

where the velocity feedback of each agent(mobile sensor) employs the diffusion term.

The initial condition associated with (1) is given by

$$\boldsymbol{x}(\theta, 0) = \boldsymbol{x}_0(\theta) \tag{3}$$

where $\boldsymbol{x}(\theta, 0) = [x_1(\theta, 0), x_2(\theta, 0)]^T, \boldsymbol{x}_0(\theta) = [x_{10}(\theta), x_{20}(\theta)]^T$, and have the mixed boundary condition

$$\boldsymbol{x}(0, t) = [0, 0]^T, \qquad \frac{\partial \boldsymbol{x}(1, t)}{\partial \theta} = \boldsymbol{u}(t) + \boldsymbol{w}(t), \tag{4}$$

where $\boldsymbol{u}(t) = [u_1(t), u_2(t)]^T$ is the controls to be designed, $\boldsymbol{w}(t) = [w_1(t), w_2(t)]^T$ is an boundary disturbance, which assumes the following:

$$\int_0^{+\infty} \boldsymbol{w}^T(t)\boldsymbol{w}(t)dt < +\infty \tag{5}$$

It's worth mentioning that the initial condition should be compatible with the boundary condition, i.e.,

$$\left.\frac{d\boldsymbol{x}_0(\theta)}{d\theta}\right|_{\theta=0} = \mathbf{0}, \qquad \left.\frac{d\boldsymbol{x}_0(\theta)}{d\theta}\right|_{\theta=1} = \boldsymbol{u}(0) + \boldsymbol{w}(0).$$

A large group of mobile sensors as a continuum by system (1), index $\theta = 1$ is the leader sensor and $\theta = 0$ is the anchor sensor, the rest of index $0 < \theta < 1$ are the follower sensors. When proper boundary control law is designed such that $\lim_{t\to+\infty} \boldsymbol{x}(\theta, t) = \bar{\boldsymbol{x}}(\theta)$ for all $\theta \in [0, 1]$, then mobile sensors is said to achieve formation consensus. Meanwhile, a large group of mobile sensors is also said to achieve consensus if $\boldsymbol{u}(t) = \mathbf{0}$ for the leader. The velocity input of diffusion-based feedback indicates all the agents utilize only local information from nearest-neighbor. A decentralized communication topology is given, which is a chain graph.

Our main purpose in this paper is to accomplish the desired formation for an array of large mobile sensor networks based on heat equation with boundary disturbance by using boundary control approach. Clearly, the 2-D formation problem can be discussed in two aspects. On the one hand, what kind of formation can be reached by a continuum of mobile sensors under diffusion-based feedback? On the other hand, how to design the boundary controller such that mobile sensors stable reaches the formation?

3. Formation Control Scheme Design

First of all, we consider the diffusion-based feedback as velocity control input in continuum of mobile sensors. According to (1) and (2), the dynamic of mobile sensors are governed by heat equation

$$\frac{\partial \boldsymbol{x}(\theta, t)}{\partial t} = D\frac{\partial^2 \boldsymbol{x}(\theta, t)}{\partial \theta^2}. \tag{6}$$

The equilibrium equation of (6) is a second ordinary differential equation

$$\frac{d^2\bar{\boldsymbol{x}}(\theta)}{d\theta^2} = \mathbf{0} \tag{7}$$

It is not difficult to solve that the solution of (7) is a family of linear functions. Those functions can be decomposed to vertical and horizontal direction, respectively. The compact form of linear functions can be described as

$$\bar{\boldsymbol{x}}(\theta) = \begin{bmatrix} \bar{x}_1(\theta) \\ \bar{x}_2(\theta) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} 1 \\ \theta \end{bmatrix} \tag{8}$$

where constants a_1, a_2, a_3 and a_4 are formation coefficients, which are picked by user to generate the shape of the target formation. The type of formation curve is determined by the basis $(1, \theta)$ which is the solution of (7). Therefore, mobile sensors which governed by (6) have the capacity of achieving linear formations or rendezvousing at the origin.

