Optimized Distance Range Free Localization Algorithm for WSN

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Abstract

DV-Hop and its various improvements overexpose hop size and hop count for localization. Whereas hop size is always erroneous and hop path is not a straight line, which leads to a faulty location estimation. The proposed model Optimized Distance Range Free (ODR) localization algorithm limits the use of hop size and hop count to approximate nearly a straight line distance between a known and an unknown node without additional hardware and without increasing extra communication. The refrain use of hop size and hop count improves localization accuracy of ODR and makes it robust against those network variables which affect the hop size accuracy adversely. In fact ODR modifies the last two steps of DV-Hop. DV-Hop finds hop size in its second step. Here ODR rectifies this hop size and then a centroid is obtained from the minimum distant anchor nodes for an unknown node. Now a minimum possible distance known as base distance is estimated with a routing table assistance. In the last step DV-Hop uses least square regression to localize, while ODR exploits linear optimization to comprehend the base distance for localization. The paper establishes ODR analytically and rugged with ranging error of the omnidirectional antenna coverage pattern experimentally.

1. Introduction:

Today we have a vast range of different applications of WSN. Among these applications [2] some of the applications are - environment monitoring, health monitoring, industrial monitoring, hazard detection, deep-sea underwater life monitoring, wildlife movement, defense operations and many more of similar nature. WSN in itself faces a lot of challenges, like- self-management [24], limited computational power [24, 2], decentralized processing, scalability [24], physical topology, transmitter and receiver's communication range, ad-hoc routing [2], etc., to prove its worth. Considering these challenges of WSN, in the above different applications, if we receive data without information about the location of its source then the received data carries no sense or simply it is worthless. This makes the localization (location estimation of the data source) a significant area of WSN and its allied field IoT (Internet of Things) [5, 3]. While using the Global Positioning System (GPS) location of a node can be obtained in the simplest way [14]. But it makes the network a cost-intensive affair, which makes it inappropriate where a good amount of sensors are required. At the same time, more energy and computational power are also needed due to satellite communication [19, 14]. On the contrary solution for WSN must be moderately scalable, energy-efficient and robust also. Therefore, in recent times researchers have been suggested various approaches to localize a sensors' position. Extensively these methodologies can be arranged in two different categories - 1) Range Based Localization and 2) Range Free Localization. There is always contention between the two categories.

Range based [13] or direct methods are dependent on some parameters, such as – a time of arrival (ToA), received signal strength indicator (RSSI) [17, 1 and 9], time difference of arrival (TDoA), and angle of arrival (AoA) [22]. Most of these methods need to incorporate additional hardware in sensors. The additional requirements make it less attractive to deal with the objectives of less energy consumption (to raise the life span), and high scalability. Therefore even with good accuracy, range based algorithms are not suitable where the repeated deployment of sensors is difficult and data collection is needed for a quite long period. Therefore considering the cost of network and energy efficiency, we need a different approach which is known as range free method. In the range free category, researchers contributed with the help of various models. These models based on their fundamental theme can be summarized like - Centroid based DV-Hop algorithm-based, and Analytical Geometric Shape analysis methods, etc. [4].

Over time DV-Hop algorithm [15] dominates among the localization methods of indirect communication. In this algorithm by using some anchor nodes (i.e. the nodes known their location using GPS) location of remaining nodes of the network is estimated. In this algorithm, the location estimation of nodes does not require any additional hardware. At the same time, it is energy efficient because of its less computational complexity. The less complexity and no requirement of additional hardware make the DV-Hop algorithm a remarkable milestone in this area. The location estimation by the DV-Hop algorithm is not precise yet it exhibits a pathway to localize a node without any extra burden on a network.

To estimate location precisely, various improvements of the DV-Hop algorithm have been suggested. Stefan Tomic and Ivan Mezei [23] improve the DV-Hop algorithm by considering three different cases. After applying the basic DV-Hop algorithm their method finds the communication range intersections of the minimum hop distant anchor nodes. The intersection coordinates of the communication range used to calculate the centroid as the location of the intended unknown node. The method suffers because of the incorporation of the hop size of DV-Hop without any correction, which is further used to calculate the radius of communication of the unknown nodes. Lu Jian Yin [12] uses the centroid method by proposing the use of half measure weight method. The method highlights the improvement by showing low localization error. The suggested weight value needs to estimate the distance between unknown and known nodes, which is still based upon the hop distance calculation as given by the DV-hop algorithm. Gulshan Kumar et al. [8] ameliorate by creating a dominating group of anchor nodes. The anchor nodes at one hop distance and with maximum degree join the dominating set. The suggested modification in itself poses a limitation to its range free architecture. Laizhong Cui et al. [10] propose a term – continuous hop count. The continuous hop count along with the average distance estimates the probable distance value for an unknown node. The method employs the average distance estimation as that of the basic DV-Hop algorithm with its shortfalls. Penghong Wang et al. [16] improves this localization algorithm as an optimization problem in terms of distance estimation, where hop size is calculated as an average hop size. The average hop size is presented as a constant multiple of communication radius. Whereas a network may have anchor nodes with different effective hop sizes. The DV-Hop algorithm is also dependent upon an average value to observe the hop size as a limitation. Wei Zhao et al. [27] contribute a method to improve the performance by a weighted method. Anchor nodes get the weights inversely proportional to their distance from the unknown node. It also comprises an experimental factor, which may vary from three hops distance onwards to get the contribution of minimum possible anchor nodes only. The algorithm offers a better localization but at the cost of computational overhead. Jing-li YANG and Qi-Shen ZHU [6] propose an improvement by defining optimization constraints. The constraints are characterized with the help of the distance between anchor nodes and the unknown nodes. The distance estimation is considered as that of the fundamental DV-Hop calculation without any up-gradation. On a similar line, Kumar and Lobiyal [21] characterize optimization constraints on the distance equations between anchor nodes and the unknown nodes. Though the paper [21] can consider the role of the error propagation in localization but unable to make any attempt to reduce it before going for the final location estimation of the nodes. Y. Chen et al. [28] carve an enhanced algorithm by finding error in hop size and calculated improved hop size of nodes. In the next step, distance is estimated with the help of a weight coefficient factor dependent upon hop counts. On the same line of action, W. Fang et al. [26] calculated the weight coefficient factor as a ratio of the estimated and actual hop counts to calculate the updated distance values.

In short, the DV-Hop algorithm and its proposed improvements calculate distances of an unknown node from anchor nodes. In the next step, this distance is used to spot the unknown node. But the distance is estimated as a summation of the hops' sizes in-between anchor node and the unknown node. This distance is not Euclidean distance. The proposed algorithm in this paper is an effort to estimate the location with the help of a minimum possible distance which is near to Euclidean distance. The distance estimated by

ODR is not exactly a straight line because after the base distance (which is a straight line) a comprehensive distance value is also estimated.

Hereafter the paper is organized in different sections. Section 2 highlights the DV-Hop algorithm and followed by a section to present its weakness by error analysis in Section 3. After this, the next section that is Section 4 discusses a significant modification in DV-Hop as Improved DV-Hop (IDV) algorithm. Subsequently, the next two sections cover the proposed model ODR analytically. Here Section 5 elaborates ODR step by step and Section 6 throws light on the cost of the suggested model (i.e. ODR) to the network. Then Section 7 establishes ODR with the help of simulation results and performance analysis. In the last, the paper concludes its achievements under Section 8.

2. DV-Hop Algorithm

Nath et al. [15] propose this algorithm. It has three steps. In the first step of the algorithm, all the anchor nodes flood the information about its location and their identification with their one-hop distant nodes only. After this, every node of the network spread out this information further by incrementing the hop count value by one to their immediate neighbours. Likewise, the whole network gets the hop count value for their neighbours. If at any instant a node receives again the information about the same node then it (i.e. receiver node) compares the hop count value with the previous value and makes the due changes in its routing table.

In the second step, each anchor node, say $'p'$, calculates its average hop size H_{size_p} by using the equation (1).

$$H_{\mathrm{size}_p} = \frac{\sum_{s\epsilon(A_n-p)} \sqrt{\left(x_p - \ x_s\right)^2 + \left(y_p - \ y_s\right)^2}}{\sum_{s\epsilon(A_n-p)} H_{\mathrm{count}_{\mathrm{ps}}}};\ \forall\ p \neq s\ and\ p, s\ \epsilon\ A_n\ (1)$$

where $H_{\mathrm{count}_{\mathrm{ps}}}$ is a minimum number of hop count between two anchor nodes $'p^{'}, 's'$ from the available anchor nodes set A_n; and$(x_p,\ y_p)\,,\ (x_s,\ y_s)$ are the coordinates of $'p^{'}, 's'$ respectively. Every anchor node spread out its average hop size value in the network. Now after obtaining average hop size value, an unknown node (U) calculates its distance $(d_i)$from respective anchor node (p) by using equation (2).

$$d_i = H_{\mathrm{size}_p} \times H_{\mathrm{count}_{\mathrm{pU}}};\ \forall i\epsilon A_n \quad amp; (2)$$

In the third step of this algorithm, an unknown node (U) estimates its location $(x,\ y)$ by using the distance equation (3).

$$\left(x_i - x\right)^2 + \left(y_i - y\right)^2 = {d_i}^2;\ \forall i\epsilon A_n \quad amp; (3)$$

where d_i is a distance calculated by $'U'$ from the equation (2) with respect to anchor nodes $i = 1,\ 2,\ \ldots,\ m$ located at$(x_i,\ y_i)$.

Further, the equation (3) is converted into a linear set of equations by subtracting all its equations of (3) by anyone equation from it, which yields equation (4).

$$\left(x_i^2 + y_i^2\right) - \left(x_m^2 + x_m^2\right) - 2\left(\left(x_i - x_m\right)x + \left(y_i - y_m\right)y\right) = d_i^2 - d_m^2 \quad amp; (4)$$

The equation (4) can be rewritten in a matrix form of $PQ = R$, where$Q^{'} =$

$\begin{bmatrix} x & amp; y \end{bmatrix}$ *and* P, R are –

$$P = 2\begin{bmatrix} (x_1 - x_m) & amp; & (y_1 - y_m) \\ (x_2 - x_m) & amp; & (y_2 - y_m) \\ \vdots & & \vdots \\ (x_{m-1} - x_m) & & (y_{m-1} - y_m) \end{bmatrix}, \quad amp; R = \quad amp; \begin{bmatrix} x_1^2 + y_1^2 - (x_m^2 + y_m^2) - d_1^2 + d_m^2 \\ x_2^2 + y_2^2 - (x_m^2 + y_m^2) - d_2^2 + d_m^2 \\ \vdots \\ x_{m-1}^2 + y_{m-1}^2 - (x_m^2 + y_m^2) - d_{m-1}^2 + d_m^2 \end{bmatrix}$$

Now, by applying the least square method the value of$Q = \left(P^{'}P\right)^{-1}P^{'}R$ is obtained and the algorithm reveals the location of the unknown node$'U'$.

