Fundamentals of Relativization

Hontas Farmer[1]

[1]Richard J. Daley College

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Abstract

A new approach to reconciling General Relativity with Quantum Field Theory is Relativization, the act of making a physical model which obeys the principles of special relativity and General Relativity. This approach immediately yields results that no other approach has. I have established the foundations and fundamentals of relativization via a set of axioms. Expressions such as appear Pμ[?]|p>=kμ[?]|p>P^{\mu}\left|p\right\rangle\,=k^{\mu}\left|p\right\rangle, or ==kμ[?]=kμ\left\langle p\right|P^{\mu}\left|p\right\r
p\right|k^{% \mu}\left|p\right\rangle\,=k^{\mu}\left\langle p\right.\left|p\right\rangle\,=% k^{\mu}. Such expressions appear in textbooks and papers but they are given a clearer interpretation in this model. Using this new approach to the problem, I will formulate the standard model as a relativized model in a curved space time with a locally valid graviton-Higgs interaction. This interaction will lead to a renormalized perturbation theory that can be summed up exactly to give the amplitude of graviton-graviton interaction as approximately C[?]o[?]s[?]h[?](p)Cosh(p). I will solve a Schrodinger equation for a gravitationally bound system and get theoretical predictions relating to the thermodynamics of Planck scale black holes. Relativization has already provided a finite quantitative prediction for the quantum corrections to the local gravitational field, and gives results compatible with established black hole thermodynamics. Further research will certainly yield new insights.

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PHYSICS

Fundamentals of Relativization

HONTAS FARMER [1 2 3]

1. Richard J. Daley College
2. Malcolm X College
3. College of Dupage

ABSTRACT

A new approach to reconciling General Relativity with Quantum Field Theory is Relativization, the act of making a physical model which obeys the principles of special relativity and General Relativity. This approach immediately yields results that no other approach has. I have established the foundations and fundamentals of relativization via a set of axioms. Expressions such as appear $P^{\mu}|p>=k^{\mu}|p>$, or $<p\,|\,P^{\mu}|p>=<p\,|\,k^{\mu}|p>=k^{\mu}<p|p>=k^{\mu}$. Such expressions appear in textbooks and papers but they are given a clearer interpretation in this model. Using this new approach to the problem, I will formulate the standard model as a relativized model in a curved space time with a locally valid graviton-Higgs interaction. This interaction will lead to a renormalized perturbation theory that can be summed up exactly to give the amplitude of graviton-graviton interaction as approximately $Cosh(p)$. I will solve a Schrodinger equation for a gravitationally bound system and get theoretical predictions relating to the thermodynamics of Planck scale black holes. Relativization has already provided a finite quantitative prediction for the quantum corrections to the local gravitational field, and gives results compatible with established black hole thermodynamics. Further research will certainly yield new insights.

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hfarmer@ccc.edu

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1 INTRODUCTION

Relativization, the process of making something relative, specifically putting a theory of physics into a form that obeys the principles of Special and General Relativity. In my last paper [7] I explained the relativization of the Klein-Gordon field. Thus uniting Quantum Field Theory (QFT) and General Relativity (GR) with one formalism in one special case. In this paper I will formalize the fundamentals of relativization with a set of axioms, and apply them to the standard model of particle physics.

Models which accomplish this unification one must use Hilbert space and Minkowski space at the same time. Expressions of the form $P^{\mu}|p>=k^{\mu}|p>$ appear in text books on Quantum Field Theory, and String/M Theory such as [12; 2] in papers on QFT in curved space time, and in textbooks that dare raise the matter [3]. Expressions which mix elements of Hilbert space and elements of Minkowski space, as above, are textbook parts of any modern relativistic quantum theory. If expressions such as $P^{\mu}|p>=k^{\mu}|p>$ make no sense to you, stop reading this paper and study something else.

Instead of treating the quantum theory as more physically fundamental, I am treating the principles of General Relativity as the more fundamental Physics. By restating the problem as one of making QFT comply with the prescripts of General Relativity my hypothesis opens a new path which can lead in a interesting new direction. In this paper I will present not only a hypothesis but a mathematical theory which makes concrete physical predictions.