Next, in general, boundary control approach is an effective way for large mobile sensor networks described by parabolic systems. Thus, we consider the leader agent($\theta = 1$) as boundary control input enable large mobile sensors reach the desired formation stably. The control law of leader can be designed as follows:

$$\boldsymbol{u}(t) = -K\boldsymbol{x}(1,t) + K\bar{\boldsymbol{x}}(1) + \frac{d\bar{\boldsymbol{x}}(1)}{d\theta}, \tag{9}$$

where $\boldsymbol{x}(1,t) = [x_1(1,t), x_2(1,t)]^T$ and $\frac{d\bar{\boldsymbol{x}}(1)}{d\theta}$ means $\frac{d\bar{\boldsymbol{x}}(\theta)}{d\theta}\big|_{\theta=1}$.

To achieve mobile sensors reach the formation, we shift the equilibrium $\bar{\boldsymbol{x}}(\theta)$ to the origin by transformation $\boldsymbol{y}(\theta,t) = \boldsymbol{x}(\theta,t) - \bar{\boldsymbol{x}}(\theta)$, the formation error dynamics for mobile sensors can be obtained from (3), (4) and (6) as follows:

$$\frac{\partial \boldsymbol{y}(\theta,t)}{\partial t} = D\frac{\partial^2 \boldsymbol{y}(\theta,t)}{\partial \theta^2} \tag{10}$$

$$\boldsymbol{y}(\theta,0) = \boldsymbol{y}_0(\theta) \tag{11}$$

$$\boldsymbol{y}(0,t) = \mathbf{0}, \quad \frac{\partial \boldsymbol{y}(1,t)}{\partial \theta} = \boldsymbol{U}(t) + \boldsymbol{w}(t) \tag{12}$$

where $\boldsymbol{y}(\theta,t) = [y_1(\theta,t), y_2(\theta,t)]^T$.

Substituting error $\boldsymbol{y}(\theta,t)$ into (4) and (8) leads to

$$\boldsymbol{U}(t) = -K\boldsymbol{y}(1,t). \tag{13}$$

In brief, the design procedure of formation deployment of large mobile sensors is listed in the following.

(1) Find equilibrium equation of dynamics model.
(2) Analyze the basis of equilibrium equation to determine the type of equilibrium curve component.
(3) Select the formation coefficients to determine the shape of equilibrium curve component.
(4) Orthogonal synthesis of the equilibrium curve of two coordinate axes to determine a target formation.
(5) Implement the control law for leader mobile sensor to achieve the formation.

4. Stability Analysis of Closed-loop System

To give our main results, we introduce the following definition and lemmas.

Definition 1[25] A parabolic system is said to be $L_2[0,1]$ stable, if

$$\lim_{t\to+\infty}\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta = \mathbf{0}$$

holds.

Definition 1 implies the closed-loop system state $\boldsymbol{x}(\theta,t)$ is $L_2[0,1]$ stable at the equilibrium curve $\bar{\boldsymbol{x}}(\theta)$.

Definition 2[26] A parabolic system is said to be globally asymptotically stable, if

$$\lim_{t\to+\infty}\boldsymbol{y}(\theta,t) = \mathbf{0}$$

holds.

Definition 2 implies $\lim_{t\to+\infty}\boldsymbol{x}(\theta,t) = \bar{\boldsymbol{x}}(\theta)$, that is to say, a large group of mobile sensors will converge into the formation $\bar{\boldsymbol{x}}(\theta)$.

Lemma 1[27] Let $z \in W^{1,2}(a,b)$ be a scalar function with $z(a) = 0$. Then

$$\int_a^b z^2(\xi)d\xi \le \frac{(b-a)^2}{2}\int_a^b\left(\frac{dz(\xi)}{d\xi}\right)^2 d\xi,$$

$$\max_{\xi\in[a,b]} z^2(\xi) \le (b-a)\int_a^b\left(\frac{dz(\xi)}{d\xi}\right)^2 d\xi.$$

Lemma 2[25] (The special case of Barbalat's Lemma) Let $f(t)$ be a non-negative function defined on $[0,+\infty)$. If $f(t)$ is differentiable on $[0,+\infty)$ and $\sup_{t\ge0}\frac{df(t)}{dt} < +\infty$, and $\int_0^{+\infty} f(t)dt < +\infty$. Then, there holds

$$\lim_{t\to+\infty} f(t) = 0.$$

The following theorems present the main outcomes of the study.

