

# On the ground states for the X-ray free electron lasers Schrödinger equation

Hangzhou Hu<sup>1</sup>, Yuan Li<sup>2</sup>, and Dun Zhao<sup>1</sup>

<sup>1</sup>Lanzhou University

<sup>2</sup>Central China Normal University

April 7, 2022

## Abstract

We consider the following X-ray free electron lasers Schrödinger equation 
$$(i\nabla - A)^2 u + V(x)u - \frac{\mu}{|x|} u = \left( \frac{1}{|x|} |u|^2 \right) u - K(x) |u|^{q-2} u, \quad x \in \mathbb{R}^3,$$
 where  $A \in L^2_{loc}(\mathbb{R}^3, \mathbb{R}^3)$  denotes the magnetic potential such that the magnetic field  $B = \text{curl } A$  is  $\mathbb{Z}^3$ -periodic,  $\mu \in \mathbb{R}$ ,  $K \in L^{\infty}(\mathbb{R}^3)$  is  $\mathbb{Z}^3$ -periodic and non-negative,  $q \in (2, 4)$ . Using the variational method, based on a profile decomposition of the Cerami sequence in  $H^1_A(\mathbb{R}^3)$ , we obtain the existence of the ground state solution for suitable  $\mu \geq 0$ . When  $\mu < 0$  is small, we also obtain the non-existence. Furthermore, we give a description for the asymptotic behaviour of the ground states as  $\mu \rightarrow 0^+$ .

## Hosted file

X-ray.pdf available at <https://authorea.com/users/474516/articles/564315-on-the-ground-states-for-the-x-ray-free-electron-lasers-schr%C3%B6dinger-equation>