2 AXIOMS OF RELATIVIZATION

The fundamentals of this model are encapsulated in the following axioms. These axioms are not all my own invention. These are based on the knowledge built up by the work of many, cited where appropriate, for how nature should behave in light of relativity and quantum field theory. The principle of relativization seems to have no precedent, and while other axiomatic formulations for QFT in curved space time have been published such as [11] they dealt with a static curved background. What follows will give a formulation which can admit a dynamical background.

F1

The principle of Relativization: All physical theories must obey the Einstein Equivalence Principle. "that for an infinitely small four-dimensional region, the relativity theory is valid in the special sense when the axes are suitably chosen." [6] In other words physical theories must be formulated in a way that is locally Lorentz covariant and globally diffeomorphism covariant. Stated with equations.

$$x^\mu = e^\mu_a x^a$$

x^a is a vector in the locally flat space near a point.

e^μ_a is a vielbien flat space to the curved manifold.

x^μ is a vector in the curved space time manifold.

F2:

Spectrum condition: All possible states of a QFT will be in the Fock-Hilbert space $\mathscr{H}$. An operator on $\mathscr{H}$ must map states to other states in $\mathscr{H}$.

F3:

Nomalization condition: The inner product on $\mathscr{H}$ must be in a set isomorphic to the division algebras $\mathbb{R}$,$\mathbb{C}$,$\mathbb{H}$, $\mathbb{O}$.[1] For example an inner product on $\mathscr{H}$ of the form $< \psi|\psi >= j^a$ with $j^a \in \mathscr{M}$ and $\forall$ $|\psi >\in \mathscr{H}$.

F4:

The principle of QFT locality: QFT interactions occur in the locally flat space at the point of interaction. The propagation of particles between interactions is governed by Relativity.

F5:

Specification condition: Relativized QFT's are defined by the above and the tensor product of their state space with Minkowski space. For a theory T, $T = \left\{ \mathscr{H}, \mathscr{H} \otimes \mathscr{M}, A(\mathscr{H}) \right\}$ (Inspired by a similar statement in [11].)

The following sections show how each of these axioms are applied.

3 HILBERT AND FOCK SPACES.

A typical textbook explanation of QFT starts with Fock Space [12]. Fock space is the Hilbert space completion composed of taking the tensor products of the Hilbert spaces of zero particles, one particle, two particles, … and so on up to n particles subspaces. As if every point in space was equipped with a quantum harmonic oscillator.

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The Hilbert space of a quantum field theory is in fact a Fock space [12]. From here [12] introduces the quantum fields, usually Klein-Gordon for it's simplicity. As it turns out a simple scalar field, the Higgs may be of fundamental importance to this issue.

3.1 HILBERT SPACE

I will take a different approach than [12]. This approach based on the algebra of the field operators $A(\mathscr{H})$ as prescribed by the specification condition. Instead of building up the space from pre-hilbert spaces let us consider the Klein-Gordon equation as if it were any other differential equation. The solutions of any differential equation of order 2 or greater form a space of solutions. This is certainly true of Klein-Gordon. I can show this by rewriting it in terms of particle creation and annihilation operators.

Unexpected text node: '3' (2)
Unexpected text node: '12' (3)

The solutions ϕ_n to the above equation are vectors in the solution space associated with this equation. To find the conjugate solutions we need the conjugate equation

Unexpected text node: '3' (4)
Unexpected text node: '12' (5)

To make a Hilbert space out of the solution space I need to define a inner product $< \, | \, >$ with certain properties. I define the inner product as

Unexpected text node: '12' (6)

In the above $\circ$ denotes the composition of functions. Does this inner product behave properly under the hermitian conjugation?

$< \phi_n | \phi_m >$ Unexpected text node: '12' (7)
Unexpected text node: '12'
$= < \phi_n | \phi_m >$

it does.

This product has to be linear in its first argument. To show this I need to define a couple of values

$a = < \phi_n | \phi_n >$ and $b = < \phi_m | \phi_m >$.

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Unexpected text node: '1'
Unexpected text node: '1'
Unexpected text node: '1' (9)

Last but not least there is the condition that the inner product of an element of the space with itself needs to be positive definite. This ensures that we may normalize probabilities to one

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For this inner product the result is

Unexpected text node: '12' (11)
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which satisfies the condition. Zero in the above would simply be the zero element in any of the appropriate division algebras which are isomorphic to $\mathbb{R}$,$\mathbb{C}$,$\mathbb{H}$, or $\mathbb{O}$.

