The Brezis-Nirenberg problem for fractional systems with Hardy potentials

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Abstract

In this work we study the existence of positive solutions to the following fractional elliptic systems with Hardy-type singular potentials, and coupled by critical homogeneous nonlinearities \begin{equation*} \begin{cases} (-\Delta)^{s}u-\mu_{1}\frac{u}{|x|^{2s}}=|u|^{2^{\ast}_{s}-2}u+\frac{\eta\alpha}{2^{\ast}_{s}}|u|^{\alpha-2} |v|^{\beta}u+\frac{1}{2}Q_{u}(u,v) \ \ in \ \Omega, \\[2mm] (-\Delta)^{s}v-\mu_{2}\frac{v}{|x|^{2s}}=|v|^{2^{\ast}_{s}-2}v+\frac{\eta\beta}{2^{\ast}_{s}}|u|^{\alpha} |v|^{\beta-2}v+\frac{1}{2}Q_{v}(u,v) \ \ in \ \Omega, \\[2mm] \ \ u, \ v>0 \ \ \ \ \ in \ \ \Omega, \\[2mm] \ u=v=0 \ \ \ \ in \ \ \mathbb{R}^{N}\backslash\Omega, \end{cases} \end{equation*} where $(-\Delta)^{s}$ denotes the fractional Laplace operator, $\Omega\subset\mathbb{R}^{N}$ is a smooth bounded domain such that $0\in\Omega$, $\mu_{1}, \mu_{2}\in [0,\Lambda_{N,s})$, $\Lambda_{N,s}=2^{2s}\frac{\Gamma^{2}(\frac{N+2s}{4})}{\Gamma^{2}(\frac{N-2s}{4})}$ is the best constant of the fractional Hardy inequality and $2^{*}_{s}=\frac{2N}{N-2s}$ is the fractional critical Sobolev exponent. In order to prove the main result, we establish some refined estimates on the extremal functions of the fractional Hardy-Sobolev type inequalities and we get the existence of positive solutions to the systems through variational methods.

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