4.1. Boundedness

Theorem 1: Under the control scheme (13), the system (10) with boundary disturbance $\boldsymbol{w}(t)$ satisfying (5) and the initial condition $\boldsymbol{y}_0(\theta)$ satisfying $\int_0^{+\infty}\boldsymbol{y}_0^T(\theta)\boldsymbol{y}_0(\theta)d\theta < +\infty$ is uniformly square integrable, i.e.,

$$\sup_{\theta\in[0,1]}\int_0^{+\infty}\boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)dt < +\infty,$$

and $\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta$ is bounded on $[0,+\infty)$ if $\sigma_2 < \sigma_1$.

proof: Consider a Lyapunov functional as

$$V(t) = \frac{1}{2}\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta. \tag{14}$$

For the system (10), the time derivative of $V(t)$ can be computed as follows

$$\begin{aligned}\frac{dV(t)}{dt} &= \int_0^1 \boldsymbol{y}^T(\theta,t)\frac{\partial \boldsymbol{y}(\theta,t)}{\partial t}d\theta \\ &= \int_0^1 \boldsymbol{y}^T(\theta,t)D\frac{\partial^2 \boldsymbol{y}(\theta,t)}{\partial \theta^2}d\theta \\ &= \boldsymbol{y}^T(\theta,t)D\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}\bigg|_0^1 \\ &\quad - \int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}\right)^T D\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}d\theta \\ &\le \boldsymbol{y}^T(1,t)D\frac{\partial \boldsymbol{y}(1,t)}{\partial \theta} \\ &\quad - \int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}\right)^T D\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}d\theta\end{aligned}$$

By employing boundary control scheme (13), we have

$$\begin{aligned}\frac{dV(t)}{dt} \le &-\boldsymbol{y}^T(1,t)DK\boldsymbol{y}(1,t) + \boldsymbol{y}^T(1,t)D\boldsymbol{w}(t) \\ &- \int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}\right)^T D\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}d\theta \\ \le &(-\sigma_1+\sigma_2)\boldsymbol{y}^T(1,t)\boldsymbol{y}(1,t) \\ &- \sigma_3\int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}\right)^T \frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}d\theta \\ &+ \frac{1}{4}\boldsymbol{w}^T(t)\boldsymbol{w}(t), \qquad (15)\end{aligned}$$

where $\sigma_1 = \lambda_{\min}(DK)$, $\sigma_2 = \lambda_{\max}(D^2)$ and $\sigma_3 = \lambda_{\min}(D)$.

Integrating (15) over $[0,t]$, we get

$$\begin{aligned}&\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta \\ &\le \frac{1}{2}\int_0^t \boldsymbol{w}^T(\tau)\boldsymbol{w}(\tau)d\tau + \int_0^1 \boldsymbol{y}_0^T(\theta)\boldsymbol{y}_0(\theta)d\theta.\end{aligned}$$

Also, integrating (15) over $[0,+\infty]$, then

$$\begin{aligned}&\int_0^{+\infty}\bigg[(\sigma_1-\sigma_2)\boldsymbol{y}^T(1,t)\boldsymbol{y}(1,t) \\ &+ \sigma_3\int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}\right)^T \frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}\bigg]d\theta \\ &\le \frac{1}{4}\int_0^{+\infty} \boldsymbol{w}^T(t)\boldsymbol{w}(t)dt + \int_0^1 \boldsymbol{y}_0^T(\theta)\boldsymbol{y}_0(\theta)d\theta.\end{aligned}$$

Considering (5), it implies that $\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta$, $\int_0^{+\infty} \boldsymbol{y}^T(1,t)\boldsymbol{y}(1,t)dt$ and $\int_0^{+\infty}\int_0^1 (\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta})^T \frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}d\theta dt$ are bounded.

By using Lemma 1, we have

$$\begin{aligned}&\int_0^{+\infty}\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta dt \\ &\le \frac{1}{2}\int_0^{+\infty}\int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}\right)^T \frac{\partial \boldsymbol{y}(\theta,t)}{\partial \theta}d\theta dt,\end{aligned}$$

and

$$\int_0^{+\infty} \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)dt$$
$$\le \int_0^{+\infty}\int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta}\right)^T \frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta} d\theta dt,$$

which implies that $\boldsymbol{y}(\theta,t)$ is uniformly square integrable. The proof of Theorem 1 is completed.