This proves that the solution space of the Klein-Gordon equation forms a Hilbert space. I have demonstrated this fact of mathematics without recourse to the specifics of any particular division algebra.

For a complete treatment of Hilbert space using geometric algebra, let me define the following outer product or ket-bra, $|><|$, as it is usually called in physics. This product gives as it's result an operator in $\mathscr{H}$ ([5; 9]) (in which it is represented with a wedge $\wedge$)

Unexpected text node: '12' (13)

With this product and the inner product one may define the Clifford product (it is always denoted without a symbol) which will always be associative[9; 5].

$$\phi_n\phi_m =< \phi_n|\phi_m > +|\phi_n > \ < \phi_m| \quad (14)$$

Quantum Mechanical theories have to use an algebra of scalars which is isomorphic to one of these four $\mathbb{R}$,$\mathbb{C}$,$\mathbb{H}$,$\mathbb{O}$[1]. Complex numbers are not the only real option [9], and for a truly relativized quantum field theory they may not be mathematically rich enough, hence the need for Clifford Algebra and Lie Algebra etc. Quantum Field Theory includes quantities that are 4-vectors in Minkowski space which leads to special considerations. To satisfy F5, the specification condition requires a word on the space time algebra.

3.2 SPACE-TIME ALGEBRA

The space-time algebra is covered in great detail in a number of references [5; 10; 8; 4; 13; 1]. The following is a summary of those materials.

Minkowski space $\mathscr{M}$, is constructed from the set of mathematical objects which vary, like vectors, under Lorentz transformations. These are four component objects which are, as a set, isomorphic to quaternions ($\mathscr{M}$ *is* isomorphic to $\mathbb{H}$). When working with quaternions we often take advantage of this fact and write them as if they were a sort of four vector. The one to one and onto correspondence between them should be clear

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The Minkowski norm is not the quaternion norm so $\mathscr{M}$ is *not* isometric to $\mathbb{H}$.

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This norm could not fulfill the requirement of being positive definite. Therefore Minkowski space is not a Hilbert space. This is not a barrier to using the algebra associated with it as the set of scalars over which one may write a Hilbert space.

It is possible to divide four vectors. There exist zero divisors and idempotent elements in the algebra of space time. These elements mean that certain operations that would be permissible in pure quantum theory are not in even standard relativistic quantum theory [12]. Minkowski space time is a pseudo-division algebra. However that is not really a problem since in the approach taken in this paper quantum mechanics, and quantum theory are subordinate to relativity. The results will be relativistically correct, that is what matters.

3.3 HILBERT SPACE OVER MINKOWSKI SPACE.

Define the Hilbert space of Klein-Gordon as being the set of all eigenfunctions of the Klein-Gordon Hamiltonian with function composition $\circ$

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Then define Minkowski space as the set of all vectors that transform as vectors under Lorentz transformation with the Clifford product (which has no symbol). As above I choose to use the basis elements of the space, which in this case are non other than the gamma matrices.

Unexpected text node: '2' (17)

With these definitions of the Hilbert space and Minkowski space we may finally combine them to get a

“Hilbert space over Minkowski space”. This is the tensor product.

Unexpected text node: '12' (18)

The tensor product of the relevant Hilbert Space with Minkowski space is defined as the set of all possible tensor products of the elements of H and M. The elements of the relevant Hilbert Space will satisfy the Klein-Gordon equation. The elements of M, x^{μ}, will have as their basis set γ^{μ} which will satisfy the equation which defines the algebra of space time.

The inner product on this tensor product space ($<,>$)is a combination of the inner products on each of the subspaces like so (shown using the basis elements of each space)

Unexpected text node: '12' (19)

The inner product on $\mathscr{H}$ and the inner product on $\mathscr{M}$ can be multiplied by each other to result in an object which is a scalar in both the Hilbert space and the Minkowski space. This need not be so. Suppose one wanted to compute the inner product on the Hilbert space yet have as a result a vector in the Minkowski space. The physical reason one would want this would be to find a probability current four vector.

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The result of that equation is a quantity which is a scalar in$\mathscr{H}$ yet also a vector in $\mathscr{M}$. It is a subtle one. Instead of just stating this let me prove it by taking a Lorentz transformation.