3. Error Analysis of DV-Hop Algorithm

The DV-Hop algorithm is fully dependent upon the calculation of the distance values obtained with the help of hop size (or average hop size is the same in the paper) of each anchor node. This hop size is an average value only. The algorithm assumes that all the hops are of the same size as an anchor node. The conversion of the hop size and the hop count into a distance value is an erroneous calculation because it has no intention to find Euclidean distance. After it, the poor estimated distance values passed to a regression method for localization. The ill-fated distance values just propagate error and yields a poor estimation of the location only. With the help of an example, the algorithm and its limitations can be analyzed. Here the Fig. 1 creates a scenario of the random positions of anchor and unknown nodes.

Fig. 1: Error analysis of the DV-Hop algorithm.

Now, applying the equation (1) we obtain the hop sizeH_{size_A} for the anchor node $'A'$ about Fig. 1, as-

$$H_{\text{size}_A} = \frac{\sqrt{\left(x_A - \ x_B\right)^2 + \left(y_A - \ y_B\right)^2} + \ \sqrt{\left(x_A - \ x_C\right)^2 + \left(y_A - \ y_C\right)^2}}{H_{\text{count AB}} + \ H_{\text{count AC}}}$$

$$= \frac{41.23 + 100}{7 + 6};$$

where $H_{\text{count AB}} = 7$ (i.e. $(A, 4)\,,\ (4,\ 3)\,,\ (3,\ 2)\,,\ (2,\ 1)\,,\ (1,\ 6)\,,\ (6,\ 5)\,,\ (5,\ B))\,;$ *similarly* $H_{\text{count AC}} = 6$

$$=> \ H_{\text{size}\,A} = 10.86,$$

$$\text{similarly } H_{\text{size}\,B} = \ 9.50 \text{ } and \text{ } H_{\text{size}\,C} = 15.71$$

By using the obtained hop size of anchor nodes $A,\ B,\ and\ C$ the unknown node $'U'$ estimates the distance from the anchor nodes A, B and C are 43.44, 28.5 and 31.42 respectively. Whereas the actual distance between unknown node $'U^{'}$ and the anchor nodes$A,\ B,\ and\ C$ are 58.31, 41.23 and 42.43 respectively. It shows a significant difference between the actual distance and the estimated distance. This inadequate distance estimation becomes a cause of poor localization.

The example is drawn with the help of Fig. 1 can showcase the error-prone nature of the DV- Hop algorithm. This error is evident because of the irregular radiation pattern of the omnidirectional antenna of a sensor apart from other factors. Ideally, the omnidirectional antenna has a figure of eight pattern [22] as shown by Fig. 2(a) but it (i.e. omnidirectional antenna) exhibits an irregular pattern [20] as shown in Fig. 2(b). This irregularity of the pattern introduces a ranging error which contributes faulty estimation of the distance between an unknown node and an anchor node.

Fig. 2: Omnidirectional radiation pattern- (a) Ideal pattern, and (b) Irregular pattern.

Therefore the erroneous nature, of the DV-Hop algorithm and its various improvements, is because of the poor distance estimation between anchor node and the unknown node. This distance is impractical because it is calculated with the support of an inadequately estimated hop size and moreover it (i.e. distance) is far away from a Euclidean distance. The proposed method in this paper is based upon the fact that the minimum distance estimation in-between anchor node and an unknown node is a close approximation of Euclidean distance in-between the two (i.e. an anchor node and an unknown node).

Consequently, to localize a node with more accuracy, a model should focus on two points- 1) approximated correction of hop size, and 2) obtain Euclidean distance up to a maximum possible extent. The proposed model ODR thrust upon these two points. To examine the proposed ODR significance, it is closely studied about a recent projected Improved DV-Hop (IDV) algorithm by S. Shen et al. [18]. The algorithm [18] employs hyperbolic function based upon the distance equations of an unknown node and an anchor node, as explained below.

4. Improved DV-Hop Algorithm (IDV) [18]

In this algorithm, localization has been improved by modifying the hop size. The correction factor ($\varnothing_p$) for hop size of an arbitrary anchor node $'p'$ is obtained by calculating a difference of the distance values obtained as per DV-Hop distance estimation method and the actual distance in-between every anchor node pair, shown by the following equation-

$$\varnothing_p = \frac{\sum_{p \neq s} \left(\left| \left(H_{\text{size}p} \times H_{\text{count}_{\text{ps}}} \right) - d_{\text{ps}} \right| \right)}{\sum_{p \neq s} H_{\text{count}_{\text{ps}}}}; \ \forall \ p \neq s \ and \ p, s \ \epsilon \ A_n;$$

where d_{ps} is an actual distance between anchor nodes$'p'$ and $'s'$.

Further to convert the hop size into the distance, IDV [18] does not consider all the anchor nodes. Instead, it finds out the hop counts which must be traversed from the unknown node to have at least three anchor nodes only. To find out the minimum number of hop counts, it employs probability based upon anchor nodes density per unit area.

Though the suggested modifications by IDV [18] can reduce the localization error considerably it draws some shortfalls also. The hop size correction factor $'\varnothing_p'$ contributes a noteworthy complexity of the order of$'m^{2}'$($if\ there\ are\ 'm'\ number\ of\ anchor\ nodes\ only$). Further, the localization accuracy is dependent upon a degree of the randomness of the nodes' distribution. The degree of randomness of the distribution of nodes will affect its performance. Because by keeping the same number of anchor nodes and unknown nodes; the anchor node's density value will remain the same but it doesn't ensure that at every instance at a distance of fix hop counts from every unknown node there will be at least three anchor nodes always. It implies that IDV [18] is less robust and unable to keep the computational complexity low but at the same time it localizes with improved accuracy in the case of dense networks only at the cost of poor latency.

The proposed model ODR in this paper improves the hop size also but with a computation requirement of the order of $'m'$ only. The ODR localizes with high accuracy and robust in comparisons to IDV [18] because it does not depend upon any of the terms which are predetermined as the density of a node of IDV [18]. Further ODR localization can calculate approximated Euclidean distance whereas IDV [18] is dependent upon the distance values obtained through discrete calculation of hop count and hop size.

5. Proposed Work: The proposed algorithm ODR has three steps. Its first step is the same as that of the DV-Hop algorithm. In the second step, the hop size is improved by calculating average hop size error. The improved hop size and the centroid of the minimum hop distant anchor nodes are used to find an approximated region in which the unknown node should exist. The approximated region is calculated by using the routing table of the respective anchor nodes. This approximated region assists to calculate a base distance which is a minimum possible distance between the unknown node and the anchor nodes($A_n - K$); where $'A_n'$ is a set of all anchor nodes and 'K' is a set of anchor nodes at a minimum hop distance from the unknown node. This base distance is a probable straight line distance between an anchor node and an unknown node. The straight line distance overcomes the drawback of the zigzag distance measured by DV-Hop and IDV algorithms. In the third step, we localize the unknown node. The base distance calculated in the second step is not a complete distance between the unknown node and the anchor nodes($A_n - K$). To make it complete we add a respective comprehensive distance value in the base distance and which is known as complete distance. Further by using linear optimization we try to minimize the comprehensive distance to get high localization accuracy.

Here we explain all the steps of ODR one by one.

Step 1.

Network Establishment: To establish the network each anchor node under its communication range supply its identification serial number, position, and hop count. The receivers (anchor and unknown nodes) update their routing table with the obtained information by adding one in the received hop count of the respective row of the routing table. A receiver may get this information repeatedly from other paths also. The receivers select the minimum hop count path by comparing the received values with the already available values in its routing table and also obtain the position of the anchor nodes.

Step 2.

Base Distance Estimation: For location estimation in DV- Hop and its improvements, hop size and hop count information is used massively. But hop size is always erroneous and the hop path is not a straight line, which is the cause of a faulty location of the unknown node. To overcome this problem, in this step we calculate the base distance. The calculation of the base distance is explained by considering Fig. 3.

Fig. 3: Minimum Base Distance estimation of unknown node nearest to a single anchor node.

As shown in this figure, the actual distance between any anchor pair (out of the available anchor nodes- 'A', 'B', and 'C') is most likely less than what can be drawn with the help of hop size and hop counts. The unknown node 'U' is needed to localize. Now instead of using hops to find out the distance, the model try to establish a probable point for the unknown node 'U' only; so as a minimum distance from some of the anchor nodes can be observed. Therefore to estimate the minimum distance, find out the anchor nodes nearest to the 'U', which are known as minimum hop distant anchor nodes from 'U'. Here, 'A' is the anchor node nearest to 'U'. Then find out the intersections from the other connected anchor nodes with the communication periphery of 'A', say$\mathrm{pt}_1,\ and\ \mathrm{pt}_2$ respectively from 'C' and 'B'. It implies that the distances B pt_2 and Cpt_1 are the minimum possible distance values from 'B' and 'C' respectively.

But still, there is a need to understand the applicability of the proposed model beyond just an elemental situation presented in Fig. 3. The real-time situation may go complex. It may be possible that the unknown node is still one hop away as a minimum hop distance, but the minimum hop distant anchor nodes are more than just one. This investigative situation is shown in Fig. 4.

Fig. 4: Minimum Base Distance estimation of unknown node nearest to multiple anchor nodes.

The anchor nodes- A1, A2, and A3 are at a distance of one hop from the intended unknown node 'U' as shown. Similarly 'B' is at a distance of three hops and 'C' is at a distance of four hops from 'U'. In this case, the anchor nodes- A1, A2, and A3 are closest to the unknown node 'U' in comparisons to the other anchor nodes. The centroid of A1, A2, and A3 is considered as a notion of the approximated location of the unknown node 'U' only, represented by Fig. 4. Now, let 'K' is a set of all anchor nodes which are at the equal hop count from an unknown node 'U' such that the hop count between 'K' and 'U' is less than the hop counts between other anchor nodes $\{A_n - K\}$ and 'U'($i.e.\ h_{\mathrm{countKU}} < h_{\mathrm{count}(A_n-K)\mathrm{U}}$). Whereas $'A_n'$ is a set of all anchor nodes. In Fig. 4-

$K = \{A1,\ A2,\ A3\}$ and $A_n = \{A1,\ A2,\ A3,\ B,\ C\}$.

Now a centroid CN($C_x,\ C_y$) is obtained by -

$$C_x = \frac{\sum_{\forall i \epsilon K} x_i}{N(K)},\ and\ C_y = \frac{\sum_{\forall i \epsilon K} y_i}{N(K)}$$

Now similarly as that of the previous case of the Fig. 3, find out the intersections from the other connected anchor nodes with the communication periphery of 'CN', say$\mathrm{pt}_1,\ and\ \ \mathrm{pt}_2$. The communication periphery of 'CN' is calculated with the help of C_r shown in Fig. 4; whereC_r is a product of the minimum hop size of$K = \{A1,\ A2,\ A3\}$ and the hop count (i.e. one) between 'K' and 'U'. Then it implies that the distances B pt_2 and Cpt_1 are the minimum possible distance from 'B' and 'C' respectively.

The situations considered in Fig. 3 and 4 show the nearest anchor nodes from 'U' are just one hop away; where the value of C_r is a hop size of the 'A' or 'CN'. But there may be the cases where the nearest anchor nodes may exist at a distance of more than one hop.