4.2. $L_2[0,1]$ stability

Theorem 2: Under the control scheme (13), and boundary disturbance $\boldsymbol{w}(t)$ is bounded on $[0,+\infty)$, the closed-loop system (11) is $L_2[0,1]$ stable, i.e.,

$$\lim_{t\to+\infty}\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta = \mathbf{0},$$

if $\sigma_2 < \sigma_1$.

proof: In this part, the $L_2[0,1]$ stability of the closed-loop system (10) to be analysis. From (15) and Lemma 1, it is easy to see that

$$\frac{1}{2}\frac{d}{dt}\left(\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta\right) \le \frac{1}{4}\boldsymbol{w}^T(t)\boldsymbol{w}(t).$$

On account of boundary disturbance $\boldsymbol{w}(t)$ is bounded on $[0,+\infty)$, there exist a constant M, such that

$$\frac{d}{dt}\left(\int_0^1 \boldsymbol{y}^T(\theta,t)\boldsymbol{y}(\theta,t)d\theta\right) \le M \tag{16}$$

holds.

By the Invariance Principle, (16) implies that the state of closed-loop system (10) is $L_2[0,1]$ stable. This completes the proof of Theorem 2.

That is to say, a large mobile sensor network is $L_2[0,1]$ stable reaches the formation $\bar{\boldsymbol{x}}(\theta)$ under the guidance of our leader.

In the following, two valuable special scenarios are considered.

4.3. Special scenarios

4.3.1. Scenario 1

When the boundary disturbance $\boldsymbol{w}(t)$ vanishes in the closed-loop system (10), we have the following result easily.

Theorem 3: Under the control scheme (13), and boundary disturbance $\boldsymbol{w}(t) = \mathbf{0}$ and $\int_0^1 \left(\frac{\partial \boldsymbol{y}_0(\theta)}{\partial\theta}\right)^T \frac{\partial \boldsymbol{y}_0(\theta)}{\partial\theta} d\theta$ is bounded, the closed-loop system (10) with auxiliary boundary condition $\frac{\partial \boldsymbol{y}(0,t)}{\partial\theta} = \mathbf{0}$ is globally asymptotically stable, i.e.,

$$\lim_{t\to+\infty}\boldsymbol{y}(\theta,t) = \mathbf{0},$$

if $\sigma_2 < \sigma_1$.

proof: To discuss the globally asymptotically stability of the closed-loop system (10), we introduce the auxiliary function as follows:

$$g(t) = \frac{1}{2}\int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta}\right)^T \frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta} d\theta$$
$$+\frac{1}{2}K\boldsymbol{y}^T(1,t)\boldsymbol{y}(1,t) \tag{17}$$

Calculating the time derivative of $g(t)$, we obtain

$$\begin{aligned}\frac{dg(t)}{dt} &\leq \int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta}\right)^T \frac{\partial^2 \boldsymbol{y}(\theta,t)}{\partial\theta\partial t} d\theta \\ &\quad + K\boldsymbol{y}^T(1,t)\frac{\partial \boldsymbol{y}(1,t)}{\partial t} \\ &\leq \left(\frac{\partial \boldsymbol{y}(1,t)}{\partial\theta}\right)^T \frac{\partial \boldsymbol{y}(1,t)}{\partial t} \\ &\quad - \int_0^1 \left(\frac{\partial^2 \boldsymbol{y}(\theta,t)}{\partial\theta^2}\right)^T \frac{\partial^2 \boldsymbol{y}(\theta,t)}{\partial\theta^2} d\theta \\ &\leq -\int_0^1 \left(\frac{\partial^2 \boldsymbol{y}(\theta,t)}{\partial\theta^2}\right)^T \frac{\partial^2 \boldsymbol{y}(\theta,t)}{\partial\theta^2} d\theta \leq \mathbf{0}.\end{aligned} \tag{18}$$

Integrating (18) over $[0,1]$, we derives

$$\begin{aligned}g(t) &\leq g(0) \\ &= \frac{1}{2}\int_0^1 \left(\frac{\partial \boldsymbol{y}_0(\theta)}{\partial\theta}\right)^T \frac{\partial \boldsymbol{y}_0(\theta)}{\partial\theta} d\theta \\ &\quad + \frac{1}{2}K\boldsymbol{y}_0^T(1)\boldsymbol{y}_0(1),\end{aligned}$$

which implies that $g(t)$ is bounded on $[0,+\infty)$.