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$$= (\phi_n{}^{-} \circ \phi_n)\gamma^a \quad (22)$$

This shows that it is clearly possible for an object in a Hilbert space over Minkowski space to be a scalar in Hilbert space while being a vector in Minkowski space. So when anyone writes expressions such as $P^{\mu}|p>= k^{\mu}|p>$, or $< p\,|\,P^{\mu}|p>=< p\,|\,k^{\mu}|p>= k^{\mu} < p|p>= k^{\mu}$ as was done in (7) it is perfectly valid physically and mathematically.

Thus the specification condition can be satisfied for Kelin-Gordon theory. Equations 16 and 18 contain all the information it takes to specify relativized Klein-Gordon Theory.

3.4 DISCUSSION

At this point I have justified the foundations and fundamentals of relativization. I have cited supporting papers and textbooks, and re-derived these expressions from first principles. *Any so-called reviewer who is not satisfied with this must show where there is an actual error not simply state their own confusions as facts and preferences as principles. If one still does not get this read the following literature* (12; 2; 3; 5; 10; 8; 4; 13; 1) *then reread* (7)*, then reread the present paper and view the accompanying PowerPoint and Mathematica files.*

4 THEORETICAL PREDICTIONS AND CONSEQUENCES.

In this section I will relativize the standard model of particle physics and include gravity. From this testable predictions will be derived.

4.1 RELATIVIZATION OF THE STANDARD MODEL OF PARTICLE PHYSICS

In my previous paper (7) I was mainly interested in products of the form

Unexpected text node: '0' (23)

Quantities such as this are of special interest for Quantum Field Theory. This subspace of$\mathscr{H} \otimes \mathscr{M}$ consist of the probability current four vectors, j^{μ}. Knowing the probability four vectors leads us to the currents due to a QFT interaction. In the terms used in this paper one may write down the Riemann curvature operator

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In equation [24] I have chosen to indicate that those bra-kets have their 4-vector components explicitly using the Unexpected text node: '0' which is an identity matrix. R_{ab} will act on the Higgs field and gives an eigenvalue equation

$R_{ab}\ \hat{}\ |\phi_m >= R_{abm}|\phi_m >$.(25)

R_{abm} is the mth curvature eigenvalue of the Riemann curvature operator, R_{ab}, for the eigenvector $|\phi_m >$.

To incorporate the standard model into this theory the simplest path is to use the Lagrangian formulation. So I will write the standard model as a QFT in curved space time in terms of a invariant Lagrangian. In other words it will be a scalar in $\mathscr{H}_{sm} \otimes \mathscr{M}$ with all indicies summed over.

Unexpected text node: '14'(26)

The term $R - \phi\gamma^a R_{ab}\phi\gamma^b$ includes the standard Einstein gravity term with a correction due to Higgs-graviton interactions. Including standard Einstein gravity means this model predicts everything that GR does. The graviton-higgs interaction will be the source of new predictions.

The Higgs interacts with gravitons in a way which will moderate (not mediate) the gravitational interaction terms. Consider the Feynman diagram rules which would relate directly to the graviton and Higgs-Higgs graviton interactions. At every loop order this hypothesis introduces terms which will balance out the graviton loops (see fig [1]) . This means that the perturbation series for graviton-graviton interaction will not run away. This series of diagrams can be summed, at least approximately. The amplitude of the graviton-graviton interaction will be approximately

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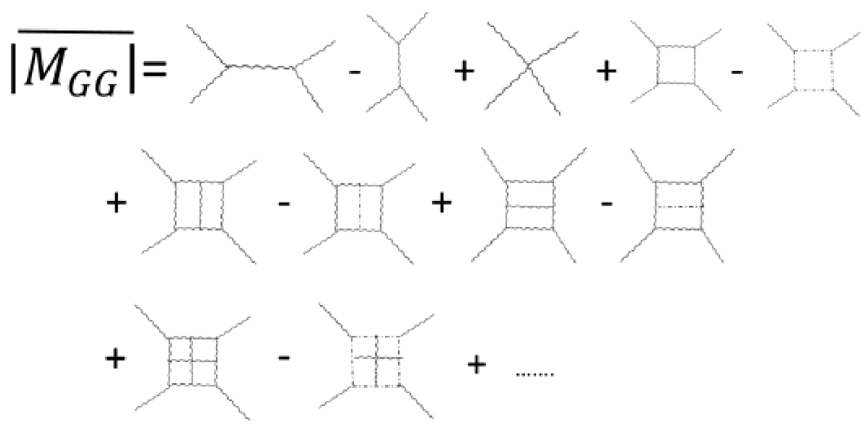

FIGURE 1: FEYNMAN DIAGRAMS FOR GRAVITON-GRAVITON INTERACTIONS MODERATED (NOT MEDIATED) BY THE HIGGS BOSON-GRAVITON DIAGRAMS.