Let 'U' exists beyond one hop count from its nearest anchor nodes set. Then the value of C_r is the product of the minimum hop size of$K = \{A1,\ A2,\ A3\}$ and the hop count between 'K' and 'U'.

The hop size of each anchor node is estimated by DV-Hop with the help of the equation (1). But in DV-Hop the calculated hop size is erroneous that needs to be rectified. So we improve the hop size of DV-Hop by removing an average error value from it. The average error($\varnothing_p$) for an arbitrary anchor node 'p' is calculated by using equation (5).

$$\varnothing_p = \frac{\left(H_{\mathrm{count}_{\mathrm{ps}}} \times\ H_{\mathrm{size}_p}\right) - d_{\mathrm{ps}}}{H_{\mathrm{count}_{\mathrm{ps}}}};\ \forall\ p, s\ \epsilon\ A_n;'\ A_n' is\ a\ set\ of\ all\ anchor\ nodes \quad (5)$$

where 's' is an anchor node located at a maximum distance from the anchor node 'p', H_{size_p} is a hop size of the anchor node $'p'$ by using DV-Hop algorithm, and$H_{\mathrm{count}_{\mathrm{ps}}}$ is a hop count in-between$'p^{'} and\ 's'$.

Therefore the corrected hop size of 'p', $CH_{\mathrm{size}p}$ is-

$CH_{\mathrm{size}p} = H_{\mathrm{size}p} -\ \varnothing_p$ (6)

Now by using equation (6), the value of C_r is calculated by equation (7)-

$C_r = \left(H_{\mathrm{count}_{\mathrm{pU}}}\ \times\ CH_{\mathrm{size}_p}\right)$; (7)

where $\mathrm{CH}_{\mathrm{size}_p} = minimum\left(CH_{\mathrm{size}_{p1}},\ \mathrm{CH}_{\mathrm{size}_{p2}}, \ldots, CH_{\mathrm{size}_{\mathrm{pt}}}\right),\ \ t = N\left(K\right),\ \ p = A\ or\ CN$and $H_{\mathrm{count}_{\mathrm{pU}}}$ is a hop count in-between the closest anchor nodes set 'K' (e.g. K = {A1, A2, A3} about Fig. 4) and 'U';

After obtaining the value of C_r a circle with radius $C_r$centered at 'A' or CN is drawn to cover the area under which the unknown node may exist. Further to get a minimum distance between an unknown node and an anchor node, we calculate appropriate intersection points$pt_1,\ pt_2$ formed by the circle and a straight line joining the points 'B' and 'C' to 'A' or CN as shown in Fig. 3 and 4. For each anchor node, there will be two intersecting points on a circle with radius C_r, because whenever a straight line passes through a circle center the equations of both the geometric shapes produce a quadratic equation.

Therefore an anchor node collects two intersecting points on the circle which is shown in Fig. 5.

Fig. 5: Representation of circle with radius Cr **and a line passing through anchor node** B and CN**.**

Here the equation of the circle represented by Fig. 5 is-

$(x - C_x)^2 + (y - C_y)^2 = {C_r}^2$ (8)

And the equation of a line passing through $'B^{'}$and CN -

$y = m_s x + c$; (9)

where $m_s = \frac{B_y - C_y}{B_x - C_x}$, and$c = C_y - m_s C_x$

On solving the equations (8) and (9), we get a quadratic equation (10)-

$x^2 + {C_x}^2 - 2xC_x + (m_s x)^2 + (c - C_y)^2 + 2m_s x(c - C_y)^2 = {C_r}^2(10)$

Now solving the quadratic equation (10), we get two points of intersection- $p_1 (x1,\ y1)\ ,\ and\ \ p_2(x2,\ y2)$, where-

$$x1 = \frac{dX + f_a}{1 + {m_s}^2};\ x2 = \frac{dX - f_a}{1 + {m_s}^2}$$

$$y1 = \frac{dY + m_s f_a}{1 + {m_s}^2};\ y2 = \frac{dY - m_s f_a}{1 + {m_s}^2}$$

where$f_a = \sqrt{{C_r}^2 (1 + {m_s}^2) - (C_y - m_s C_x - c)^2}$,$dX = C_x + C_y m_s - c m_s$ and

$dY = c + C_x m_s + C_y {m_s}^2$.

The selection of a point out of the two pointsp_1 and p_2 , as shown in Fig. 5, is based upon a routing table of the anchor node which takes part in the centroid calculation and draws a route to the unknown node$'U'$. Here, the proposed method employs the same routing table which is populated during the network establishment. The anchor node which takes part in a route establishment as an intermediate node to the unknown node from another anchor node is considered as a ‘participative node’. The routing table helps to draw the decision to consider any one of the intersecting points. The intersecting point $\mathrm{pt}_i = p_2$ is selected for the anchor node $'B'$ if and only if a route $'\mathrm{rt}'$exist in a routing table $'T'$ from $'B'to\ 'U'$ and the route $'rt'$consists any of the participative node from $'K'$. Similarly, if a route does not exist on the same line then the intersecting point$\mathrm{pt}_i = p_1$ is selected. The different possibilities about the Fig. 5 are summarised in equation (11) where$d(B, p_1$or $p_2)$ represents the distance between$'B'$ $and\ 'p_1$ or p_2'.

$$\mathrm{pt}_i = \begin{cases} p_2 |\ d(B, p_1) < d(B, p_2)\ \ and\ rt \in T(B, U)\ ;\ \forall B \in A_n\& \\ p_1 |\ d(B, p_1) < d(B, p_2)\ \ and\ rt \notin T(B, U)\ ;\ \forall B \in A_n \end{cases} \quad (11)$$

The selection of point $p_1$from an anchor node $'B'$produces the minimum possible distance but the selection of point $p_2$is not suitable to find the minimum possible distance. Instead of the point$p_2$is able to signify that the unknown node ‘U’ is beyond the centroid point CN (as shown in Fig. 5) only. So the pointp_2 in the equation (11) must be replaced by CN, therefore equation (11) can be rewritten as-

$$\mathrm{pt}_i = \begin{cases} CN\ |\ d(B, p_1) < d(B, p_2)\ \ and\ rt \in T(B, U)\ ;\ \forall B \in A_n\& \\ p_1 |\ d(B, p_1) < d(B, p_2)\ \ and\ rt \notin T(B, U)\ ;\ \forall B \in A_n \end{cases} \quad (12)$$

The equation (12) signifies that the unknown node $'U'$ is somewhere in a region surrounded by CN and p_1. There must be multiple $p_1$with respect to each of their anchor nodes as shown in Fig. 6 are- $p_1^l,\ p_1^m,\ p_1^n,\ and\ p_1^o$.

Fig. 6: Expected region formed by CN and intersecting points$\mathbf{p_1^l}$, $\mathbf{p_1^m}$, $\mathbf{p_1^n}$, and, $\mathbf{p_1^o}$from their respective anchor nodes l, m, n, and o.

Now the base distance values from the anchor nodes$(A_n - K)$are obtained, which are Euclidean values, but still the unknown node $'U'$ may exist beyond their respective$p_1$or $'CN'$, as shown by the shaded area in Fig. 6. So the location is still needed to be estimated, which is described in the next step- Location Estimation.

Step 3.

Location Estimation: After obtaining the points$p_1^l,\ p_1^m,\ ...,\ p_1^o,\ and\ CN$ (as shown in Fig. 6) still the unknown node $U(x,y)$may exist somewhere beyond the points $p_1^l,\ p_1^m,\ ...,\ p_1^o,\ and\ CN$. After getting the base distance value, now we calculate comprehensive distance value by using base distance value to localize the unknown node $'U'$. So, for location estimation of unknown node $'U'$, considerD_i $(where\ D_i = D_1,\ D_2,\ \ldots,\ and\ so\ on)$ as a distance value in-between the anchor nodes $(A_n - K)$ and the respective intersecting points p_i. Since the unknown node $'U'$may be beyond the intersecting points, therefore we add the respective comprehensive distance e_i in D_i, where$e_i = e_1, e_2,\ \ldots,\ and\ so\ on$. Here e_i is a necessary comprehensive value to reach on a consensus from the anchor nodes with the calculated distance D_i. The centroid CNis also considered as a reference point to participate in the location estimation. It implies that the unknown node $'U'$ may exist at a nominal distance, say e_n, from $CN(C_x,\ C_y)$. Nowe_i updates with e_n, therefore$e_i =\ e_1, e_2,\ \ldots, and\ e_n$. Hence e_i is a necessary error required to calculate; such that$\sum e_i \approx 0$. Consequently by considering errore_i in D_i a system of Euclidean distance between anchor nodes and the unknown node $U(x,y)$ can be written by equation (13)-

$$\left.\begin{array}{c} (x-x_1)^2 + (y-y_1)^2 = (D_1+e_1)^2 \\ (x-x_2)^2 + (y-y_2)^2 = (D_2+e_2)^2 \\ \vdots \\ (x-x_{n-1})^2 + (y-y_{n-1})^2 = (D_{n-1}+e_{n-1})^2 \end{array}\right\} \quad amp;(13)$$

where $(n-1) = N(A_n - K)$

Similarly, the distance equation between unknown node $'U'$ andCN can be written by equation (14)-

$(x - C_x)^2 + (y - C_y)^2 = (e_n)^2$ (14)

Now, subtract the equation (14) from every equation of (13) one by one, we get equation (15)-

$$\left.\begin{array}{c} 2(C_x-x_1)x + 2(C_y-y_1)y - 2D_1e_1 + \left(e_n^2 - e_1^2\right) = \left(C_x^2 + C_y^2\right) - \left(x_1^2 + y_1^2\right) + D_1^2 \\ 2(C_x-x_2)x + 2(C_y-y_2)y - 2D_2e_2 + \left(e_n^2 - e_2^2\right) = \left(C_x^2 + C_y^2\right) - \left(x_2^2 + y_2^2\right) + D_2^2 \\ \vdots \\ 2(C_x-x_{n-1})x + 2(C_y-y_{n-1})y - 2D_{n-1}e_{n-1} + \left(e_n^2 - e_{n-1}^2\right) = \left(C_x^2 + C_y^2\right) - \left(x_{n-1}^2 + y_{n-1}^2\right) + D_{n-1}^2 \end{array}\right\} \quad amp;(15)$$

Further divide system of equation (15) by BL^2 to minimize a term $(e_n^2 - e_i^2)$ where BL is a border length of the region under considerations.