Accordingly, $\int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta}\right)^T \frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta} d\theta$ and $\boldsymbol{y}(1,t)$ are bounded on $[0,+\infty)$.

Moreover, by using Lemma 1, we have

$$\max_{\theta\in[0,1]} |\boldsymbol{y}(\theta,t)| \leq \sqrt{\int_0^1 \left(\frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta}\right)^T \frac{\partial \boldsymbol{y}(\theta,t)}{\partial\theta} d\theta},$$

and thus $\boldsymbol{y}(\theta,t)$ is uniformly bounded.

Noting that $\boldsymbol{y}(\theta,t)$ is once continuously differentiable for any $t\in[0,+\infty)$, we can conclude from Lemma 2 that $\lim_{t\to+\infty}\boldsymbol{y}(\theta,t)=\mathbf{0}$.

In other words, the state $\boldsymbol{y}(\theta,t)$ of closed-loop system (10) converges to zero. In this case, a large mobile sensor network is global asymptotic stability reaches to the formation $\bar{\boldsymbol{x}}(\theta)$ under the guidance of our leader.

4.3.2. Scenario 2

In the following, we consider the case when the system (10) has no control law of leader, i.e. the boundary condition of system (10) satisfies Dirichlet boundary condition $\boldsymbol{y}(0,t)=\boldsymbol{y}(1,t)=\mathbf{0}$. From what has been discussed above, the corollary is easily obtained from Theorem 3.

Corollary 1: Under boundary disturbance $\boldsymbol{w}(t)=\mathbf{0}$ and $\int_0^1 \left(\frac{\partial \boldsymbol{y}_0(\theta)}{\partial\theta}\right)^T \frac{\partial \boldsymbol{y}_0(\theta)}{\partial\theta} d\theta$ is bounded, the closed-loop system (10) with Dirichlet boundary condition is globally asymptotically stable, i.e.,

$$\lim_{t\to+\infty}\boldsymbol{y}(\theta,t)=0,$$

if $\sigma_2<\sigma_1$.

That is when no leader in a large mobile sensor network, mobile sensors will reach consensus at zero globally asymptotically stable under the condition of Corollary 1.

5. Simulation example

In this section, we present the simulation results to justify the Theorem 2 and Corollary 1 obtained above. In the following examples, we consider the mobile sensor networks including 10 mobile sensors to deployment. The boundary control law (13) is implemented by the leader sensor. In this manner, the mobile sensor θ can stable move to the desired formation rapidly.

In simulation, the initial positions of mobile sensors are sampled from the Gaussian distribution $\mathcal{N}(0,1)$ and boundary disturbance $\boldsymbol{w}(t) = [e^{-\frac{t}{2}}, \frac{1}{2t+1}]^T$. The coefficient of diffusion operator $D = 0.015I$.

Our first scenario focuses on the mobile sensors in a straight line under control law of leader. The feedback gain of boundary controller is considered as follows $K = \mathrm{diag}[5,8]$. Fig. 1 depicts the spatio-temporal trajectories of the mobile sensors, which show that 10 agents move onto desired straight line in time interval $[0,40]$. Fig. 1 is also projected onto the plane x_1x_2 to show the motion of the mobile sensors on 2D clearly. The simulations above verify the Theorem 2.

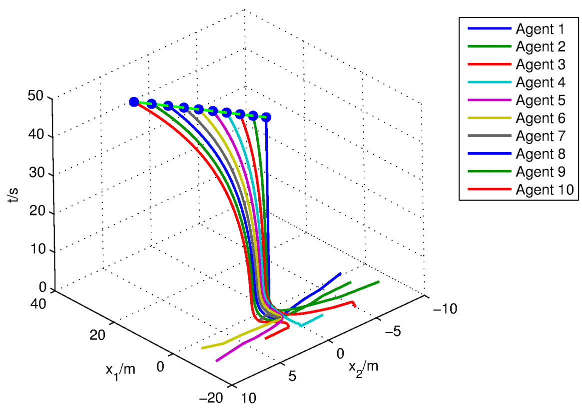

Figure 1: The spatio-temporal trajectory of mobile sensors.