The gravitational correction to the amplitude of the probability 4-current for Higgs-Higgs interactions would be tiny. The gravitational interaction can be safely ignored in any particle physics experiment at currently accessible energies. However at cosmological scales we may see the effects of these interactions.

4.2 STATES ENCLOSED IN A GRAVITATIONALLY INDUCED EVENT HORIZON.

The graviton-graviton self interaction is very interesting because it is this interaction which would dominate any region where gravity is strong enough to create a relativistic horizon, such as the interior of a black hole, or the whole of the universe. In classical GR nothing can escape a black holes horizon, and nothing may escape the universe. While boundary effects may cause Hawking radiation at any relativistic horizon due in part to the fact that even these astronomical objects ,while huge, are not of infinite extent.

With equation [27] providing the shape of the potential well one may attempt to set up and solve a Schroedinger equation to obtain a set of energy eigenstates for the gravitationally bound system. I did this in momentum space where the momentum is Unexpected text node: '2'. I assumed the mass of the system was proportional to the Planck mass and Unexpected text node: '1'.

The command I gave Mathematica was (In which $m = m_p$ as in the Planck mass).

Unexpected text node: 'Solve' (28)

The output was, verbatim

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Written in traditional form

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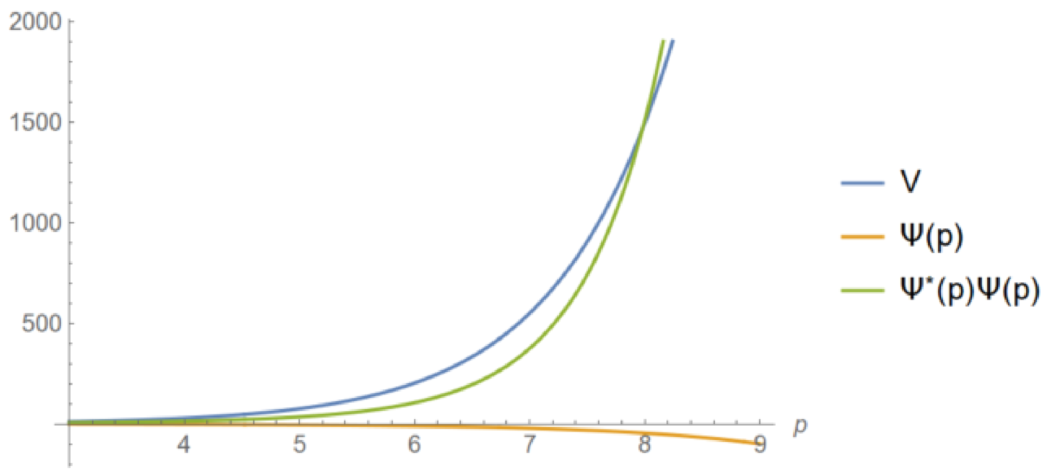

FIGURE 2: MOMENTUM SPACE POTENTIAL, SOLUTION, AND SQUARE OF THE SOLUTION. THIS IS A GRAPH OF THE MOMENTUM SPACE POTENTIAL DUE TO RELATIVIZED GRAVITY (V),EQUATION [27], AS WELL AS THE SOLUTION FOR THE MOMENTUM SPACE QUANTUM STATE OF THE SYSTEM $\psi(p)$, EQUATION [30]. IN THIS PLOT Unexpected text node: '1'. THERE IS A SMALL REGION WHERE THE SQUARED WAVE FUNCTION LIES OUTSIDE THE POTENTIAL WELL. THIS REPRESENTS A SMALL PROBABILITY OF ESCAPE FROM THE BLACK HOLE. FROM THIS I WILL FIND THE INTENSITY OF RADIATION FROM A BLACK HOLE AS IN EQUATION [33]. THIS AGREES WITH THE FINDINGS OF BLACK HOLE THERMODYNAMICS AND SHOWS THAT IT IS POSSIBLE FOR INFORMATION TO LEAK OUT OF A BLACK HOLE. AT SHORT DISTANCES BLACK HOLES DO HAVE HAIR.