After dividing the equation (15) by BL^2, we get the equation (16).

$$\left.\begin{array}{c} \frac{2(C_x-x_1)x}{\mathrm{BL}^2} + \frac{2(C_y-y_1)y}{\mathrm{BL}^2} - \frac{2D_1e_1}{\mathrm{BL}^2} + \frac{e_n^2-e_1^2}{\mathrm{BL}^2} = \frac{\left(C_x^2+C_y^2\right)-\left(x_1^2+y_1^2\right)+D_1^2}{\mathrm{BL}^2} \\ \frac{2(C_x-x_2)x}{\mathrm{BL}^2} + \frac{2(C_y-y_2)y}{\mathrm{BL}^2} - \frac{2D_2e_2}{\mathrm{BL}^2} + \frac{e_n^2-e_2^2}{\mathrm{BL}^2} = \frac{\left(C_x^2+C_y^2\right)-\left(x_2^2+y_2^2\right)+D_2^2}{\mathrm{BL}^2} \\ \vdots \\ \frac{2(C_x-x_{n-1})x}{\mathrm{BL}^2} + \frac{2(C_y-y_{n-1})y}{\mathrm{BL}^2} - \frac{2D_{n-1}e_{n-1}}{\mathrm{BL}^2} + \frac{e_n^2-e_{n-1}^2}{\mathrm{BL}^2} = \frac{\left(C_x^2+C_y^2\right)-\left(x_{n-1}^2+y_{n-1}^2\right)+D_{n-1}^2}{\mathrm{BL}^2} \end{array}\right\} \quad amp;(16)$$

Since $\mathrm{BL}^2 >> e_{1,2,\ldots,\ n}^2$, *the term*$\frac{e_n^2 - e_{1,\ 2,\ \ldots,\ n-1}^2}{\mathrm{BL}^2}$

will be negligible and the modified form of equation (16) can be written in a matrix form of $AE = t$, as-

$$\left.\begin{bmatrix} a_1 & amp; b_1 & amp; c_1 \;\; 0 & amp; \ldots & amp; 0 \\ a_2 & amp; b_2 & amp; 0 \;\; c_2 & amp; \ldots & amp; 0 \\ \vdots & amp; \vdots & amp; \vdots \;\; \vdots & amp; \ddots & amp; 0 \\ a_{n-1} & b_{n-1} & 0 \;\; 0 & \ldots & c_{n-1} \end{bmatrix} \begin{bmatrix} x \\ y \\ e_1 \\ e_2 \\ \vdots \\ e_{n-1} \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n-1} \end{bmatrix} \right\} \quad amp; (17)$$

where$a_i = \frac{2(C_x - x_i)}{\mathrm{BL}^2}$, $b_i = \frac{2(C_y - y_i)}{\mathrm{BL}^2}$, $c_i = \frac{-2(D_i)}{\mathrm{BL}^2}$, *and* $t_i = \frac{(C_x^2 + C_y^2) - (x_i^2 + y_i^2) + D_i^2}{\mathrm{BL}^2}$; $\forall i \in (1,\ 2,\ \ldots,\ n-1)$.

Here our objective is to minimize the error, so we solve the equation (17) in such a way that net error must be tended to zero i.e.-

$\sum_{i=1}^{n-1} e_i \approx 0$ (18)

Although the net error can't be zero; but reduce it to close to zero or as minimum as possible.

So the equation (18) can be written by equation (19)-

$$\text{minimize} \left(\begin{bmatrix} 0 & amp; 0 & amp; \overbrace{1 \quad amp; 1 \quad amp; \ldots \quad amp; 1}^{(n-1)\text{times}} \end{bmatrix} \begin{bmatrix} x \\ y \\ e_1 \\ e_2 \\ \vdots \\ e_{n-1} \end{bmatrix} \right) \quad amp; (19)$$

The equation (19) can be rewritten as ($O'E$) such that the equation (17) must be true, where $O^{'} =$

$\begin{bmatrix} 0 & amp; 0 & amp; 1 & amp; 1 & amp; \ldots & amp; 1 \end{bmatrix}$, and E' =

$\begin{bmatrix} x & amp; y & amp; e_1 & amp; e_2 & amp; \ldots & amp; e_{n-1} \end{bmatrix}$.

To find out the minimum value of the summation of errors as shown by equation (18) and (19), some bounded values for the matrix $'E^{'}$are also required. Let 'lb' and 'ub' are the lower and upper bounds respectively such as-

$\mathrm{lb}^{'} =$

$\begin{bmatrix} 0 & amp; 0 & amp; 0 & amp; 0 & amp; \ldots & amp; 0 \end{bmatrix}_{1\times(2+(n-1))}$, and$\mathrm{ub}^{'} =$

$\begin{bmatrix} \mathrm{BL} & amp; \mathrm{BL} & amp; 2Rc & amp; 2Rc & amp; \ldots & amp; 2Rc \end{bmatrix}_{1\times(2+(n-1))}$. In a summarized form the objective and its constraints can be defined as-

$\mathrm{EsuchthatAE} = \mathrm{tandlb} \leq \mathrm{E} \leq \mathrm{ub}\,(20)$

The solution of the equation (20) leads to an estimation of the unknown node's coordinates (x, y) as-

$x = E\,(1)\,,\ and\ y = E(2)$.

6. Algorithm Cost

The applicability of any model depends upon its cost to a network. In the case of localization, the cost refers to the cost of communication and computation.

Here the cost of communication is the cost incurred by the network to spread out the information about the hop size between every connected pair of the nodes. In the DV-Hop algorithm, each anchor node out of the total anchor nodes $'m'$ has to inform all the nodes $'N'$ of the network about its hop size value. So every node informs every other node in the network about the hop size of an anchor node. Repeatedly this controlled flooding takes place the same number of times as that of the number of anchor nodes [11]. Therefore the communicational cost is$O\left(mN^2\right)$. Since the proposed model ODR and IDV [18] follow the same process as that of the DV-Hop algorithm to communicate the hop size, so the communication complexity is also the same as summarized by Table 1. The other causal factor in the cost is computational complexity. DV-Hop [15] employs the least square method to estimate the location in its last step. It (i.e. least square method) needs matrix multiplication three times and inversion of a matrix one time.

Table 1: Communication cost of DV-Hop algorithm, IDV, and ODR.

Communication Cost	DV-Hop Algorithm [15]	IDV [18]	ODR
First Step	-	-	-
Second Step	$O\left(\mathrm{mN}^2\right)$	$O\left(\mathrm{mN}^2\right)$	$O\left(\mathrm{mN}^2\right)$
Third Step	-	-	-

All the four operations (i.e. three matrix multiplications and one matrix inversion) contribute a complexity of $O\left(m^3\right)$. The other algorithm that is IDV [18], adopts a two-dimensional hyperbolic method for location estimation. The significant operations for complexity analysis of the two-dimensional hyperbolic method are matrix multiplication and matrix inversion. It (i.e. two-dimensional hyperbolic method) needs matrix multiplication and matrix inversion three times and one time respectively also. Therefore the computational complexity of IDV [18] for the last step to localize is also same as that of DV-Hop algorithm that is $O\left(m^3\right)$. Here it is worthwhile to consider that IDV [18] contributes one more cost component of $O\left(m^2\right)$ by each anchor, node to calculate the hop size correction factor. In IDV [18], since each anchor node corrects its hop size by $O\left(m^2\right)$ therefore the net complexity of correction factor becomes $O\left(m^3\right)$ for the whole network. The proposed algorithm ODR is dependent upon linear programming in its last step, which contributes$O\left(m^{3.5}\right)$ as suggested by Karmarkar [7]. The optimized solution of a linear programming problem is also dependent upon the number of constraints (Con). It implies that based on constraints the computational complexity of proposed ODR is $O(Con)$[5, 3] where $Con \leq m$.

Table 2: Computation cost of DV-Hop algorithm, IDV, and ODR.

Computation Cost	DV-Hop Algorithm [15]	IDV [18]	ODR
First Step	-	-	-
Second Step	$O\left(m^2\right)$	$O\left(m^3\right)$	$O\left(m^2\right)$
Third Step	$O\left(m^3\right)$	$O\left(m^3\right)$	$O\left(m^{3.5}\right)$

Therefore the computational complexity of the proposed method is little more than the DV-Hop algorithm due to linear programming as shown in Table 2. At the same time, it is sensible to mention that ODR considers some of the anchor nodes only to estimate the unknown nodes' location at the final step to keep the complexity low in comparison to some other similar variations like- Advanced DV-Hop algorithm [21], and Improved DV-Hop algorithm [25]. Thus ODR strives a balanced tradeoff between localization accuracy and its cost.

7. Simulation Results and Performance Analysis

This section establishes the proposed model ODR in comparison to the DV-Hop algorithm [15] and IDV [18] with the help of simulation performed on Matlab R2008b. The simulation parameters are shown in Table 3. For a comparative study among the proposed model ODR and the referenced algorithms' (i.e. DV-Hop and IDV) localization error is obtained as an average value after performing the simulation one hundred times over every experiment drawn by the Table 3.

Table 3: Simulation parameters and values.

Parameters	**Constant Value**	**Variable Value(s)**
Area Covered (Ar)(**A**)	100 x 100	Not Applicable (NA)
Communication Range (**R**)	NA	15, 20, 25, 30
Ranging Error slabs	NA	0-10%, 0-20%, 0-30% of **R**
Total Number of Nodes (**N**)	NA	200, 230, ..., 500
Anchor Nodes (**m**)	NA	5%, 10%, ..., 25% of N
Unknown Nodes ($\mathbf{U_n}$)	NA	$\mathbf{N - m}$

The average value is calculated after each setup during an experiment illustrates localization error (LE) as defined by the equation (21).

$$\left.\begin{array}{l} LE = \frac{\sum \text{Distance between estimated localized poition and actual position}}{R \times total\ number\ of\ unknown\ nodes} \times\ 100\% \\ \quad = \frac{\sum_{i=1}^{U_n} \sqrt{(x_{\text{ai}}-x_{\text{ei}})^2+(y_{\text{ai}}-y_{\text{ei}})^2}}{RU_n} \times\ 100\%; \end{array}\right\} \quad amp; (21)$$

where$(x_{\text{ai}},\ y_{\text{ai}})$ *and* $(x_{\text{ei}}, y_{\text{ei}})$are the actual and estimated coordinates respectively of the unknown node $'i'$.

The localization error definition, from equation (21), can show its relationship with the communication range. Therefore a substantial value of the communication range may improve the localization accuracy dramatically. But in real life, the communication range is never able to cover a region uniformly in all the directions as shown in Fig. 2 also. So it is inappropriate to analyze the performance of any model based on a regular-shaped coverage area. Rather the communication range should bear a random effect of attenuation during simulation experiments. To exhibit the effect of random attenuation of communication range on localization accuracy, the concept of ranging error is introduced in the simulation environment. The experiments performed show the performance of DV-Hop, IDV, and ODR under the environment when there is no ranging error as well as when the network is affected by ranging error. The ranging error is considered under three different slabs i.e. 0- 10%, 0- 20%, and 0- 30% of the communication range. The communication range got attenuated by taking value from the ranging error slabs in a random fashion during an experiment.

After the communication range; the number of unknown nodes is another term that affects the performance of the localization error as shown by the equation (21). Hence it is a need to perform experiments for the analysis of the proposed model ODR along with the referenced model (i.e. DV-Hop and IDV) based on two variables, where one variable is the unknown nodes' percentage and the other variable is communication range.

Therefore to cover all possibilities three experiments are simulated. In Experiment 1, the relationship between anchor nodes and localization error is discussed. The second, Experiment 2, marks the effect of communication range on the localization error. The last simulation setup studies the localization error due to some variations in the total number of nodes, in Experiment 3.

7.1. Experiment 1. To study the effect of anchor nodes' variation

Experiment 1 performed to study the effect of the anchor nodes' percentage on the total number of nodes in a network. By keeping a total number of nodes constant at 200, any percentage variation in the anchor nodes also alters the percentage of the unknown nodes in the network but in the opposite direction only. Here the simulation is performed in a bounded region of 100×100. The communication range of each node is kept constant at 15 units.