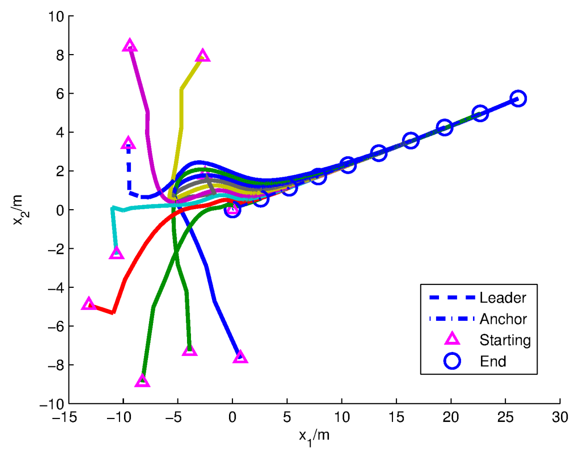

Figure 2: 2D trajectory of mobile sensors.

Next, Fig. 3 is formed using parameter $K = \mathbf{0}$, which can verify the Corollary 1. It is also clear that distributed parameter multi-agent system under feedback input $\boldsymbol{v}(\theta,t) = \frac{\partial^2 \boldsymbol{x}(\theta,t)}{\partial \theta^2}$ will rendezvous at the origin if no boundary controller implemented by the leader. Fig. 4 demonstrate 2D trajectory of mobile sensors. As can be seen from it, 10 agents rendezvous at the origin on the projection plane x_1x_2.

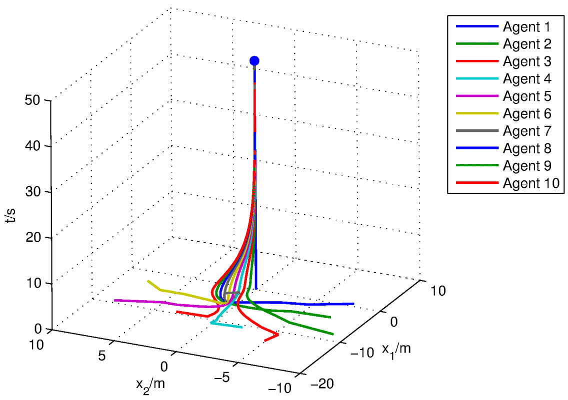

Figure 3: The spatio-temporal trajectory of mobile sensors.

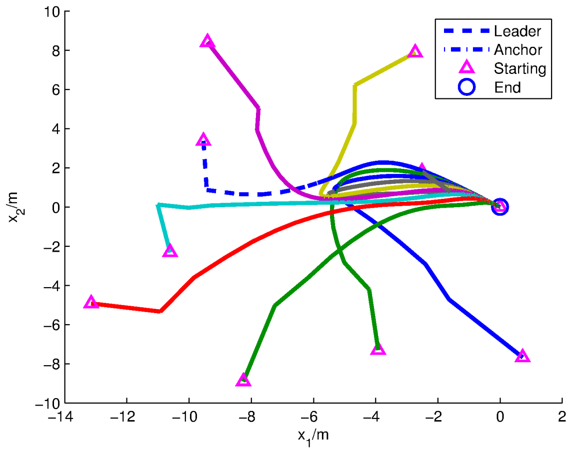

Figure 4: 2D trajectory of mobile sensors.

6. Conclusions

This paper has studied the formation problem for an array of large mobile sensor networks. A new framework of mobile sensor networks has been introduced by modeling the dynamic of mobile sensors as a continuum which described by parabolic system with boundary disturbance such that the model of system more conform to the realities of situation in the nature. In this model, the communication topology of agents is a chain graph and fixed. By utilizing the Lyapunov functional method and boundary control approach, leader feedback laws which are designed in a manner to the boundary control of large mobile sensor networks allow the mobile sensors stable achieve the formation. In fact, the large mobile sensor network addressed can be used for agricultural applications, the formation of unmanned aerial systems, and so on. Simulation results further illustrated the effectiveness of our results.

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