Interpretation of these last few equations is that the state function $\psi(p)$, as seen in figure [2], goes to negative infinity at zero momentum. Where will the momentum be zero in a black hole? The location of zero momentum is the event horizon. Black holes may be quantum mechanically all surface and no inside.

The region where the squared wave function is outside of the potential well represents a range of momenta and energies over which Hawking Radiation may be emitted. This sets a theoretical maximum on the momentum of the particles being emitted of about eight times the Planck Momentum

Unexpected text node: '8' (31) will be approximately 1500 times the Planck energy

Unexpected text node: '1500' (32)

If these numbers are per Planck time then the following equation for the intensity of the radiation emitted will be true.

Unexpected text node: '2.934' (33)

I have arrived at a relationship between the mass of a black hole an the intensity of the Hawking radiation. Black hole thermodynamics a topic worthy of separate discussion follows from here.

5 DISCUSSION

What does it mean for a space to be defined over another space? The first thing to realize is that a Hilbert space and the normed division algebra it is defined over are separate things. For example the Hilbert space of spin 1/2 particles is used to learn the basics in graduate school usually represented with 2x2 complex matrices, it could be represented with quaternions and 4x4 matrices with elements that are all real [(8)].

The underlying normed division algebra need not itself be a Hilbert space. The underlying normed division algebra need not consist of mathematical objects representable by 1x1 matrices. i.e. a quaternion which can be written as if it were a four vector, can be a scalar with respect to a Hilbert space. In a sense the Hilbert space is where all the actual physics takes place which is separated from the normed division algebra, which is also separate from the particular representation we choose for the operators and vectors in the Hilbert space, or the representation of the algebra.

However, we need those tools in order to translate abstract statements into concrete predictions. In the case of QFT we need to combine $\mathscr{H}$ with $\mathscr{M}$ to get the QFT we recognize. Then we need to combine $\mathscr{H}$ with $\mathscr{M}$ with a Riemannian manifold to get the General Relativized theory.

Physically equations [25] and [26] make perfect sense. Interactions involving the Higgs field give mass to the particles of the standard model. Gravity couples into the mass of a particle.The inclusion of the graviton-Higgs interaction leads to a finite number of counter terms at every loop order. They sum to a well known exact function. The gravitational effect will be proportional to the mass, so said both Newton and Einstein. It makes sense then that the field that gives mass could be an eigenvector of the gravitational field with a tensorial Eigenvalue. While the effect of this on the Lagrangian and the Feynman diagrams, as in figure [1]) is to moderate and control the divergences in the perturbation expansion in terms of Feynman diagrams. The result is the amplitude for the graviton graviton interaction [27].

The graviton graviton interaction figure [1] will dominate in regions that are bound primarily by gravity, not only black holes but the whole universe, especially near the big bang. Figure [2] shows the shape of the gravitational potential. It is a well with infinitely high walls, this theory comports with the simple observation that on may not escape th universe, or the interior of a black hole. One may even make a hypothesis about the nature of the big bang, perhaps it was a transition from one excited state to another within this potential well. That analysis leads to the prediction of equation [33].

This model removes the classical big bang/ blackhole singularity and replaces it with a semi-relativized quantum field theoretical reality. This is all only possible when one relativizes the standard model with the sensible assumption that the Higgs field which gives mass to all things also plays a special role in gravity.

All of that said I do not propose that what is exposed in this paper is a fundamental and final model. The portion dealing with the standard model is not fully relativized, it does not explain just how the standard model arises, it does not predict what other particles will arise between the Higgs scale and the Planck scale. Any fundamental theory will derive the standard model as one of its consequences.

There is a program which can provide the tools to do such a thing, M-theory. Only M-Theory once formulated in a background free, diffeomorphism covariant way , could provide a final relativized theory of everything.

Relativization has already provided a finite quantitative prediction for the quantum corrections to the local gravitational field, and gives results compatible with established black hole thermodynamics. Further research will certainly yield new insights.

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