Fig. 7 shows the performance of DV-Hop, IDV, and ODR; when the communication range is not affected by any attenuation i.e. under the ideal situation where no ranging error is applicable. Similarly, figures 9, 10, and 11 exhibits the performance with the raging error of 0- 10%, 0- 20%, and 0- 30% respectively.

In this experiment, it is illustrated that DV-Hop, IDV, and ODR improves its localization accuracy or decreases the localization error with the increase of the anchor nodes' percentage. This reduction in localization error is evident just because of the improvement in hop size. A better-estimated hop size can assess the distance between an anchor node and an unknown node with more accuracy.

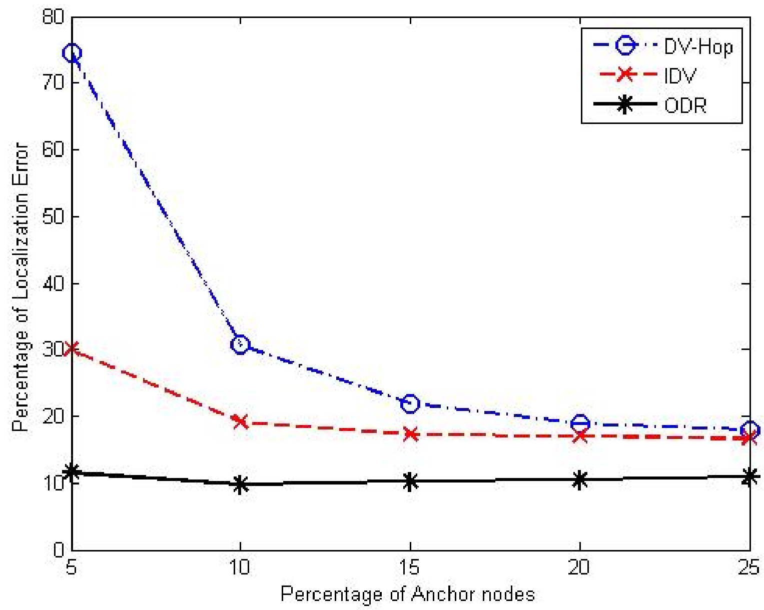


Fig. 7: Percentage of localization error vs. percentage of anchor nodes with no ranging error.

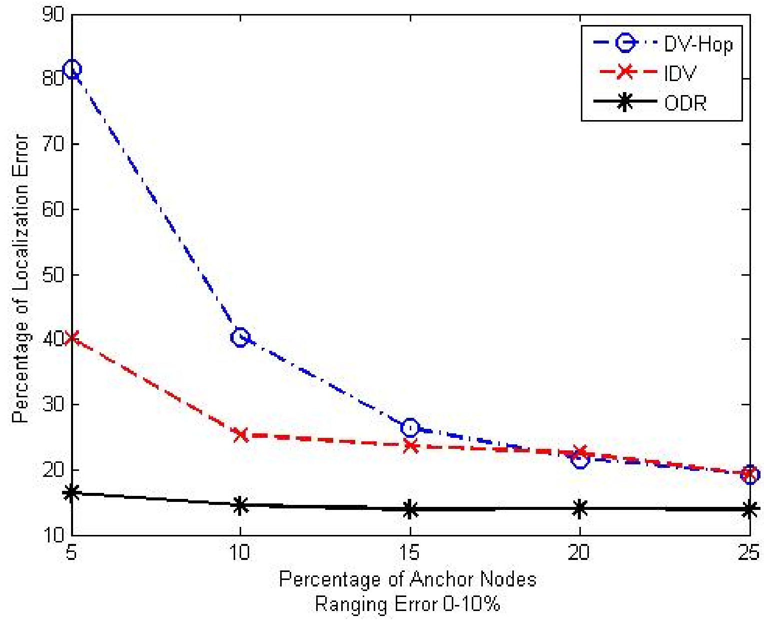


Fig. 8: Percentage of localization error vs. percentage of anchor nodes with 0-10% ranging error.

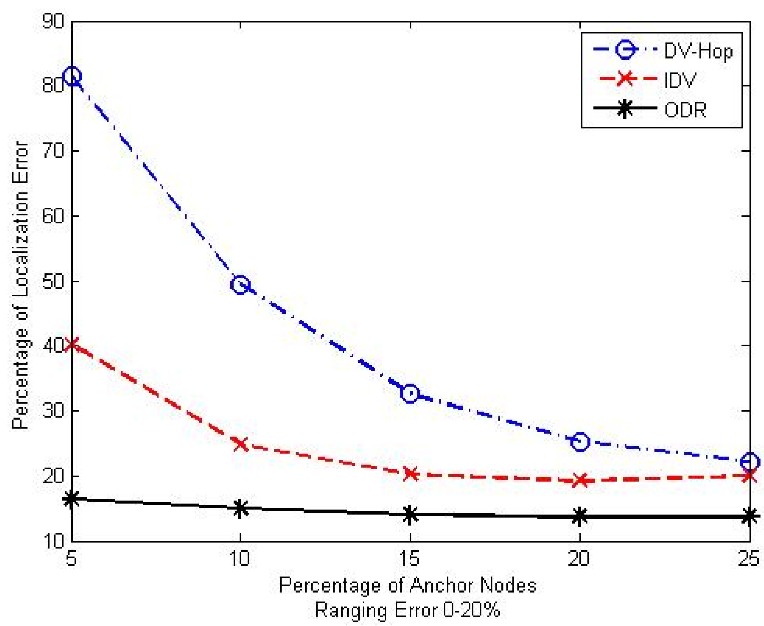


Fig. 9: Percentage of localization error vs. percentage of anchor nodes with 0-20% ranging error.

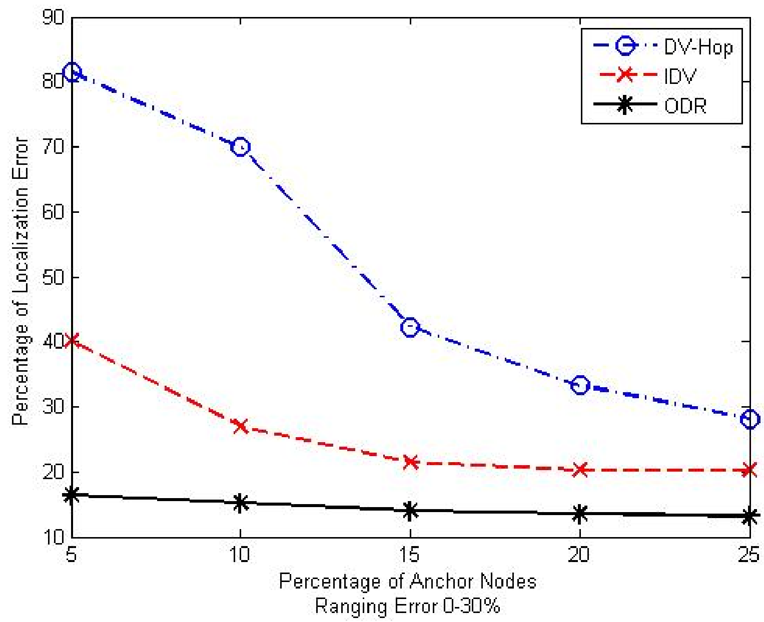


Fig. 10: Percentage of localization error vs. percentage of anchor nodes with 0-30% ranging error.

For better understanding, equation (1) is rewritten as equation (22).

$$H_{\text{size}_p} = \frac{\text{Total distance between p and other anchor nodes}}{\text{Total HopCounts between p and other anchor nodes}} \quad (22)$$

Here as per the equation (22), the hop size of an anchor node is dependent upon the hop counts only because the distance in between anchor nodes' pair is an unmanaged value. If the hop counts value is less than the required value then the hop size is more than the necessary value. It implies that the distance estimated with the help of hop size is more than the exact straight line distance between a node pair. On the other side, if the hop counts value is more than the requisite value than the hop size leads to a less distance estimation. Therefore hop size calculation gets improved with an increase in the anchor nodes up-to a certain extent, which reflects improvement to localize a node. Since ODR is nearly free from equation (1) to estimates the distance between an anchor node and an unknown node as well as put efforts to approximate Euclidean distance, therefore, the effect of hop size assessment does not affect much in comparison to DV-Hop algorithm and IDV.

Fig. 7 shows the performance of the models when communication is not affected by ranging error, here ODR improves the results by more than 22% and 9% in comparison to DV-Hop and IDV respectively. The simulation results obtained through Fig. 8 to Fig. 10 shows the localization error in the presence of the random effect of ranging error. The simulation using ranging error shows the localization error for ODR is smaller by 23%, 27%, and 36% than DV-Hop as shown by figures 8, 9, and 10 respectively. Whereas at the same time experiment exhibits the reduction of error due to ODR is more than 11%, 10%, and 11% in comparison to IDV as plotted by Fig. 8, 9, and 10 respectively.

Here Experiment 1 infers a finding that the increases in ranging error increase the localization error also for all the models- DV-Hop, IDV, and ODR. But in all the results plotted by Fig. 7 to Fig. 10, it is also apparent that the proposed model ODR is more robust and less error-prone.

7.2. Experiment 2. To study the effect of communication range

In the next experiment, Experiment 2 analyses the effect of the communication range variation on ODR in reference to DV-Hop and IDV. A simulation setup of a total of 200 nodes in a bounded region of100×100 with a variable communication range of$\{15,\ 20,\ 25,\ 30\}$ is drawn. Under this experiment (i.e. Experiment 2), the number of anchor nodes kept constant at 10% of the total number of nodes (i.e.$10\%\ of\ 200\ is\ 20$).

The simulation results are drawn through Fig. 11 to Fig. 14 show the effect of the variable communication range on the model ODR in comparison to DV-Hop, and IDV.

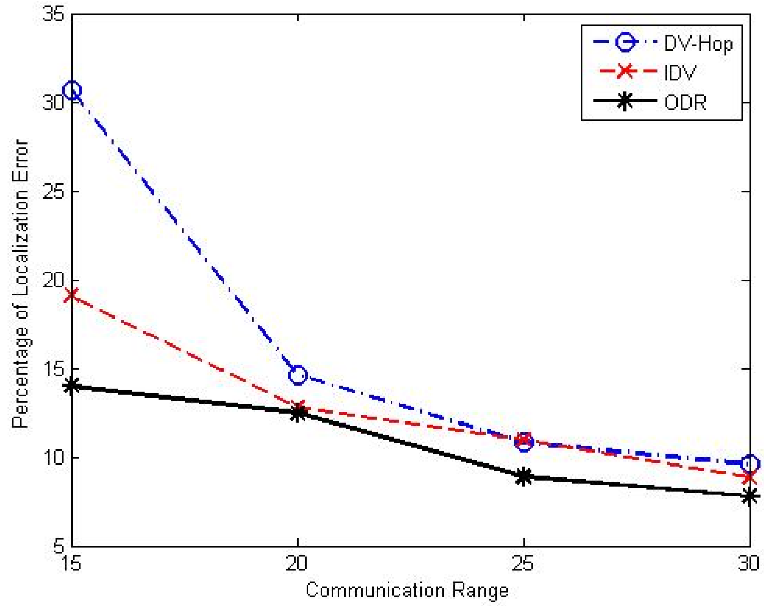


Fig. 11: Percentage of localization error vs. percentage of communication range with no ranging error.

