Algebraic Lq-norms and complexity-like properties of Jacobi polynomials-Degree and parameter asymptotics

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Abstract

The Jacobi polynomials $\hat{P}_n^{(\alpha,\beta)}(x)$ conform the canonical family of hypergeometric orthogonal polynomials (HOPs) with the two-parameter weight function $(1-x)^\alpha (1+x)^\beta, \alpha,\beta>-1,$ on the interval $[-1,+1]$. The spreading of its associated probability density (i.e., the Rakhmanov density) over the orthogonality support has been quantified, beyond the dispersion measures (moments around the origin, variance), by the algebraic $\mathfrak{L}_{q}$-norms (Shannon and R\'enyi entropies) and the monotonic complexity-like measures of Cram\'er-Rao, Fisher-Shannon and LMC (L\'opez-Ruiz, Mancini and Calbet) types. These quantities, however, have been often determined in an analytically highbrow, non-handy way; specially when the degree or the parameters (α,β) are large. In this work, we determine in a simple, compact form the leading term of the entropic and complexity-like properties of the Jacobi polynomials in the two extreme situations: ($n\rightarrow \infty$; fixed α,β) and ($\alpha\rightarrow \infty$; fixed n,β). These two asymptotics are relevant \textit{per se} and because they control the physical entropy and complexity measures of the high energy (Rydberg) and high dimensional (pseudoclassical) states of many exactly, conditional exactly and quasi-exactly solvable quantum-mechanical potentials which model numerous atomic and molecular systems.

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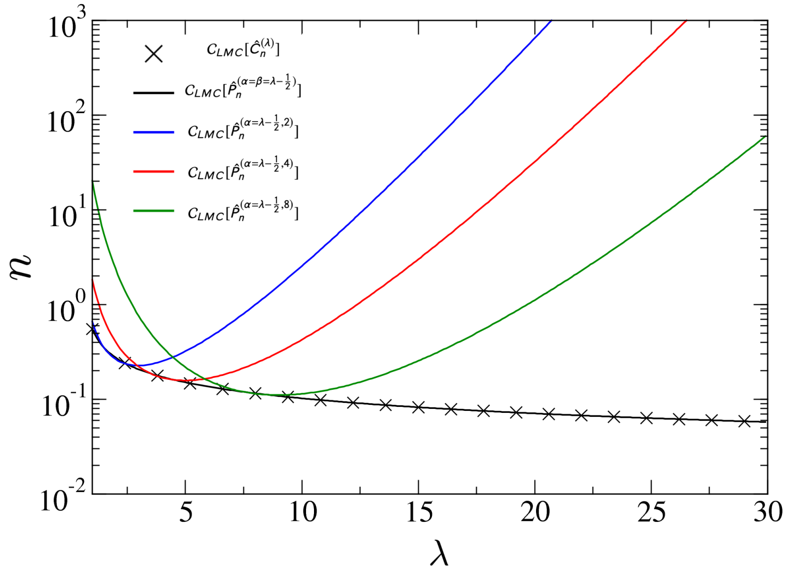
$C_{LMC}[\hat{C}_n^{(\lambda)}]$
$C_{LMC}[\hat{P}_n^{(\alpha=\beta=\lambda-\frac{1}{2})}]$
$C_{LMC}[\hat{P}_n^{(\alpha=\lambda-\frac{1}{2},2)}]$
$C_{LMC}[\hat{P}_n^{(\alpha=\lambda-\frac{1}{2},4)}]$
$C_{LMC}[\hat{P}_n^{(\alpha=\lambda-\frac{1}{2},8)}]$
n
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
5
10
15
20
25
30
λ