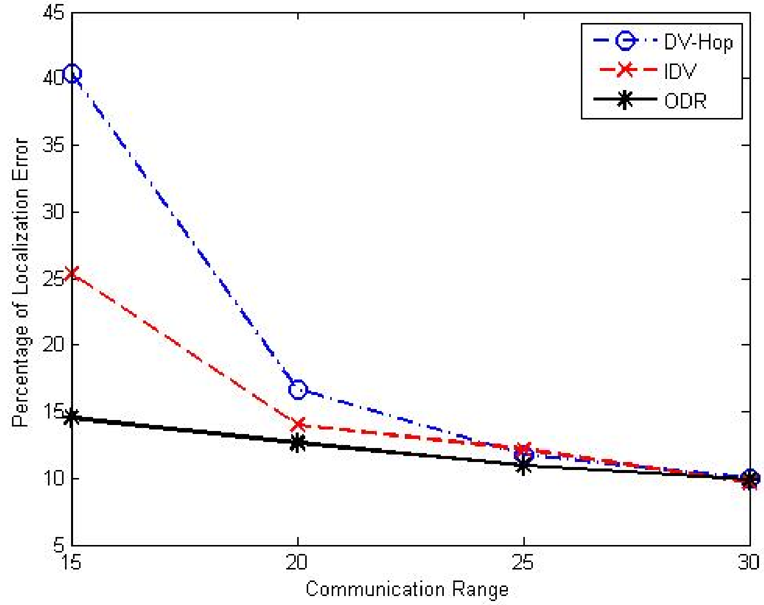


Fig. 12: Percentage of localization error vs. communication range with 0-10% ranging error.

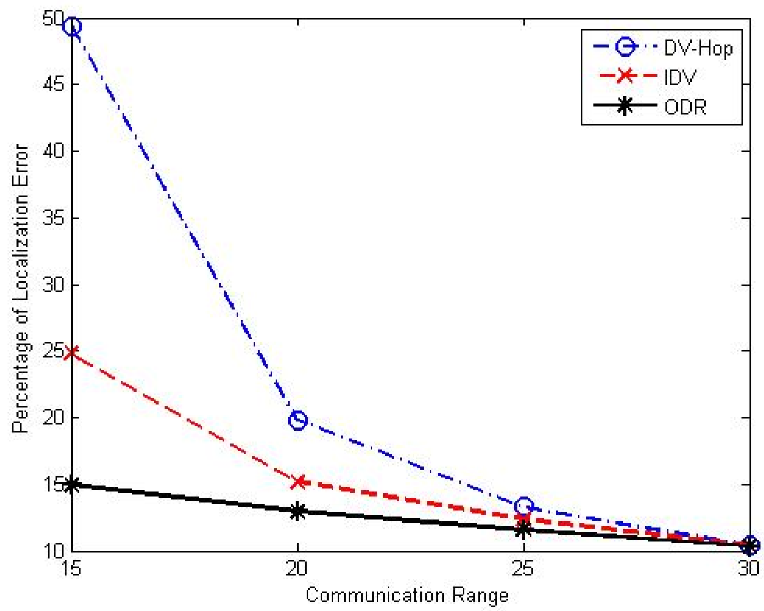


Fig. 13: Percentage of localization error vs. communication range with 0-20% ranging error.

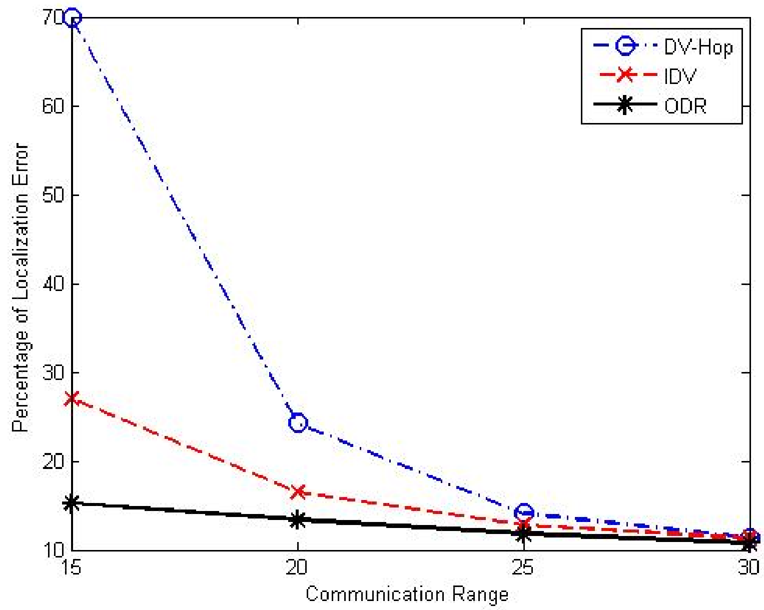


Fig. 14: Percentage of localization error vs. communication range with 0-30% ranging error.

It is shown with the help of Fig. 11 to Fig. 14 that the localization error falls with the rise of communication range for ODR and as well as for DV-Hop and IDV also. The localization gets better with the increase in communication range because an unknown node comes closer by some hop counts to more anchor nodes. As more and more anchor nodes cover an unknown node with lesser hops the distance estimated between them is calculated with less number of hops. The lesser number of hops contributes less error because hop size is estimated as an average value only (equation (1)) with an inherent error. This localization improvement can be answered by the equation (2). Here the equation (2) is rewritten as equation (23).

$$d_i = Hop\ Size\ per\ unit\ hop\ \times number\ of\ hops \quad (23)$$

The localization accuracy is dependent upon the correctness of distanced_i between an anchor node and an unknown node. As the number of hop counts value reduces with the increase in the communication range so as the value d_i also reduces. Therefore with less number of hop counts up-to a certain value the distance d_i is estimated with more accuracy.

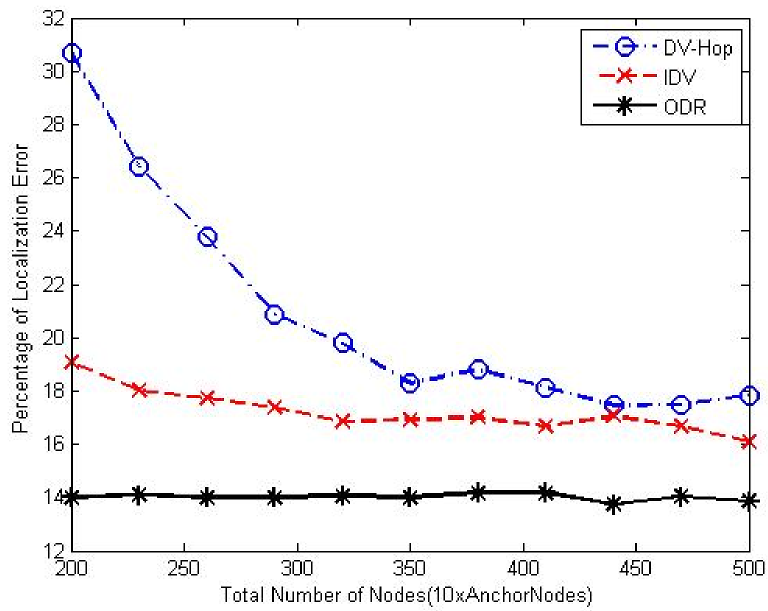
Here the equation (2) and (23) is applicable for the DV-Hop algorithm and IDV only. Therefore the correction in d_i with an increase in communication-range improves the localization accuracy for these two models gently. But IDV is completely away from the equations (2) and (23). IDV gets the benefit of the increased communication range just because of the improvement in calculating the pointCN $\left(\frac{\sum_{\forall i \epsilon K} x_i}{N(K)}, \quad \frac{\sum_{\forall i \epsilon K} y_i}{N(K)}\right)$ due to the increase in the arity of $'K'$.

The proposed model ODR is able to perform better than DV-Hop and IDV by 7% and 3% respectively when the network is considered to be immune to any kind of ranging error effect as shown by Fig. 11. The adverse effects of different ranging errors slabs (i.e. 0-10%, 0-20%, and 0-30%) are highlighted through Fig. 12 to Fig. 14. The network simulation under the influence of different ranging error slabs shows the localization error for ODR is lesser by 24%, 28%, and 37% than DV-Hop as shown by Fig. 12, Fig. 13, and Fig. 14 respectively. However, on the same configurations, the experiment (i.e. Experiment 2) exhibits the reduction of error by 12%, 11%, and 12% because of ODR in comparison to IDV as plotted by Fig. 12, Fig. 13, and Fig. 14 respectively.

Although Experiment 2 establishes the adverse effect of ranging error similar to Experiment 1 also but still ODR can localize the unknown nodes with better accuracy than DV-Hop and IDV. Furthermore, it is demonstrated that ODR is more robust even in the presence of ranging error and yields less localization error.

7.3 Experiment 3. To study the effect of the total number of nodes' variation

The third experiment examines the effect of a total number of nodes' variation on ODR in reference to the DV-Hop algorithm and IDV. The trial simulates with a constant communication range at 15 of all the nodes in a bounded region of 100×100. The total number of nodes varies like 200, 230,. . . , and so on up to 500 by taking anchor nodes equal to 10% of the total number of nodes.



The outcome of the experiment is marked with the help of Fig. 15 through Fig. 18. It is demonstrated that the localization error reduces with the increase in the total number of nodes. The algorithms DV-Hop and IDV improve their accuracy drastically with every add-on step for the total number of nodes whereas IDV shows a good tolerance against any variation in the total number of nodes.

Fig. 15: Percentage of localization error vs. Total Number of Nodes with no ranging error.

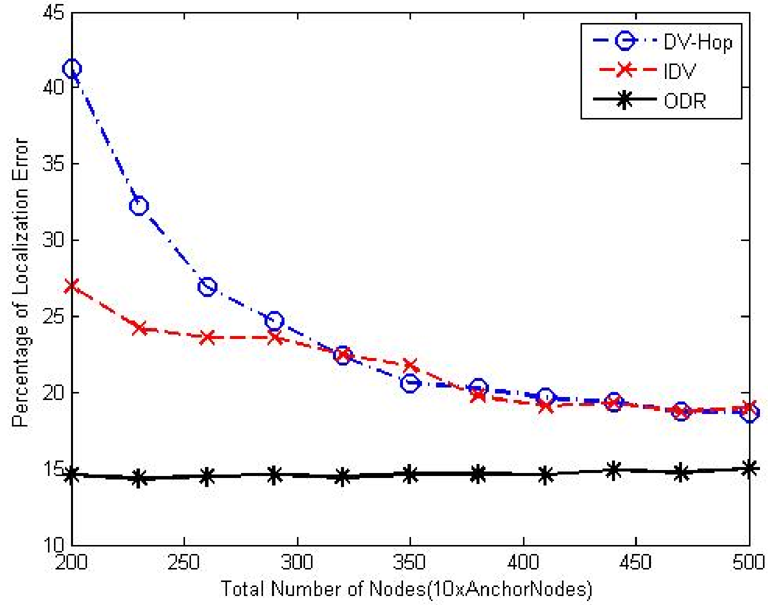


Fig. 16: Percentage of localization error vs. Total Number of Nodes with 0-10% ranging error.

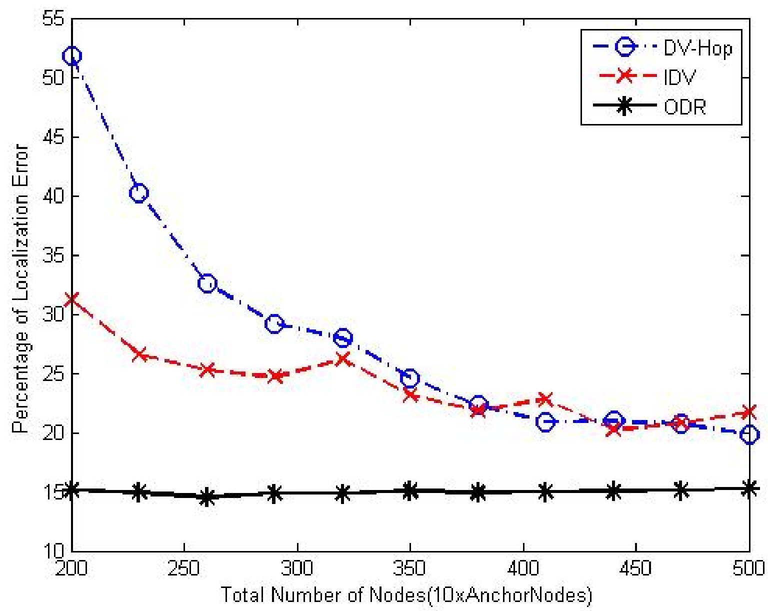


Fig. 17: Percentage of localization error vs. Total Number of Nodes with 0-20% ranging error.

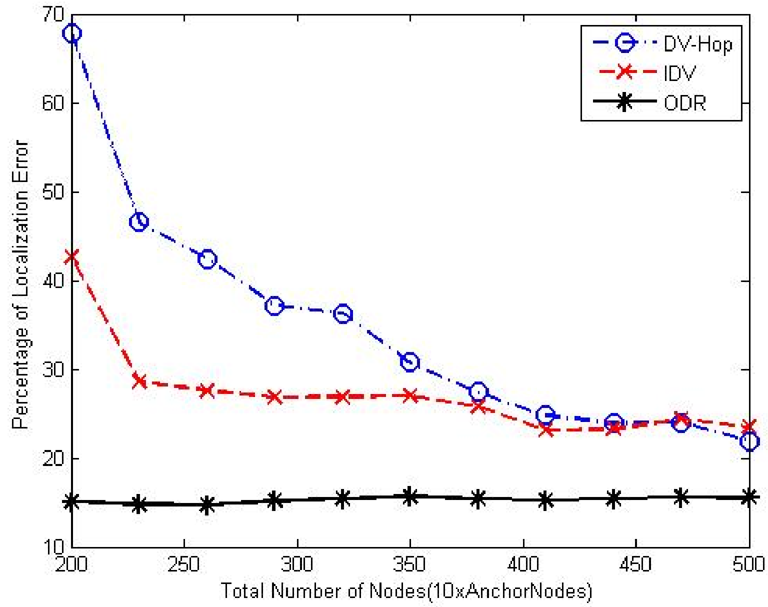


Fig. 18: Percentage of localization error vs. Total Number of Nodes with 0-30% ranging error.

As node density per unit area increases with an increase in the total number of nodes, it makes the unknown nodes closer to more anchor nodes. This way the lesser hop counts determines the distance d_i between the anchor node and the unknown node with less error is governed by equation (2) and (23). The more appropriate distance estimationd_i contributes a better location estimation for the DV-Hop algorithm and IDV. Similarly, ODR improves due to more anchor nodes fall in the minimum distant anchor nodes set $'K'$ and hence the centroidCN can indicate a probable location for an intended unknown node with more precision.

The Experiment 3 reveals that the proposed model ODR is better than DV-Hop algorithm and IDV by approximately 7% and 4% respectively in the ideal case as shown by Fig. 15 while the improvement is approximately 10%, 14%, 20% and 8%, 10%, 12% in comparison to DV-Hop algorithm and IDV respectively by considering the ranging error slabs of 0-10%, 0- 20%, and 0-30% respectively as presented through Fig. 16, 17, and 18.

7.4 Analytical Examination of Results

DV-Hop algorithm and IDV noticeably portray the localization error inversely proportional to the percentage of anchor nodes, communication range, and the total number of anchor nodes (i.e. nodes' density) but ODR shows moderate resistance to such changes as exposed through Fig. 10 to Fig. 18.

Table 4: Relationship among hop counts, anchor nodes percentage, and localization error.

Anchor Nodes % (m)	Average Hop Counts (Hop_{count})	Percentage of Localization Error (LE)	Percentage of Localizatio
		DV-Hop	IDV
5	4.28	74.52	30.03
10	4.56	30.69	19.08
15	4.67	21.97	17.21
20	4.64	18.93	16.96
25	4.73	17.92	16.70

Through Fig. 7 to 10 it is obtained that the localization error reduces with an increase in anchor nodes‘percentage. This reduction is caused due to an increase in hop counts as evident from Table 4 of the data collected through the simulation performed about the Fig. 7 and shown from Fig. 19. The increase in average count$\mathrm{Ho}p_{\mathrm{count}}$ is obvious with the increase in the more anchor nodes.

To establish the effect of the variation in anchor nodes and its ripple in terms of the hop counts is studied over localization error in respect of the three models- DV-Hop, IDV, and ODR. Here consider that the localization error (LE) is a dependent variable depends upon the number of anchor nodes (m), and hop counts$\mathrm{Ho}p_{\mathrm{count}}$ values as written by equation (24)-

$$LE = \lambda + \beta_1 m + \beta_2 \mathrm{Ho}p_{\mathrm{count}} + \eta; \;\; (24)$$

where ‘λ' is an intercept, $'\eta'$ is a residual,$\beta_1$and β_2 are slopes with respect to $'m'$,$\mathrm{Ho}p_{\mathrm{count}}$ such that-

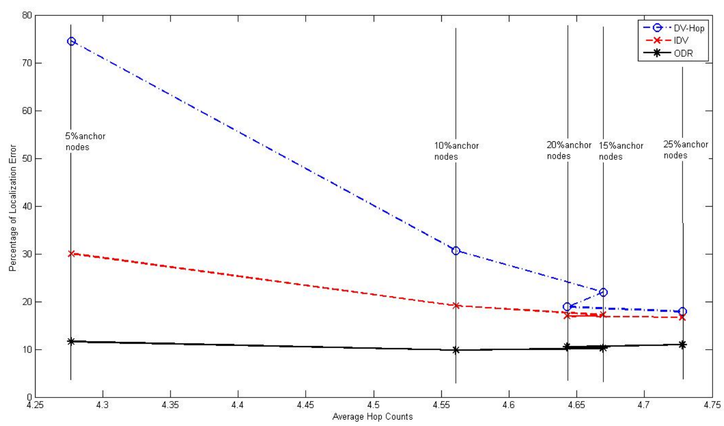


Fig. 19: Percentage of localization error vs. Average Hop Counts vs. anchor nodes.

$$\beta_1 = \frac{\left(\sum \mathrm{Hop_{count}}^2\right)\left(\sum mLE_o\right) - \left(\sum \mathrm{m\ Ho}p_{\mathrm{count}}\right)\left(\sum \mathrm{Hop_{count}\ L}E_o\right)}{\left(\sum m^2\right)\left(\sum \mathrm{Hop_{count}}^2\right) - \left(\sum \mathrm{m\ Ho}p_{\mathrm{count}}\right)^2}, \; and$$

$$\beta_2 = \frac{\left(\sum m^2\right)\left(\sum \mathrm{Hop_{count}}LE_o\right) - \left(\sum \mathrm{m\ Ho}p_{\mathrm{count}}\right)\left(\sum \mathrm{m\ L}E_o\right)}{\left(\sum m^2\right)\left(\sum \mathrm{Hop_{count}}^2\right) - \left(\sum \mathrm{m\ Ho}p_{\mathrm{count}}\right)^2};$$

where LE_o is LE observed from Table 4.

The localization error LE is analyzed by a linear equation only just to reduce the complexity. By using equation (24), the localization error is summarized as equation (25)-

$$LE = \begin{cases} 0.21851\ m\ -\ 149.80688\ Hop_{\mathrm{count}}\ +\ 711.76707; for\ DV-Hop \\ 0.09028\ m\ -\ 38.57935\ Hop_{\mathrm{count}}\ +\ 193.82679; for\ IDV \\ 0.06698\ m\ -\ 7.41643\ Hop_{\mathrm{count}}\ +\ 42.53416; for\ ODR \end{cases} \quad (25)$$

From equation (25) it is marked that the localization error is pulled down by the hop counts. The equation (25) needs analysis along with Table 4. The first two columns of Table 4 shows that an increment in the number of anchor nodes, the $\mathrm{Ho}p_{\mathrm{count}}$ also, increases but the equation (25) shows the effect of$\mathrm{Ho}p_{\mathrm{count}}$ is several times more than the $'m'$ to reduce the localization error. At the same time, the equation (25) shows that the effect of the contributory factors $'m'$,$\mathrm{Ho}p_{\mathrm{count}}$ and residual value $'\eta'$ in localization error is very less for ODR in comparison to DV-Hop and IDV, which establishes the claim of robust performance of the proposed model ODR.

Similarly, the effect of communication range on localization error is studied analytically. Consider Fig. 20 obtain through a simulation carried (in reference to Fig. 11 collected) in Table 5.

Table 5: Relationship among hop counts, communication range, and localization error.

Communication Range (R)	Average Hop Counts ($\mathrm{Ho}p_{\mathrm{count}}$)	Percentage of Localization Error (LE)	Percentage of Loca
		DV-Hop	IDV
15	4.56	30.69	13.99
20	3.25	14.63	12.46
25	2.60	10.83	8.89
30	2.22	9.56	7.77

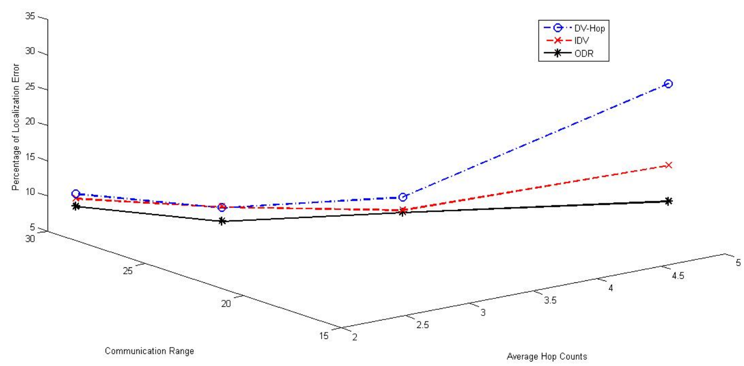


Fig. 20: Percentage of localization error vs. Average Hop Counts vs. Communication Range.

For the analysis of the data presented by Table 5, the equation (24) modifies by replacing a variable $'m^{'}$ by another variable$'R^{'}$(*i.e. communication range*) and another equation (26) is obtained-

$$LE = \begin{cases} 1.1429\ R\ +\ 16.21056\ Hop_{\mathrm{count}}\ -\ 60.4726; for\ DV-Hop \\ 0.08077\ R\ +\ 4.77295\ Hop_{\mathrm{count}}\ -\ 3.97544; for\ IDV \\ -0.43927R\ +\ 0.03471\ Hop_{\mathrm{count}} +\ 20.55158; for\ ODR \end{cases} \quad (26)$$

The equation (26) along with Table 5 exposes the cause behind the reduction in localization error due to the increase in the communication range. Table 5 demonstrates an interesting trend that with the rise in communication range the hop counts start falling. Therefore $'R'$ and$\mathrm{Ho}p_{\mathrm{count}}$ are inversely proportional to each other. Further all the three prepositions of the equation (26) state that the effect of $\mathrm{Ho}p_{\mathrm{count}}$ is many more times than the$'R^{'}$. Hence if at any stage $'R'$ is enhanced, the drop in$\mathrm{Ho}p_{\mathrm{count}}$ becomes strongly prominent to pull back the localization error several times.

Further, the investigation of the equation (26) shows that the ODR is more and more dependent upon $'R'$ rather than$\mathrm{Ho}p_{\mathrm{count}}$, which establishes its efficiency to exploit the antenna capabilities of a sensor more than the DV-Hop and IDV, because in DV-Hop and IDV the prominent factor is$\mathrm{Hop}_{\mathrm{count}}$ rather than $'R'$. Even the equation (26) shows that the localization error will remain quite low in contrast with DV-Hop and IDV.

8. Conclusion

In the range free WSN, the localization of nodes is proposed by many models. Here, DV-Hop algorithm rules over other algorithms due to its simplicity only. But it (DV-Hop algorithm) is unable to spot a node with high accuracy. So it (DV-Hop algorithm) highlights a gap for a method to localize with high precision. To improve localization accuracy, the proposed model ODR spots the location of unknown nodes with high accuracy and without any requirement of additional hardware. Also, ODR improves the localization accuracy without any additional burden on the network in terms of message communication. The paper highlights the need for Euclidean distance to determine the distance between the anchor and unknown nodes. Here ODR paves a way to find out Euclidean distance (between the anchor and unknown nodes) approximately and establishes the proposed model to localize with high accuracy. Although the exact Euclidean distance is not obtained an essential error value in this distance (between the anchor and unknown nodes) is defined and estimated with the help of optimized linear programming.

The other algorithm IDV improves the localization but at the cost of additional computational requirements due to the hop size correction factor. The suggested model ODR improves the localization results further better than both DV-Hop and IDV algorithms but no extra cost is needed to improve the hop size. The communication requirements among the nodes are the same in ODR, DV-Hop, and IDV.

Hence the paper is able to establish analytically as well as experimentally the proposed method ODR to localize unknown nodes with more accuracy by estimating approximate Euclidean distance. Also, it (i.e. ODR) can contribute a method to improve the hop size without any extra cost. Moreover, through simulation results and analytical reasoning ODR establishes itself as the least affected algorithm by the network variables (i.e. anchor nodes' percentage, communication range, and nodes' density) in comparison to its other contenders- DV-Hop algorithm and IDV.

In the future, the localization should be further improved by reducing the values of WSN constraints' limits.

Conflict of Interest

The authors declare no potential conflict of interest.

References

1. Adeniran Ademuwagun, Verdicchio Fabio, Reach Centroid Localization Algorithm, *Wireless Sensor Network* , Volume 9, Issue 2, 2017, 87-101, DOI: 10.4236/wsn.2017.92005
2. Ali A, Ming Y, Chakraborty S, Iram S., A Comprehensive Survey on Real-Time Applications of WSN, *Future Internet* . Volume 9, Issue 7, 2017, DOI: 10.3390/fi9040077
3. Christin D, Reinhardt A, Mogre PS, Steinmetz R, Wireless sensor networks and the internet of things: Selected challenges,*Proceedings of 8th GI/ITG KuVS Fachgesprch* , Drahtlose Sensornetze, 2009,1-3.
4. Elhadi Shakshukia, Abdulrahman Abu Elkhailb, Ibrahim Nemerb, Mumin Adamb, Tarek Sheltami, Comparative Study on Range Free Localization Algorithms, *Procedia Computer Science* , Volume 151, 2019, 501-510, ISSN: 1877-0509, DOI: 10.1016/j.procs.2019.04.068
5. Fekher Khelifi, Abbas Bradai, Abderrahim Benslimane, Priyanka Rawat, and Mohamed Atri, A Survey of Localization Systems in Internet of Things, Mobile Networks and Applications, Volume 24, Issue 3, 761-785, 2019, DOI: 10.1007/s11036-018-1090-3
6. Jing-li YANG and Qi-shen ZHU, An Improvement of DV-Hop Localization Algorithm Based on Harmony Search Algorithm, *Computer Science and Technology* . 2017, 109-116, DOI: 10.1142/9789813146426_0013
7. N. Karmarkar, A new polynomial-time algorithm for linear programming.*In Proceedings of the sixteenth annual ACM symposium on Theory of computing (STOC '84).* ACM, New York, NY, USA, 1984, 302-311, DOI: https://doi.org/10.1145/800057.808695

8. Kumar G., Rai, M. K., Saha, R., and Kim H. An improved DV-Hop localization with minimum connected dominating set for mobile nodes in wireless sensor networks. *International Journal of Distributed Sensor Networks* . 2018, 10.1177/1550147718755636.
9. Kwang-Yul Kim and Yoan Shin, A Distance Boundary with Virtual Nodes for the Weighted Centroid Localization Algorithm, *Sensors* , Volume 18, Issue 4, 2018, DOI: 10.3390/s18041054
10. Laizhong Cui, Chong Xu, Genghui Li, Zhong Ming, Yuhong Feng, Nan Lu, A High Accurate Localization Algorithm With DV-Hop And Differential Evolution For Wireless Sensor Network, *Applied Soft Computing,*Volume 68, 2018, 39-52, DOI: 10.1016/j.asoc.2018.03.036
11. Liu Y., Yang Z., Location, Localization, and Localizability Location-awareness Technology for Wireless Networks, Springer; New York, NY, USA, 2011, 55–74.
12. Lu Jian Yin, A New Distance Vector-Hop Localization Algorithm Based on Half-Measure Weighted Centroid, *Mobile Information Systems* , Volume 2019, Article ID 9892512, 2019. DOI: 10.1155/2019/9892512.
13. Mekelleche, F. and Haffaf, H., Classification and comparison of range-based localization techniques in wireless sensor networks,*Journal of Communications* , Volume 12, Issue 4, 2017, 221-227, DOI: 10.12720/jcm.12.4.221-227
14. Merry K., Bettinger P., Smartphone GPS Accuracy Study In An Urban Environment, *PloS one* , Volume 14, Issue 7, 2019, DOI: 10.1371/journal.pone.0219890
15. Niculescu, Dragoș and Nath, Badri, Ad hoc positioning system (APS),*Proceedings of IEEE Global Telecommunications Conference* , Volume 5, 2001, 2926 - 2931 DOI: 10.1109/GLOCOM.2001.965964.
16. Penghong Wang, Fei Xue, Hangjuan Li, Zhihua Cui, Liping Xie, and Jinjun Chen, A Multi-Objective DV-Hop Localization Algorithm Based on NSGA-II in Internet of Things, *Mathematics 2019* , Volume 7, Issue 2, 2019, DOI: 10.3390/math7020184
17. Rui Jiang, Xin Wang, and Li Zhang, Localization Algorithm Based on Iterative Centroid Estimation for Wireless Sensor Networks,*Mathematical Problems in Engineering* , Volume 2018, Article ID 5456191, 2018, DOI: 10.1155/2018/5456191
18. S. Shen, B. Yang, K. Qian, Y. She and W. Wang, On Improved DV-Hop Localization Algorithm for Accurate Node Localization in Wireless Sensor Networks, *Chinese Journal of Electronics* , Volume 28, Issue 3, 2019, 658-666, DOI: 10.1049/cje.2019.03.013.
19. S. Vatansever and I. Butun, A broad overview of GPS fundamentals: Now and future, *Proceedings of 2017 IEEE 7th Annual Computing and Communication Workshop and Conference (CCWC),* Las Vegas, NV, 2017, 1-6, DOI: 10.1109/CCWC.2017.7868373
20. Shahin Farahani, RF Propagation, Antennas, and Regulatory Requirements, ZigBee Wireless Networks and Transceivers, 2008, 171-206, DOI: 10.1016/B978-0-7506-8393-7.00005-4
21. Shrawan Kumar, D. K. Lobiyal, An Enhanced DV- Hop Localization Algorithm for Wireless Sensor Network, *International Journal of Wireless Networks and Broadband Technologies* , volume 2, Issue 2, 2012, 16-35.
22. Singal, *Analog and Digital Communication* , Tata McGraw-Hill Education, ISBN 1259084523, 9781259084522, 2012, 33.
23. Stefan Tomic and Ivan Mezei. Improvements of DV-Hop localization algorithm for wireless sensor networks. *Telecommunication System* , Volume 61, Issue 1, 2016, 93-106. DOI: 10.1007/s11235-015-0014-9
24. Torres-Ruiz, M., Lytras, M. D., and Mathkour H., Innovative services and applications of wireless sensor networks: Research challenges and opportunities, *International Journal of Distributed Sensor Networks,* Volume 14, Issue 5, 2018, DOI: 10.1177/1550147718772973
25. W. R. Shi, C. J. Jia, and H. H. Liang, An Improved Dv-Hop Localization Algorithm For Wireless Sensor Networks, *Chinese Journal of Sensors Actuators* , Vol.24, No.1, 2011, 83–87.
26. W. Fang, H. Xu, and G. Yang, Improved DV-Hop Algorithm Based on Minimum Hops Correction and Reevaluate Hop Distance, *Proceedings of 5th International Conference on Information Management (ICIM)* , Cambridge, United Kingdom, 2019, 102-107, DOI: 10.1109/INFOMAN.2019.8714669
27. Wei Zhao, Shoubao Su, Fei Shao, Improved DV-Hop Algorithm Using Locally Weighted Linear Regres-

sion in Anisotropic Wireless Sensor Networks, Wireless Personal Communications: An International Journal Archive, Volume 98 Issue 4, 2018, 3335-3353, DOI: 10.1007/s11277-017-5017-2

28. Y. Chen, X. Li, Y. Ding, J. Xu, and Z. Liu, An improved DV-hop localization algorithm for wireless sensor networks, *Proceedings of 13th IEEE Conference on Industrial Electronics and Applications (ICIEA)* , Wuhan, 2018, 1831-1836, DOI: 10.1109/ICIEA.2018.8398006